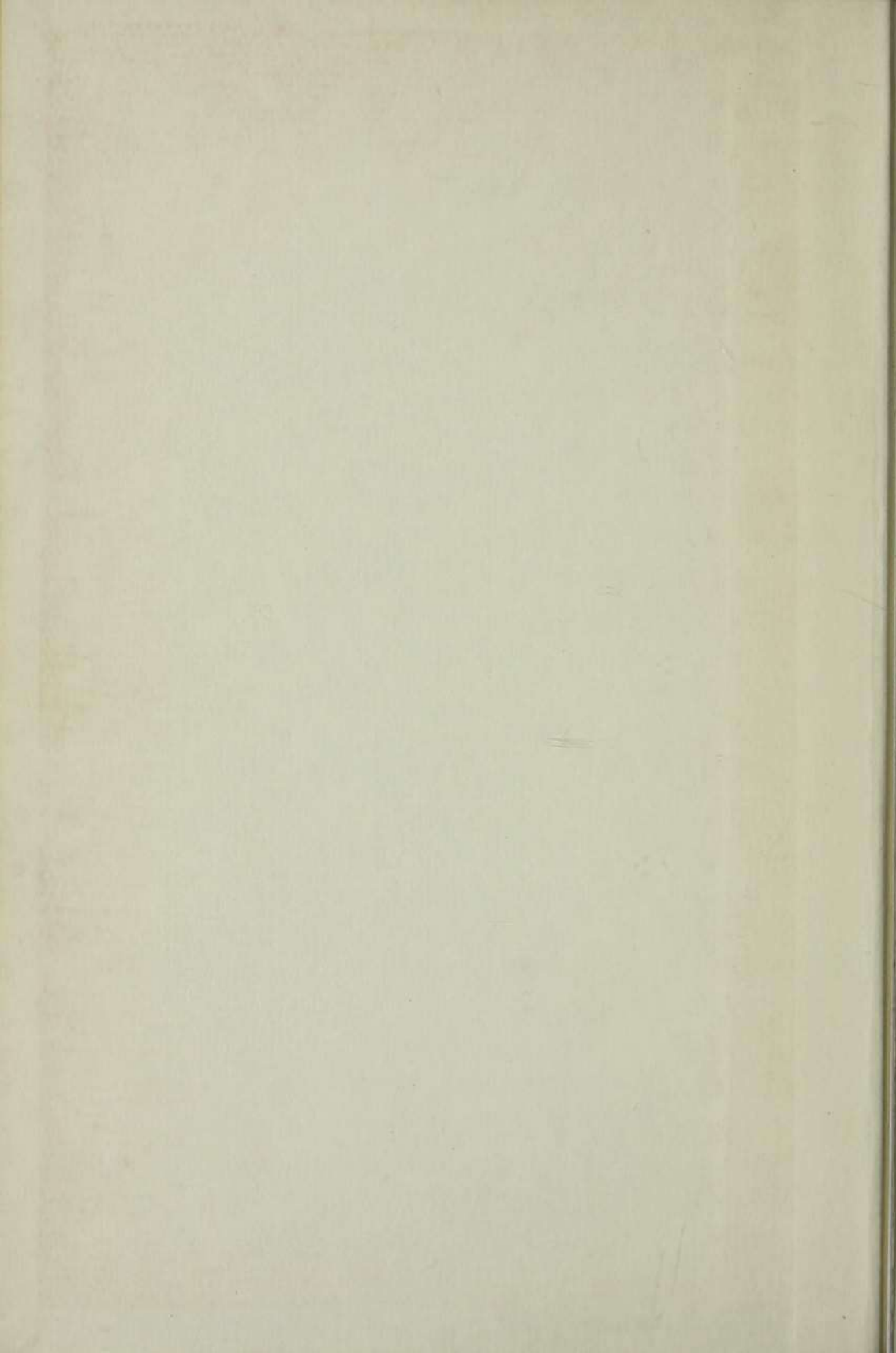
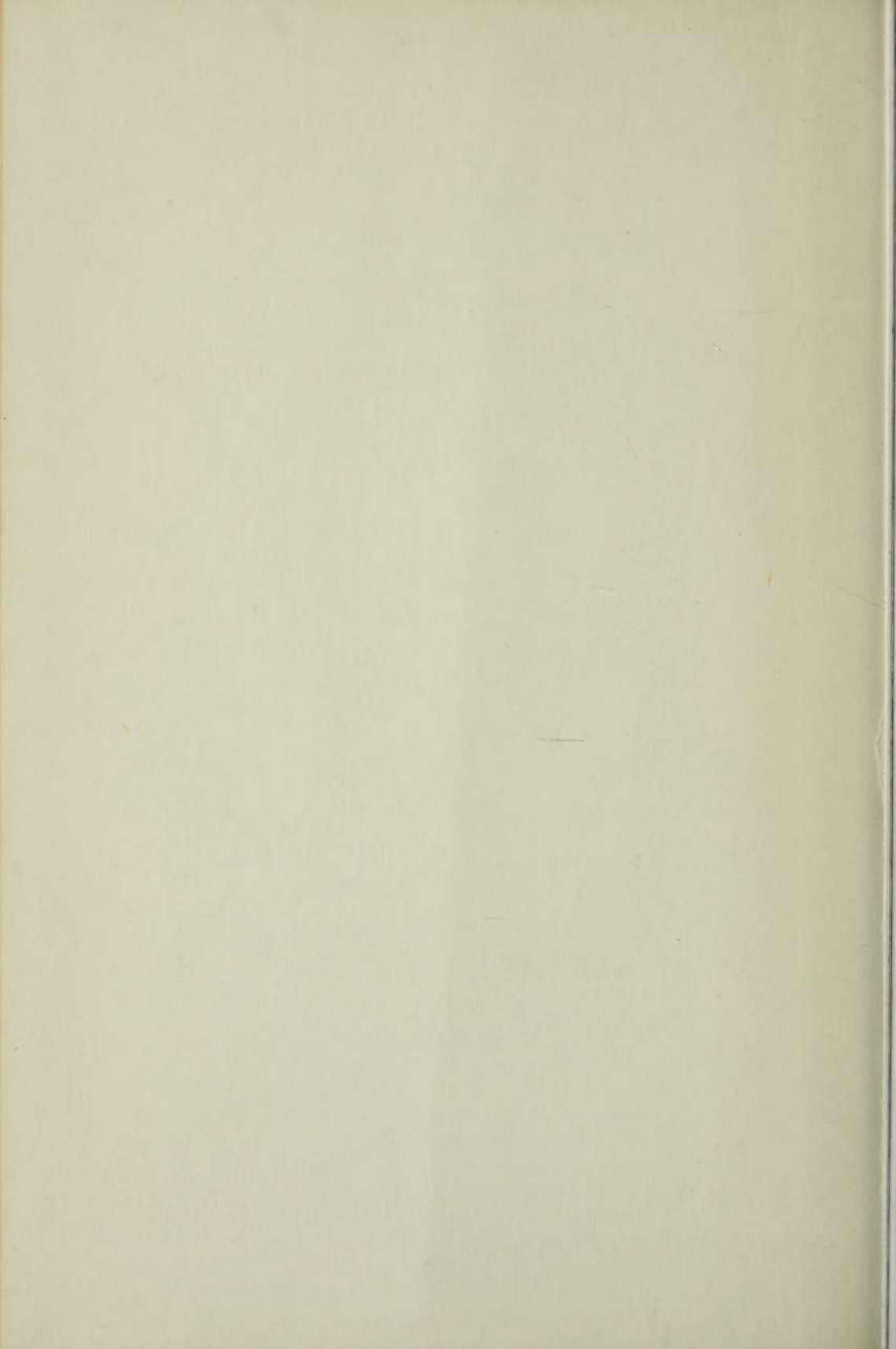




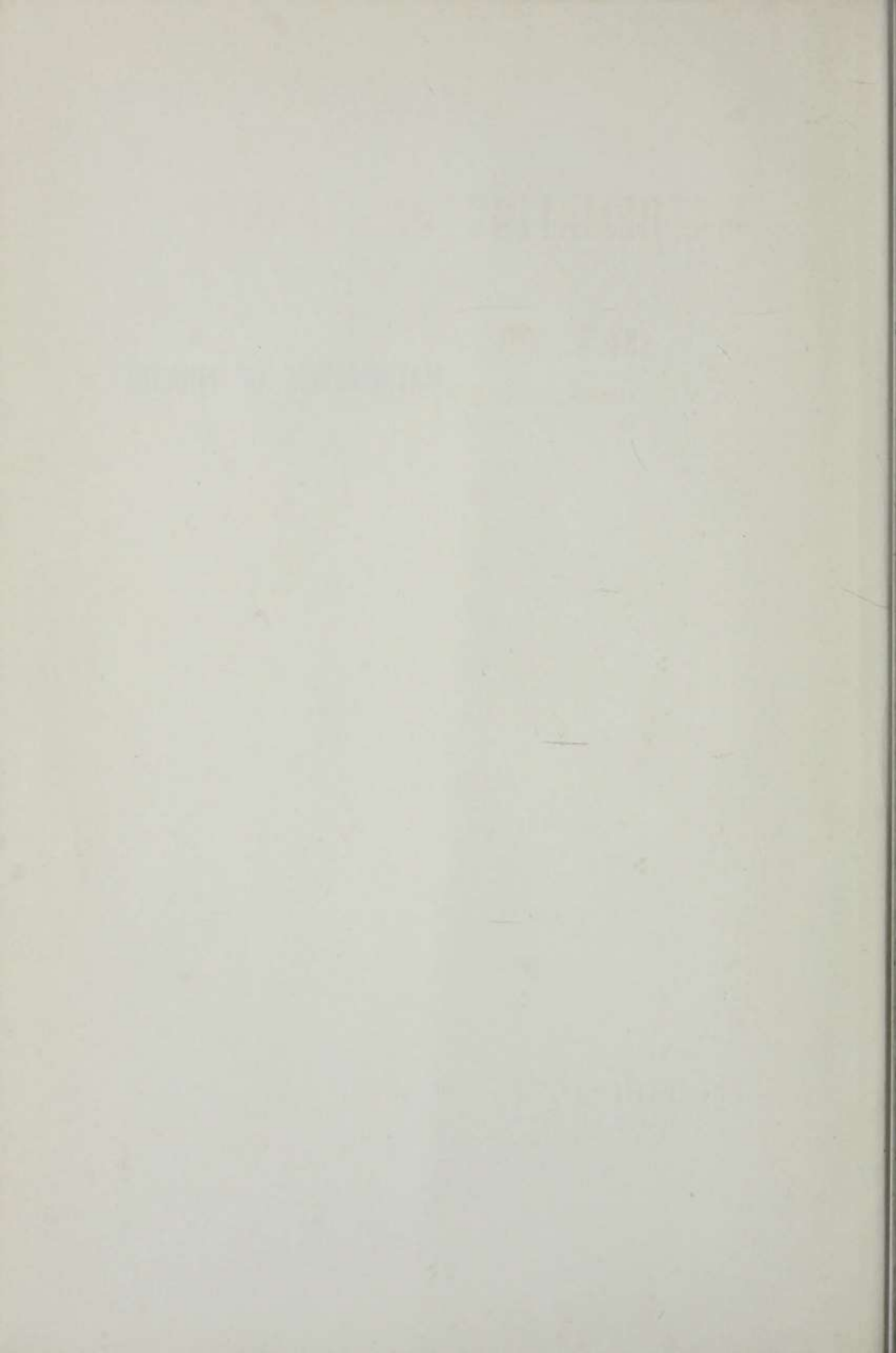
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MATHEMATICS OF FINANCE



MATHEMATICS OF FINANCE

BY T. HOYLE LEE

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UNIVERSITY OF SOUTH CAROLINA

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PREFACE

This book is written to provide a practical course in the mathematics of finance, not only for the student of commerce and business administration, but also for all students who desire to have a better understanding of the financial transactions necessary in dealing with their fellowmen. Each, regardless of his occupation, can more intelligently engage in the financial business of everyday life if he has a knowledge of the principles given here.

Rather than list a large number of formulas to be memorized, and promptly forgotten, emphasis is placed on an understanding of the basic principles underlying financial transactions. The use of line diagrams and the equation of value is stressed as the basis of all work. Only eight formulas are used in the treatment of the entire book exclusive of the two chapters on life annuities and life insurance. Numerous detailed illustrative problems are carefully worked in the text. These show how the eight formulas can be used to reach the solution of all the basic transactions involving simple interest, bank discount, compound interest, annuities, perpetuities, and depreciation.

A chapter on logarithms precedes the chapter on compound interest. Even if the student has had previous work in logarithms, this chapter will give a valuable review of the logarithmic techniques he will use in solving many of the problems in the following chapters.

The general case of annuities certain is treated in a much simpler manner than it is done in many texts. This treatment is given immediately after the simple case and is shown to follow from the simple case without the use of any additional formulas. This early discussion makes the general case available for use in the applications that follow throughout the book.

In the discussion of depreciation, every attempt has been made to use the terminology that is in keeping with modern accounting practice. There is a brief reference to a special application first sanctioned by the 1954 tax code.

In the two brief chapters on life annuities and life insurance, emphasis again is placed on the use of the equations of value and an understanding of the principles involved rather than on numerous formulas. No knowledge of probability is assumed. The latest stand-

ard actuarial notation and symbols are used along with the mortality tables currently in use by insurance companies.

A very important part of any text in mathematics of finance is the tables. Every effort has been made to provide the best tables available. Six-place mantissas of logarithms are given. Seven-place mantissas are included for numbers from 10,000 to 11,009. Not only are tables given for compound interest and the standard annuity symbols (including the reciprocal of $s_{\overline{n}|i}$ for certain fractional values of n), but printed with these tables are seven-place logarithms of each of the numbers in the compound interest and annuity tables. These are so arranged that the reader can see the numerical value on the left-hand page and its logarithm on the page facing it. The Commissioners' 1941 Standard Ordinary (CSO) Mortality Table is given with commutation columns based on $2\frac{1}{2}$ per cent. This mortality table and interest rate are currently used by most insurance companies.

Acknowledgments of the sources of these tables and some references to more extensive tables are given in the introductory remarks preceding the tables.

The author wishes to express appreciation for the helpful suggestions of his associates who have taught the preliminary edition of this text at the University of South Carolina for the past two years.

T. HOYLE LEE

COLUMBIA, SOUTH CAROLINA
February, 1957

TABLE OF CONTENTS

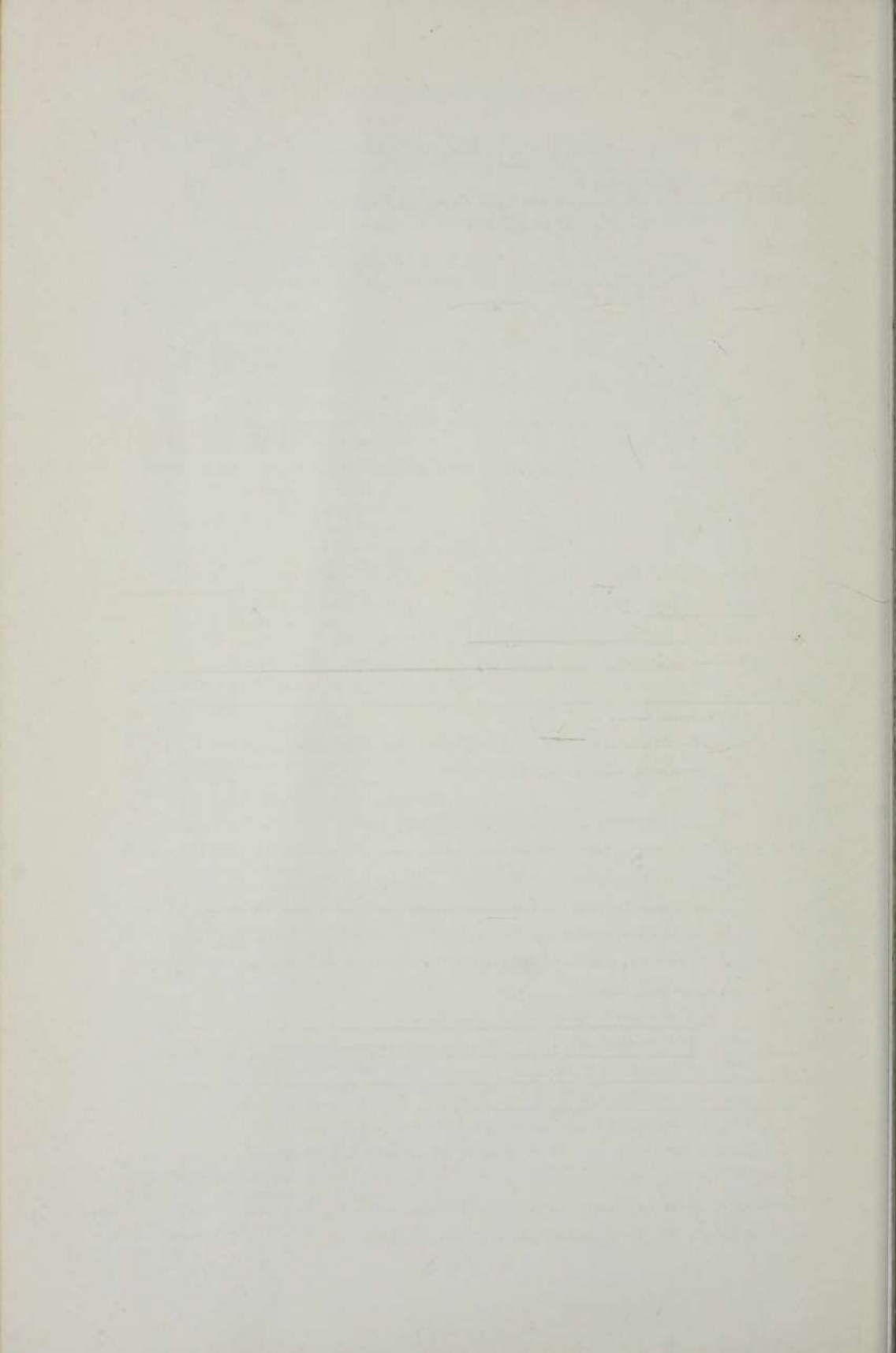
CHAPTER	PAGE
1. SIMPLE INTEREST AND BANK DISCOUNT	1
1.1 Introduction	1
1.2 Simple Interest	2
1.3 Exact and Ordinary Interest	4
1.4 Exact and Approximate Time	5
1.5 Bank Discount	7
1.6 Promissory Notes	9
1.7 Relations between Simple Interest Rates and Bank Discount Rates	12
2. LOGARITHMS	17
2.1 Introduction	17
2.2 Definition	17
2.3 Laws of Logarithms	18
2.4 Numerical Value of the Logarithm of a Number	19
2.5 Finding the Characteristic	20
2.6 Finding the Mantissa	22
2.7 Finding a Number if Its Logarithm Is Given	23
2.8 Interpolation	24
2.9 Computation by Means of Logarithms	26
3. COMPOUND INTEREST	30
3.1 Compound Interest	30
3.2 Compound Interest Formula	31
3.3 Terminology	33
3.4 Use of Logarithms in the Compound Interest Formula	34
3.5 Equivalent Rates of Interest	37
3.6 Present Value	40
3.7 Finding the Maturity Value for Terms Involving a Fraction of a Period	43
3.8 Finding the Present Value for Terms Involving a Fraction of a Period	44
3.9 Solving for i and n	46
3.10 Equations of Value	49
4. ANNUITIES	56
4.1 Definitions	56
4.2 Amount of Ordinary Annuities Certain	57
4.3 Present Value of Ordinary Annuities	61
4.4 Finding the Periodic Payment R	66
4.5 Finding an Unknown Interest Rate	70
4.6 Finding the Size of a Final Payment	72
4.7 Finding the Number of Payments	72
4.8 Special Terminology	75
4.9 General Case of Annuities	78
5. AMORTIZATION AND SINKING FUNDS	87
5.1 Amortization	87
5.2 Amortization Schedule	89

CHAPTER	PAGE
5.3 Finding the Outstanding Principal without a Schedule	91
5.4 Finding the Amount of Interest or Principal Included in Any Payment	93
5.5 Amortization of a Bonded Debt	95
5.6 Sinking Fund Method for Retiring a Debt	97
5.7 Net Indebtedness	99
5.8 Growth of the Sinking Fund	99
5.9 Comparison of the Amortization and Sinking Fund Methods	102
6. BONDS	106
6.1 Introduction	106
6.2 Price of a Bond Bought on an Interest Date	108
6.3 The Premium-Discount Method	112
6.4 Amortization of the Difference between the Purchase Price and the Redemption Value	113
6.5 Purchase Price and Book Value between Bond Interest Dates	117
6.6 Finding the Yield Rate	121
7. DEPRECIATION, PERPETUITIES, AND CAPITALIZED COST	127
7.1 Introduction	127
7.2 The Straight Line Method	128
7.3 The Sinking Fund Method	129
7.4 The Constant Percentage Method	131
7.5 The Sum of Digits Method	135
7.6 Perpetuities	137
7.7 Perpetuity Formula	138
7.8 Capitalized Cost	140
7.9 Comparison of the Cost of Two Assets with Different Life Spans	141
8. LIFE ANNUITIES	144
8.1 Introduction	144
8.2 Mortality Tables	144
8.3 Net Premiums and Loading Factor	146
8.4 Mortality Table Symbols	146
8.5 Whole Life Annuities	147
8.6 Other Types of Life Annuities	151
8.7 Life Annuities Purchased by Making a Set of Equal Payments	156
9. LIFE INSURANCE	160
9.1 Life Insurance	160
9.2 Single Premium, Whole Life Insurance	160
9.3 Annual Premium, Whole Life Insurance	162
9.4 Limited Payment Life Insurance	164
9.5 Term Insurance	166
9.6 Endowment Insurance	167
9.7 Brief Summary	168
9.8 Reserves	170
TABLES	177
Introduction and Acknowledgments	177
1. Logarithms. Six-Place Mantissas	180
2. Logarithms. Seven-Place Mantissas	200
3. $(1+i)^n$ and $\log(1+i)^n$	204
4. $(1+i)^{-n}$	230
5. $(1+i)^{\frac{1}{p}}$ and $\log(1+i)^{\frac{1}{p}}$	244
6. $s_{\overline{n} i}$ and $\log s_{\overline{n} i}$	248
7. $a_{\overline{n} i}$ and $\log a_{\overline{n} i}$	274
8. $\frac{1}{a_{\overline{n} i}}$	300

TABLE OF CONTENTS

ix

9. $\frac{1}{s_1^{\frac{1}{p}} _i}$ and $\log \frac{1}{s_1^{\frac{1}{p}} _i}$	314
10. CSO Mortality Table	318
11. Commutation Columns for the CSO Table, $2\frac{1}{2}$ Per Cent	322
12. The Number of Each Day of the Year	326
INDEX.	327
ANSWERS.	331



SIMPLE INTEREST AND

BANK DISCOUNT

1.1 Introduction. Interest is a fee paid for the use of another person's money just as rent is a fee paid for the use of property. The original amount of the debt (that is, the amount of the money being used) is called the principal of the debt, and it is usually denoted by the letter P . The amount of the interest is denoted by I .

The original debt may have been incurred by a loan of money, or it could have been created by the purchase of goods or property. For example, suppose the local bank loans Mr. Jones \$100 in cash and Mr. Jones later pays the bank \$110 as a complete settlement of this debt. There has been an interest charge here of \$10 for the use of the bank's money. On the other hand, Mr. Smith may purchase a bill of goods amounting to \$1,000. Instead of paying cash, he signs a note promising to pay \$1,050 at some specific later date. In effect, Mr. Smith has borrowed \$1,000 from the seller of the goods and has agreed to make an interest payment of \$50 for the use of this money over the allotted time.

Usually, the exact amount of interest is not stated in the contract. Instead, it is the custom to tell what percentage of the principal will equal the interest to be paid for each unit of time. This percentage is commonly called the rate of interest. It would probably be better to call it the *ratio of interest*, since the decimal equivalent of the percentage is the ratio of the amount of interest to the principal. The term "rate" has become firmly established in the business world and, therefore, will be used here. An attempt will be made to use the terminology of the businessman even though there are good reasons,

in many cases, why other terms would more accurately describe the transactions.

1.2 Simple Interest. In simple interest, the amount of interest is always computed on the original principal no matter how long a time is involved in the contract. Thus, it is computed (1) on the original principal, and (2) only once during the life of the transaction. These two characteristics will help to distinguish simple interest from the other two types of interest studied in this text. For example, in compound interest, the interest is computed at the end of each period and added to the preceding principal in order to get a new principal for the next period. Thus the interest itself earns other interest during the next period. This method will be studied in Chapter III.

Simple interest is normally used only for short terms, usually a year or less. It will be seen later that compound interest is better adapted to longer terms.

P has already been defined as the original principal or present value. The rate of simple interest per year will be denoted by r , while t will represent the time in years. Thus t is usually a fraction. The total amount of interest I is gotten by multiplying the original principal P by the rate r and the time t , that is: $I = Prt$. If this interest is added to the original principal, the final amount or maturity value of this debt is given. Let S represent the final amount or maturity value. Then

$$\underline{S = P + I = P + Prt = P(1 + rt) .}$$

This gives the first formula, the simple interest formula:

$$(1) \quad \underline{S = P(1 + rt) .}$$

S = final amount or maturity value.

P = original principal or present value.

r = rate of simple interest per year.

t = time in years.

NOTE: Some people prefer to use the simple interest formula in two parts: $S = P + I$, where $I = Prt$. The combination of these two will always give the same results as formula (1).

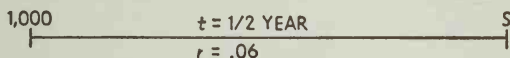
This can be represented diagrammatically on what is called a *time line*. That is, a straight line will represent time. On it, any item that occurs later in time than another item will be placed to the right of

the earlier item. For simple interest, this time line representation will appear as follows:



Illustration 1. Mr. Jones borrows \$1,000 to be repaid at the end of 6 months, along with simple interest at the rate of 6 per cent per year. Find the amount Mr. Jones must repay.

Solution. The use of the time line will be an invaluable help in the solution of problems in financial mathematics, especially later in the text. Thus it is recommended that the student form the habit of making such a diagram for each problem. In this example, $P = 1,000$, $t = \frac{1}{2}$ year, $r = .06$, and the time diagram is:

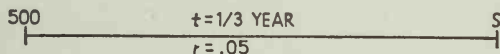


By formula (1),

$$\begin{aligned} S &= P(1 + rt), \\ &= 1,000[1 + (.06)(\frac{1}{2})], \\ &= 1,000(1.03), \\ &= \$1,030. \end{aligned}$$

Illustration 2. Find the maturity value of \$500 for 4 months at 5 per cent simple interest.

Solution. Since 4 months $= \frac{4}{12}$ year $= \frac{1}{3}$ year, $t = \frac{1}{3}$, $P = 500$, and $r = .05$.



$$S = 500[1 + (.05)(\frac{1}{3})]$$

Since $(.05)(\frac{1}{3})$ has an unending decimal representation, $.0166666 \dots$, the question arises, "How many decimal places are necessary for an answer correct to dollars and cents?" This question may be avoided here if the portion in the brackets is added first (by getting a common denominator) and postponing the division as long as possible. This can be done as follows:

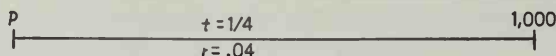
$$\begin{aligned} S &= 500\left[\frac{3}{3} + \frac{.05}{3}\right] = 500\left[\frac{3.05}{3}\right] = \frac{1,525}{3}, \\ &= \$508.33. \end{aligned}$$

Finding S as in the above illustration is called accumulating the principal P at the simple interest rate r for t years. Since formula (1) contains four letters, it is necessary to know only three (any three) in order to find the fourth. If the maturity value S is given and the

present value P is to be found, it is said that S is being discounted at the simple interest rate r for t years. Thus "discount" means to bring back in time, while "accumulate" means carry forward in time.

Illustration 3. Discount \$1,000 at 4 per cent simple interest for 3 months.

Solution. Here, P is to be found, $S = 1,000$, $r = .04$, and $t = \frac{1}{4}$.



By formula (1),

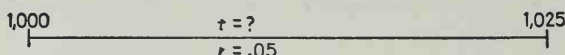
$$1,000 = P[1 + (.04)(\frac{1}{4})],$$

$$1,000 = 1.01P,$$

$$P = \frac{1,000}{1.01} = 990.099 = \$990.10 \text{ to the nearest cent.}$$

Illustration 4. How long will it take \$1,000 to accumulate to \$1,025 at 5 per cent simple interest?

Solution. $P = 1,000$, $S = 1,025$, $r = .05$, and $t = \text{time in years}$.



Using formula (1) again,

$$1,025 = 1,000[1 + .05t],$$

$$1,025 = 1,000 + 1,000(.05)t,$$

$$25 = 50t,$$

$$t = \frac{1}{2} \text{ year.}$$

By a similar process r may be calculated when P , S , and t are known. Unless something else is said to the contrary, all interest rates will be calculated accurate to four decimal places. This usually means that the divisions must be carried to five decimal places and then rounded off to the fourth place. In this text, if the part deleted in rounding off the decimal is equal to or greater than a half (i.e., the next digit is 5 or larger), then the last digit kept will be increased by one. If the deleted part is less than a half, it shall simply be marked off. For example, .05235 and .05237 will both round off to .0524 correct to four decimal places, while .05234 will become .0523.

1.3 Exact and Ordinary Interest. In the previous examples, the time has been given in months, and it is easy to get t by dividing the

number of months by twelve. Quite frequently the time is given in days. In this case there are two common methods for getting t . If we use as a denominator the exact number of days in a year, 365 (except for leap years), we have what is called *exact interest*. Thus for 45 days,

$$t = \frac{45}{365} = \frac{9}{73}.$$

The second method uses as the denominator 360 (12 months of 30 days each). When 360 is used as the denominator, we have ordinary interest. This is the more common practice because in this case the fraction t will frequently reduce to simple fractions. For 45 days, $t = \frac{45}{360} = \frac{1}{8}$. Incidentally, t is larger when 360 is used as a denominator than when 365 is the denominator. Thus ordinary interest gives a larger interest fee than exact interest.

Unless otherwise stated, ordinary interest will be used in all simple interest problems in this text.

1.4 Exact and Approximate Time. There are also two ways of getting the numerator of the fraction t when the time between two specific dates is to be found. Exact time is the actual number of days between the two dates, counting one end day but not both. It is considered preferable in practice to count the last day and not the first. Note that in leap years, February has 29 days, otherwise 28 days. (Leap years are those years for which the number of the year is divisible by 4 except that the year representing the end of a century must be divisible by 400 in order to be a leap year. Thus 1944, 1972, and 2000 are leap years, while 1900, 1951, and 1979 are not.) For example, the exact time from February 10, 1956, to June 10, 1956, is 121 days since 1956 is a leap year. For the same dates in 1957, the exact time would be 120 days.

The second method of counting time is called *approximate time* (sometimes called *ordinary time*). In this method, from any date of one month to the same date of the next month is counted as 30 days regardless of the actual number of days involved. Thus, from February 10, 1956, to June 10, 1956, would be 120 days by approximate time.

Now either of these two methods of computing time may be used with either method of finding the denominator discussed in Section 1.3. This gives four possible methods of getting the fraction t .

1. Ordinary interest and exact time.
2. Ordinary interest and approximate time.

3. Exact interest and exact time.
4. Exact interest and approximate time.

Except for some periods which include (or begin on) the last day of February and, in addition, include the first day of March, ordinary interest and exact time will give the largest (or at least as large) t of any of the four methods. This is the method most commonly used in practice. Unless otherwise stated, ordinary interest and exact time will be the method employed in all problems in this book.

Exercise 1

- ✓ 1. Accumulate \$500 for 6 months at 6 per cent simple interest.
- ✓ 2. Accumulate \$750 for 3 months at 4 per cent simple interest.
- ✓ 3. Accumulate \$600 for 4 months at 7 per cent simple interest.
- ✓ 4. Accumulate \$800 for 8 months at 5 per cent simple interest.
- ✓ 5. Discount \$1,000 for 5 months at 6 per cent simple interest.
- ✓ 6. Discount \$750 for 7 months at 6 per cent simple interest.
- ✓ 7. Discount \$200 for 8 months at 4 per cent simple interest.
8. Discount \$250 for 4 months at 7 per cent simple interest.
9. At what rate of simple interest will \$500 amount to \$512 in 8 months?
10. Find the rate of simple interest used to accumulate \$600 for 7 months if the maturity value is \$610.17.
11. \$5,000 is invested at 5 per cent on June 4, 1957. Find the maturity value on December 21, 1957, if you use (a) ordinary interest and exact time, (b) ordinary interest and approximate time, (c) exact interest and exact time, and (d) exact interest and approximate time.
12. Accumulate \$3,000 from March 8, 1955, to November 23, 1955, at 3 per cent simple interest, if you use (a) ordinary interest and exact time, (b) ordinary interest and approximate time, (c) exact interest and exact time, and (d) exact interest and approximate time.
13. Mr. E. W. Smith borrows \$575 on January 1, 1956, at 5 per cent simple interest. How much should he repay on April 30, 1956?
14. Mr. O. T. Jones purchases a stove worth \$300 on February 11, 1960. He agrees to pay for this purchase on June 10, 1960. If simple interest at 6 per cent is added for this period of time, find the size of his payment on June 10, 1960.
15. How long will it take for \$2,900 to amount to \$2,948.32 at 5 per cent simple interest?
16. How long will it take \$653.25 to amount to \$675.46 at 3 per cent simple interest?
17. a) \$1,000 is borrowed on March 3, 1963. If 6 per cent ordinary simple interest is used, find how much is due on July 1, 1963.
b) If 6 per cent exact simple interest were used in part (a), find the amount due on July 1, 1963.

18. a) Find the present value of \$100 due 6 months from now at 4 per cent ordinary simple interest.

b) Discount \$2,000 for 8 months at 5 per cent ordinary simple interest.

19. Mr. A. P. Brown has a note for \$1,000 due on June 15, 1964. He sells this note to his bank on March 7, 1964, for \$975. Find the rate of ordinary simple interest used.

20. A bank pays \$96.00 for a note of \$100 due 6 months from now. Find the rate of simple interest being charged.

21. How long will it take \$3,476.00 to amount to \$4,000 at 4 per cent ordinary simple interest?

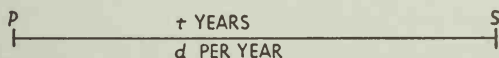
22. Solve Problem 21 if 4 per cent exact simple interest is used.

23. On January 23, 1960, Mr. Will Osborne borrows a certain sum of money from his bank. He promises to pay back \$1,000 on March 23, 1960. If the bank charges a 6 per cent simple interest rate, how much does Mr. Osborne receive from the bank on January 23, 1960?

1.5 Bank Discount. Another type of interest often confused with simple interest is called *bank discount* or *interest in advance*.¹ In fact, the terminology often used helps to add to the confusion of these two methods. So, note carefully the distinctions. Bank discount is (1) computed on the final amount or maturity value (simple interest is computed on the original principal), and (2) it is computed once during the life of the transaction. Once the bank discount is computed, it is subtracted from the maturity value to give the present value or principal. Let P , S , and t be defined as in simple interest, but let d denote the bank discount rate. Then $I = Sdt$ and $P = S - I$. Thus $P = S - Sdt = S(1 - dt)$. This gives formula

$$(2) \quad P = S(1 - dt),$$

where



P = present value.

S = final amount or maturity value.

t = time in years.

d = bank discount per year.

As in the case of simple interest, given any three of the values P , S , t , and d , the fourth may be found by using formula (2), the bank discount formula. The same methods for finding the numerator and

¹ Some writers also call this simple discount, while others use simple discount to mean "discount at simple interest." It is recommended that the use of this term be avoided.

denominator for t are to be used here as were used in simple interest.

The difference between S and P is called the interest if it is thought of as being added to P in order to get S . It is called the discount if it is thought of as being subtracted from S to get P . For the interest and the discount to be the same numerically, as here, the simple interest rate must be somewhat higher than the bank discount rate.

Illustration. Find the present value of \$500 due at the end of 120 days if interest is at a bank discount rate of 6 per cent.

Solution. $S = 500$, $t = \frac{120}{360} = \frac{1}{3}$, and $d = .06$.



By formula (2),

$$\begin{aligned} P &= 500[1 - (.06)(\frac{1}{3})], \\ &= 500[1 - .02], \\ &= 500(.98), \\ &= \$490. \end{aligned}$$

Exercise 2

- Find the present value of \$785 due at the end of 5 months if interest is at 6 per cent bank discount.
- Discount \$800 for 4 months at 7 per cent bank discount.
- Discount \$1,000 for 8 months at 5 per cent bank discount.
- Find the present value of \$575 due at the end of 180 days if interest is at 5 per cent bank discount.
- Mr. Smith has promised to pay \$1,625 on June 20. On April 21, the local bank buys this contract (or note). What does the bank pay if it charges 6 per cent bank discount?
- Mr. Oliver has promised to pay \$6,500 on August 10. If he desires to pay this debt on May 12, how much should he pay? It is agreed that interest will be computed at 8 per cent bank discount.
- A bank which charges 5 per cent bank discount loans a man \$580. If the man agrees to repay \$600, find the time to elapse before the \$600 is to be repaid.
- How long will it take \$2,300 to accumulate to \$2,500 at 6 per cent bank discount?
- Find the bank discount rate if the present value of \$900 due in 240 days is \$865.
- Find the bank discount rate if a loan association agrees to give Mr.

Smith \$3,041.34 today and Mr. Smith is to repay \$3,200 at the end of 14 months.

11. The Regal Bank charges 6 per cent "interest in advance." How much will Mr. E. J. Phipps receive on a 90-day loan if he is to pay the bank \$1,500 on the maturity date?

12. Mr. Jones wishes to receive \$500 on a 90-day loan. If the bank charges 5 per cent "interest in advance," for how much should he ask in order to get \$500?

13. Accumulate \$5,000 for 4 months at 6 per cent bank discount.

14. If the present value of a loan is \$7,500, find the maturity value due at the end of 3 months. Interest is computed at 5 per cent bank discount.

15. Find the bank discount rate if \$700 is the present value of \$750 due at the end of 4 months.

16. Find the bank discount rate if \$595 is the present value of \$625 due at the end of 18 months.

17. Find the amount due at the end of 6 months if the present value is \$1,200 and the interest is at (a) 5 per cent simple interest, and (b) 5 per cent bank discount.

18. Find the present value of \$1,000 due at the end of 20 years if the interest is at 5 per cent bank discount. Can you explain your result? Work the same problem for 25 years. Then work each of these problems using 5 per cent simple interest. Using simple interest, could the present value ever be zero? Negative? Why?

19. \$3,000 is borrowed on March 3. If 6 per cent bank discount is used, find the amount due on July 1 of the same year.

20. Find the present value of \$500 due 6 months from now at 4 per cent bank discount.

21. Mr. William Jones has a note for \$1,000 due on June 15, 1965. He sells this note to his bank on March 7, 1965, for \$975. Find the rate of bank discount used.

22. A bank pays \$96.00 for a note of \$100 due 6 months from now. Find the bank discount rate used.

23. How long will it take \$3,476.00 to amount to \$4,000.00 at 4 per cent bank discount rate?

24. Solve Problem 23 if a 6 per cent bank discount rate is used.

25. On January 23, 1964, you borrow a sum of money from a bank and promise to pay back \$1,000 on March 23, 1964. If the bank charges a 6 per cent bank discount rate, how much do you receive from the bank on January 23, 1964?

1.6 Promissory Notes. Promissory notes are written promises to pay a fixed sum of money at some fixed date. The note *matures* on the date specified as the payment date. The *face* of the note is the sum of money specifically mentioned in the note. The *maturity value* is the

amount of money paid on the date the note matures. Usually the maturity value includes interest at a rate specified in the note and has to be calculated from the information in the note. Such a note is called an *interest-bearing* note. At other times, the maturity value is given as a definite amount in the note, and no mention is made of an interest rate. Such a note is called a *noninterest-bearing* note. This does not mean that no interest is being paid on the debt. It simply means that the interest rate is not mentioned. In any note, the wording will make clear which is the case. The signer of a note is called the *maker*.

In any problem concerning a note, no matter what rate or kind of interest is used, the maturity value of this note should be found first.

Illustration 1. Consider the following note:

<u>Columbia, S. C.</u>
<u>August 5</u> , 19 <u>55</u>
<u>One hundred twenty</u> days after date, I promise to
pay to the order of <u>Mr. S. I. Jones</u> ,
<u>\$1,000.00</u> , together with simple interest at <u>6</u> per cent
per year.
(Signed) <u>C. B. Smith</u>

Find the maturity value and maturity date of this note.

Solution. The face of this note is \$1,000 (here, the present value of the debt), and the maturity value has to be calculated at 6 per cent simple interest for $t = \frac{1}{5}$ year. Thus

$$S = 1,000[1 + (.06)(\frac{1}{5})] = \$1,020.$$

This note matures 120 days after August 5, 1955, or on December 3, 1955.

This is an example of an *interest-bearing* note since the interest rate is given in the note. This note could have been worded as in the next illustration. In that case it would be what is technically called a *noninterest-bearing* note.

Illustration 2.

	<u>Columbia, S.C.</u>
	<u>August 5</u> , 19 <u>55</u>
<u>One hundred twenty</u>	days after date, I promise to
pay to the order of	<u>Mr. S. L. Jones</u> ,
<u>\$1,020.00</u> .	
(Signed)	<u>O. B. Smith</u>

In this note, the face is \$1,020, here the maturity value. No indication is given as to what the original amount of the loan was.

Generally, notes are negotiable instruments, that is, they can be bought and sold at any time during the life of the note. When they are bought, the purchaser is buying a promise to pay the maturity value due on the maturity date, and he can purchase this to yield him any interest rate agreed upon by him and the seller. It does not have to be the rate mentioned in the note (if any). Furthermore, he is buying this note for the time it still has to run before its due date. Thus, there are three important things to remember in finding the purchase price of such a note:

1. Find the maturity value of the original note.
2. Find the time from the purchase date to the maturity date.
3. Calculate the purchase price, using the maturity value, the time from the purchase date to the maturity date, and the yield rate of interest for the purchaser.

Finding the purchase price of a note is called *discounting the note*. It may be discounted at a simple interest rate or a bank discount rate.² The most common practice is to use a bank discount rate.

Illustration 3. Suppose the note of Illustration 1 was bought by the Gervais Street Bank on September 4, 1955. Find the purchase price if the bank receives (a) 6 per cent bank discount on the investment, and (b) 6 per cent simple interest on its investment.

² Later, when compound interest is studied, along with long-term notes, the same procedure will be used there in calculating the purchase price of such a note.

Solution. The maturity value of this note must be calculated first. This has already been done in Illustration 1, and $S = 1,020$ on December 3, 1955. Since this note is purchased on September 4, 1955, the time involved will be from September 4 to December 3, or 90 days. Thus $t = \frac{1}{4}$ year.



In part (a), $d = .06$. Using formula (2),

$$\begin{aligned} P &= 1,020[1 - (.06)(\frac{1}{4})], \\ &= 1,020[1 - .015], \\ &= 1,020(.985), \\ &= \$1,004.70. \end{aligned}$$

Since P was calculated by the bank discount method, the purchase price \$1,004.70 is quite frequently called the *proceeds* of the sale.

In part (b), S and t are the same as in part (a). But here, $r = .06$. Therefore, the simple interest formula is to be used.

$$\begin{aligned} 1,020 &= P[1 + (.06)(\frac{1}{4})] \\ 1,020 &= (1.015)P \\ P &= \frac{1,020}{1.015} = \$1,004.93 \end{aligned}$$

NOTE: The purchase price at 6 per cent bank discount rate is 23 cents less than the price at 6 per cent simple interest. Thus, it is to the banker's advantage to purchase the note at a 6 per cent bank discount rate.

1.7 Relations between Simple Interest Rates and Bank Discount Rates. The solutions to parts (a) and (b) of the preceding illustration show that there is a difference between simple interest and bank discount. Just how can the two rates be related to each other?

Two rates of interest are said to be equivalent provided they give the same results, that is, if the same amount of money is invested under each rate for the same length of time, the two final amounts should be the same. This principle will enable us to find a simple interest rate equivalent to a bank discount rate, or conversely. It can easily be shown that the amount invested under each rate will have no effect on the equivalence of the rates. A few examples will show that in comparing simple interest and bank discount rates, different times will give different equivalent rates. For example, two rates that are equivalent for 6 months will not be equivalent for 4 months, and

so on. (Later, it will be seen that the time does not make a difference when getting equivalent compound interest rates.)

In finding a simple interest (or bank discount) rate equivalent to a given bank discount (or simple interest) rate for a given time, proceed as follows: Arbitrarily choose some amount of money for either P or S (either one will do), and calculate the other for the given rate and the given time. Then, using this set of values of P , S , and t , calculate the other rate.

Illustration. Find the simple interest rate equivalent to 4 per cent bank discount if the time is 90 days.

Solution. Let $S = \$1,000$ (you could have let $P = \$1,000$), $t = \frac{1}{4}$.

$$\begin{array}{ccc} P & & 1,000 \\ | & \xrightarrow[r = 1/4]{d = .04} & | \end{array}$$

$$P = 1,000[1 - (.04)(\frac{1}{4})] = 1,000(.99) = 990$$

Next, use $P = 990$, $t = \frac{1}{4}$, $S = 1,000$, and calculate r , the simple interest rate.

$$\begin{array}{ccc} 990 & & 1,000 \\ | & \xrightarrow[r = ?]{t = 1/4} & | \end{array}$$

$$990[1 + (\frac{1}{4})(r)] = 1,000$$

$$990 + \frac{990r}{4} = 1,000$$

$$\frac{990r}{4} = 10$$

$$r = \frac{40}{990}$$

$$r = .0404+$$

Thus, for 90 days, a simple interest rate of .0404+ is equivalent to a bank discount rate of .04.

Exercise 3

1. Find the simple interest rate equivalent to 4 per cent bank discount rate (a) if the time is 1 month, (b) if the time is 6 months, and (c) if the time is 1 year.

Compare these answers with the answer to the preceding illustration. This problem shows clearly that the time element does make a difference in the answer.

2. What bank discount rate is equivalent to 6 per cent simple interest if the time is 8 months?

3. In discounting for 180 days, what bank discount rate is equivalent to 5 per cent simple interest?

4. Calculate the simple interest rate equivalent to 3 per cent bank discount if the loan is for (a) 2 months, (b) 6 months, and (c) 1 year.

5. Find out which is to the advantage of the borrower, to borrow money at 4 per cent bank discount or at 4.1 per cent simple interest if the time of the loan is (a) 30 days, (b) 90 days, and (c) 1 year.

6. A note that will mature in 60 days for \$968 is sold to a bank using a 5 per cent simple interest rate. What is the purchase price?

7. Mr. Smith holds the following note:

Augusta, Maine
October 17, 1965

180 days after date, I promise to pay to the order
of Mr. E. L. Smith,
\$5,000.00, and simple interest at 4 per cent.
(Signed) Otto Plumer

On February 14, 1966, Mr. Smith sells this note to his bank. Find the price the bank pays for this note if it is discounted at (a) 5 per cent bank discount, and (b) 5 per cent simple interest.

8. On March 10, 1965, Mr. H. E. Holzman sells the following note to the Apex National Bank:

November 5, 1964

One year from date, I promise to pay to the order
of Mr. H. E. Holzman,
\$10,000.00, and simple interest at 5 per cent.
(Signed) Bill Murray

a) If the Apex National Bank uses a 6 per cent bank discount rate, what are the proceeds?

b) What simple interest rate did the bank realize on this transaction?

9. The Downstate Bank charges 5 per cent bank discount on its loans. It loans a sum of money to Mr. J. P. Epps who agrees to pay back \$2,500 at the end of 120 days. The Downstate Bank immediately sells this note to the Upstate Bank at a 5 per cent simple interest rate. What is the gross profit made by the Downstate Bank?

10. Mr. Willie Jones holds a note due in 8 months with a maturity value of \$5,000. Mr. W. E. Soakem offers to buy this note at 6.25 per cent simple interest, while Mr. I. Cheatem offers to buy it at 6 per cent bank discount. Which is the better offer and how much does Mr. Jones make by taking the better offer?

11. On January 5, 1964, John Smith borrows \$1,000 from you and gives you a note promising to repay this \$1,000 with simple interest at 5 per cent on September 1, 1964. On July 3, 1964, you sell this note to a bank which charges 6 per cent bank discount. How much do you receive from this bank?

12. Consider the following note:

On April 23, 1965, I promise to pay to the
order of A. B. Jones,
\$500.00, with simple interest from today at 4
per cent.

(Signed) J. M. Brook

a) If this note is sold on February 22, 1965, find its purchase price using 6 per cent simple interest.

b) If this note is sold on February 22, 1965, find its purchase price using 6 per cent bank discount.

c) Find the simple interest rate equivalent to the 6 per cent bank discount rate used in part (b).

13. Mr. Jones has a note for \$1,000 due on July 20, 1965. On May 21, 1965, he sells this note to a bank whose bank discount rate is 6 per cent.

a) Find the amount Mr. Jones receives from the bank.

b) What simple interest rate is Mr. Jones paying for the use of this money he receives from the bank?

14. A bank discounts at 6 per cent simple interest a noninterest-bearing note for \$1,000 brought to it 60 days before maturity. Find the bank's profit if it immediately rediscounts this note at another bank whose bank discount rate is $4\frac{1}{2}$ per cent.

15. On 60-day loans a certain bank charges a 6 per cent bank discount rate. At what simple interest rate are they loaning money?

16. A 90-day note for \$500 is dated July 1, 1967, and bears simple interest at 6 per cent. On July 31, 1967, this note is discounted at a bank whose bank discount rate is 4 per cent. Find the proceeds.

17. Find the simple interest rate equivalent to a bank discount rate of 5 per cent (a) on a 90-day note, and (b) on a 1-year note.

18. A debt of \$1,000 is due in 3 months with simple interest at 6 per cent. Find the present value at (a) a 7 per cent simple interest rate, and (b) a 7 per cent bank discount rate.

19. Mr. Olds borrowed from a bank and actually received \$1,000. He signed a 6 months' note. If the bank used a bank discount rate of 6 per cent, find (a) the amount Mr. Olds promised to pay back, and (b) the simple interest rate equivalent to this 6 per cent bank discount rate.

20. On January 5, 1967, you sign a note for \$100 due in 180 days with simple interest at 5 per cent. On March 6, 1967, this note is sold to a bank whose bank discount rate is 6 per cent.

a) What does the bank pay for this note?

b) What simple interest rate is equivalent to this bank discount rate of 6 per cent?

21. Consider the following note:

	<u>October 1</u>	, 19 <u>64</u>
<u>120</u>	days after date, I promise to pay to the order	
of the	<u>E. F. Loan Company</u>	,
<u>\$1,000.00</u>	, together with simple interest at <u>6</u>	
per cent.		
(Signed)	<u>Joe Broke</u>	

a) On October 31, 1964, this note was sold. Find the purchase price if the buyer gets 5 per cent simple interest on his money.

b) Find the purchase price on October 31, 1964, if the buyer gets 5 per cent bank discount on his money.

c) Find the simple interest rate equivalent to the 5 per cent bank discount rate used in part (b).

LOGARITHMS

2.1 Introduction. In this chapter, logarithms will be defined and the methods and techniques of their use in arithmetical computation will be given. No attempt will be made to present a rigorous treatment of the theory of logarithms. The purpose here is to develop a facility in handling common logarithms which will enable the reader to save much tedious arithmetic computation. There are also certain operations in which logarithms are essential. Such problems will be met in Chapter 7.

2.2 Definition. The logarithm x of any number N to the base a is defined as the exponent or the power to which the base a has to be raised to give the number N . This way may be written in symbols as follows:

$$\log_a N = x \quad \text{means} \quad a^x = N.$$

$\log_a N$ is read "logarithm of N to the base a ."

It is possible to prove that any positive number except 1 may be used as the base of a system of logarithms. Furthermore, for each base a , the logarithm of every positive number exists. The proof of both of these statements is beyond the scope of this book.

Although any positive number except 1 could be used as a base, only two numbers are commonly used. One of these is the transcendental number denoted by the letter e , which equals approximately 2.71828 · · · . A system of logarithms with e as a base is called a *Natural* or *Naperian system*. Natural logarithms are indispensable in advanced mathematics and in its applications, but they are not well adapted to arithmetic computation. Natural logarithms will not be used here.

The other base commonly used is 10. This system of logarithms is called *common logarithms*. Common logarithms are well suited to arithmetic computation. They shall be used exclusively in this book. Whenever the term *logarithm* is used henceforth, it will be understood that the base is 10 and $\log N$ will be used to mean $\log_{10} N$. Thus, for base 10, the definition may be written:

$$\log N = x \quad \text{means} \quad 10^x = N.$$

For any positive number N , there exists just one real number x such that $10^x = N$. That is to say, the real logarithm of every positive number exists and is unique. Zero does not have a logarithm. The logarithms of negative numbers do exist, but a negative number does not have a unique logarithm. Furthermore, the logarithms of a negative number involve the imaginary unit $\sqrt{-1}$. These two facts prevent us from using the logarithms of negative numbers in arithmetical computation. Thus, whenever $\log N$ is used in this book, it is to be understood that N is a positive number.

2.3 Laws of Logarithms. It was seen in the definition that logarithms are exponents. Therefore, they obey the same rules of combination as do exponents. These rules of exponents expressed in logarithmic language are called the laws of logarithms. There are three such laws, and they may be stated thus:

$$\text{I} \qquad \log MN = \log M + \log N.$$

The logarithm of a product of two numbers is equal to the sum of the logarithms of the two numbers.

$$\text{II} \qquad \log \frac{M}{N} = \log M - \log N$$

The logarithm of a quotient is equal to the logarithm of the numerator minus the logarithm of the denominator.

$$\text{III} \qquad \log M^k = k \log M$$

The logarithm of a number raised to a power is the same as the exponent indicating the power times the logarithm of the number.

NOTE: Law I is stated for a product of two factors. It also holds for a product of any finite number of factors. In Law III, k may be a fraction. Thus $\log \sqrt[n]{M} = \log M^{\frac{1}{n}}$ is covered by this law.

In order to prove these laws, let

$$\begin{aligned}\log N &= x, \quad \text{i.e., } 10^x = N; \\ \log M &= y, \quad \text{i.e., } 10^y = M.\end{aligned}$$

Thus

$$\log MN = \log (10^y)(10^x) = \log 10^{y+x}.$$

But

$$\log 10^{y+x} = y + x \text{ by the definition of a logarithm.}$$

$$\text{Also, } x = \log N \text{ and } y = \log M$$

$$\therefore \log MN = \log 10^{y+x} = y + x, \text{ so that}$$

$$\log MN = \log M + \log N,$$

and Law I is proved.

Likewise, for Law II,

$$\log \frac{M}{N} = \log \frac{10^y}{10^x} = \log 10^{y-x} = y - x,$$

$$\log \frac{M}{N} = \log M - \log N.$$

For Law III,

$$\begin{aligned}\log M^k &= \log (10^y)^k = \log 10^{ky} = ky, \\ \log M^k &= k \log M.\end{aligned}$$

2.4 Numerical Value of the Logarithm of a Number. It has been stated above that the logarithm of every positive number exists, but no method has been given to find the numerical value of this logarithm. By inspection, it is seen that

$\log 100 = 2$	since $10^2 = 100$,
$\log 10 = 1$	since $10^1 = 10$,
$\log 1 = 0$	since $10^0 = 1$,
$\log .1 = -1$	since $10^{-1} = .1$,
$\log .01 = -2$	since $10^{-2} = .01$,
	etc.

But for numbers, like 473, which are not integral powers of 10, finding the logarithm is a long and tedious process. Extensive tables have been constructed, and we shall make use of such tables.

In general, the logarithm of a number consists of two parts. One part is either a positive or negative whole number or zero. This part is called the *characteristic* of the logarithm of the number. For base 10

logarithms it can be found by inspection. The other part can be approximated by a positive decimal fraction. This decimal part is called the *mantissa*. It is the mantissa that is found in the tables.

Illustrations. $\log 473 = 2.674861$. The characteristic is 2, while the mantissa is .674861 correct to six decimal places. $\log .00560 = -3 + .748188$. The characteristic is -3 , and the mantissa is .748188.

2.5 Finding the Characteristic. The characteristic of the logarithm of a positive number can be found by inspection, using a rather simple rule. This rule may be obtained as follows:

In any number, the first significant digit is the first nonzero digit reading from left to right. If the decimal point is immediately to the right of this first significant digit, it is said to be in *standard position*. Thus, the decimal in 4.063 is in standard position immediately to the right of the first nonzero digit 4, reading from left to right. For reasons which will become clear as we proceed, we shall call this position in a number the *starting point*. That is, the starting point is immediately to the right of the first nonzero digit reading from left to right. It will be found helpful, at first, to mark this position by a small arrow: $4 \downarrow 06.3$. The use of this arrow marking should be discontinued as soon as the system is well established.

From the preceding listing of the logarithms of the powers of ten, it is seen that the logarithm of 1 is 0, while the logarithm of 10 is 1. From this it appears (and it can be proved, but the proof will not be given here) that for any number between 1 and 10, its logarithm will lie between 0 and 1, i.e., its logarithm will be 0 plus a decimal part. Thus the logarithm of any such number would have a characteristic 0. Also, it may be noted that any number between 1 and 10 has its decimal point in what we call the standard position or starting point. In like manner, any number between 10 and 100 will have its logarithm between 1 and 2, i.e., 1 plus a decimal part, giving a characteristic of 1. In this number between 10 and 100, it will be noted that the decimal point lies one place to the right of the starting point. A similar discussion can be given for numbers lying between 100 and 1,000, etc.

Now consider any number between 1 and .1. From the above listing, its logarithm will lie between 0 and -1 , i.e., it can be represented as -1 plus a positive decimal part. This gives a characteristic of -1 for the logarithm of any number between 1 and .1. It should be noted that in each number between 1 and .1, the decimal

point lies one place to the left of the starting point. A similar discussion may be made for numbers lying between .1 and .01, etc.

The results of the above discussion can be summed in the following rule for finding the characteristic:

Beginning at the starting point, count the number of digits crossed in going to the decimal point. If you move to the right to reach the decimal point, this number of digits crossed is the characteristic of the logarithm of this number. If you move to the left to reach the decimal point, the negative of this number of digits crossed is the characteristic of the logarithm of this number.

Illustration 1. The characteristic of the logarithm of 570.6 is 2. In this number, the starting point is immediately to the right of 5. If it is marked by the small arrow, $5\uparrow 70.6$, it is clear that you must move to the right two places in order to reach the decimal point.

Illustration 2. The characteristic of the logarithm of .004063 is -3 . In this number, the starting point is just to the right of 4, $.004\uparrow 063$, and you must move to the left three places to reach the decimal.

Illustration 3. The characteristic of the logarithm of 5.603 is 0. Here, the starting point and the decimal point coincide $5\uparrow .603$. Thus, you cross no digits to reach the decimal point.

The rule for finding the characteristic may be stated more briefly as follows:

The characteristic of the logarithm of a number is the number of places you move from the starting point to the decimal point, positive if you move to the right and negative if you move to the left.

Be sure to go from the starting point to the decimal point.

Since the mantissa is always positive, a problem arises as to how to write a logarithm with a negative characteristic. For example, if the characteristic is -2 and the mantissa is .432761, it could be written $-2 + .432761$, or better $.432761 - 2$. It will appear later that many times it is desirable to have a positive number to the left of the mantissa. This can be done by expressing -2 as, for example, $5 - 7$. Then the logarithm can be written as $5.432761 - 7$. It is customary to have the subtracted part either 10 or a multiple of 10. This is not at all necessary, but it is often found convenient. Thus this logarithm would be written $8.432761 - 10$. Any combination is permissible just so the sum of the two integers is actually the characteristic, here -2 .

NOTE: It would be definitely incorrect to write this logarithm as -2.432761 since this way of writing the number says that the decimal part is also negative. The mantissa is always a *positive* decimal fraction, and extreme care should be used to see that it is expressed as a *positive* decimal fraction.

Exercise 4

Give the characteristic of the logarithms of the following numbers.

- | | |
|------------|----------------|
| 1. 45.62 | 8. .0000503 |
| 2. 37 | 9. .050201 |
| 3. 2561 | 10. 630,000.87 |
| 4. 3.172 | 11. 4.573 |
| 5. .000453 | 12. .3021 |
| 6. 23.62 | 13. 4865.0004 |
| 7. 36,000 | 14. 10,000 |

2.6 Finding the Mantissa. As was seen in the preceding section, the characteristic is completely determined by the position of the decimal points. The mantissa is not affected at all by the position of the decimal point. That is, the mantissa of the logarithm of each of the following numbers will be the same:

.0003425, .3425, 3.425, 342.5, 3,425,000.

That this is true may be shown as follows:

$$\begin{aligned}
 \log 342.5 &= \log [(3.425)(10^2)], \\
 &= \log 3.425 + \log 10^2, \\
 &= \log 3.425 + 2; \\
 \log .3425 &= \log [(3.425)(10^{-1})], \\
 &= \log 3.425 + \log 10^{-1}, \\
 &= \log 3.425 - 1.
 \end{aligned}$$

In like manner, the logarithm of each of these numbers can be shown to be equal to $\log 3.425$ plus or minus an integer. Now the characteristic of the $\log 3.425$ is 0; that is to say, the $\log 3.425$ is the same as the mantissa of $\log 3.425$.

Therefore, in finding the mantissa, the decimal point may be completely ignored. Furthermore, zeros may be added on to either end of a number without affecting its mantissa.

As was stated earlier, the mantissas will be found in a table. In such tables, the decimal point is omitted, but it is understood to belong to the left of the first digit in the mantissa, and should be placed there when it is written down.

In the table of mantissas used here, the mantissas are given only

for numbers containing four or fewer "significant" digits. The first significant digit has already been defined. The last significant digit (in the sense used here) is the last nonzero number reading from the left to right. That is to say, the "significant" digits are all those digits from the first nonzero digit to the last nonzero digit inclusive. (Zeros on the right of a number may be "significant" in the sense that they indicate the accuracy of an approximate number. But that is not the sense of significance used here.) The following illustrations will show how to use these tables.

Illustration 1. Find the logarithm of 3472.

Solution. First, find the characteristic of the logarithm of 3472. By the rule it is 3. In order to find the mantissa, locate the first three digits on the left, 347, in the column headed by *N*. Then go horizontally across from *N* to the column headed by the fourth digit, 2. The number opposite 347, and in the column headed by 2, is 540580. Thus the mantissa is .540580, and

$$\log 3472 = 3.540580.$$

Illustration 2. Find $\log .572$.

Solution. By the rule, the characteristic is -1 , or $9 - 10$. To find the mantissa, the decimal may be ignored. Now 572 contains only three digits. In order to have four digits, a zero is annexed on the right, giving 5720. We now look opposite 572 and in the column headed by 0. Only four digits 7396 appear there. In front of these we must place two other digits, the first two digits of the last six-digit figure preceding this number in the column headed by 0. Here these two digits are 75. Thus the mantissa is .757396.

$$\begin{aligned} \log .572 &= .757396 - 1, \\ \text{or } \log .572 &= 9.757396 - 10. \end{aligned}$$

2.7 Finding a Number if Its Logarithm Is Given. If we know the logarithm of a number, the number itself can be found from these same tables. Such a process is called finding the antilogarithm. The following illustrations will give the method used.

Illustration 1. Find x if $\log x = 2.786325$.

Solution. Since only the mantissas are found in the table, we shall look for 786325 (the decimal part) within the body of the table. When it has been located in the table, the first three significant digits of our number will be the digits under *N* opposite 786325 and the fourth digit will be the number at the top of the column containing 786325. To the left of 786325 and in column *N* are the digits 611, and 4 is at the top of the column

containing 786325. Thus the significant digits of our number x are 6114. Now the location of the decimal point is determined by the characteristic (the whole number part) of our $\log x$. This is $+2$. Thus the decimal point is two places to the right of the starting point, $6 \uparrow 11.4$.

Thus

$$x = 611.4 .$$

Illustration 2. Find y if $\log y = 4.786325$.

Solution. Since the mantissa here is the same as in Illustration 1, the significant digits of y will be the same, 6114. But the decimal point will come at a different place. Since the characteristic is $+4$, we must move 4 places to the right of the starting point to get to the decimal point. Since there are only 3 digits to the right of 6, we annex a zero and then put the decimal point, $6 \uparrow 1140$.

Thus

$$y = 61140.$$

Illustration 3. Find x if $\log x = 7.563006 - 10$.

Solution. Finding the mantissa .563006 within the body of the table, we read the four significant digits as 3656. Since the characteristic is $7 - 10$, or -3 , we must move 3 places to the left of the starting point to find the decimal point. Again we have an insufficient number of digits and must supply extra zeros on the left, .003 $\uparrow$ 656.

Thus

$$x = .003656 .$$

2.8 Interpolation. In order to find the logarithm of a five-digit number or to find an antilogarithm when the mantissa does not appear in the table, interpolation is to be used. If you have ever used interpolation in any type of table, you will find the process to be the same here. The following illustrations will indicate the procedure to be followed.

Illustration 1. Find $\log 234.56$.

Solution. The characteristic is 2. To find the mantissa, the decimal point may be ignored. Thus, the mantissa of the $\log 23456$ is desired. Now, 23456 lies between 23450 and 23460, and the mantissa of its logarithm should lie between the mantissas of 23450 and 23460. It should occupy approximately the same relative position between the two mantissas as does 23456 between 23450 and 23460. Since zeros on either end of a number do not affect the value of the mantissa, the mantissas of the logarithms of 23450 and 23460 may be read directly from the table by ignoring the zeros. The work may be arranged as follows:

	<i>Digits</i>	<i>Mantissa</i>	
10	$\left[\begin{array}{c} 23450 \\ 6 \left[\begin{array}{c} 23450 \\ 23456 \end{array} \right] \\ 23460 \end{array} \right]$	$\left[\begin{array}{c} 370143 \\ ? \\ 370328 \end{array} \right]$	185
		c	

All decimal points are ignored for the present. The differences between numbers connected by brackets are shown, with the unknown difference being represented by c , the correction to be made to the mantissa 370143 in order to get our desired mantissa. By proportional parts, we have

$$\frac{c}{185} = \frac{6}{10}, \quad \text{or} \quad c = \frac{6(185)}{10} = 111.0.$$

If c had not been a whole number, it would have been rounded off to the nearest whole number.

Now c , or 111, is the correction to be made to 370143 in order to get the mantissa for 23456. The answer must lie between 370143 and 370328. Therefore 111 must be added to 370143 giving 370254 as the digits of the desired mantissa.

Thus

$$\log 234.56 = 2.370254.$$

Illustration 2. Find x if $\log x = 3.534217$.

Solution. Here the mantissa .534217 is not found in the body of the table but it lies between the numbers .534153 and .534280 which correspond to the digits 3421 and 3422, respectively. Ignoring decimal points and annexing a zero to both 3421 and 3422 for convenience sake, the work may be arranged thus:

	<i>Digits</i>	<i>Mantissa</i>	
10	$\left[\begin{array}{c} 34210 \\ c \left[\begin{array}{c} 34210 \\ ? \end{array} \right] \\ 34220 \end{array} \right]$	$\left[\begin{array}{c} 534153 \\ 534217 \\ 534280 \end{array} \right]$	127
		64	

By interpolation,

$$\frac{c}{10} = \frac{64}{127}, \quad \text{or} \quad c = \frac{(64)(10)}{127} = 5.04.$$

The correction c is rounded off to the nearest integer, 5 here, and this is added to 34210 to give the digits 34215 for x . Since the characteristic is 3,

$$x = 3421.5.$$

Exercise 5

Find $\log x$ if—

- | | |
|--------------------|---------------------|
| 1. $x = 52.16$ | 11. $x = 47.035$ |
| 2. $x = 123$ | 12. $x = 5,610.2$ |
| 3. $x = .04785$ | 13. $x = 21.377$ |
| 4. $x = 3.512$ | 14. $x = .98431$ |
| 5. $x = 9.8$ | 15. $x = .00062314$ |
| 6. $x = .00043$ | 16. $x = 176.43$ |
| 7. $x = 4,000$ | 17. $x = .037956$ |
| 8. $x = 5,213,000$ | 18. $x = 6.2138$ |
| 9. $x = .3216$ | 19. $x = 4,792,900$ |
| 10. $x = .4891$ | 20. $x = .51324$ |

Find x if—

- | | |
|------------------------------|------------------------------|
| 21. $\log x = 3.583879$ | 31. $\log x = 2.956081$ |
| 22. $\log x = 7.870053 - 10$ | 32. $\log x = 7.563047 - 10$ |
| 23. $\log x = 2.048442$ | 33. $\log x = .590000$ |
| 24. $\log x = .394452$ | 34. $\log x = 4.672951$ |
| 25. $\log x = 6.400020 - 10$ | 35. $\log x = 6.077215$ |
| 26. $\log x = 8.857091 - 10$ | 36. $\log x = 3.866037$ |
| 27. $\log x = 5.999957$ | 37. $\log x = 8.573560 - 10$ |
| 28. $\log x = 9.974005 - 10$ | 38. $\log x = 3.428175$ |
| 29. $\log x = 6.865282 - 10$ | 39. $\log x = .513076$ |
| 30. $\log x = 3.825621$ | 40. $\log x = 8.615042$ |

2.9 Computation by Means of Logarithms. As indicated by the laws of logarithms, the arithmetic operations of multiplication, division, raising to a power, and extracting roots may be performed by use of logarithms. The fact that the logarithms of the two sides of an equation may be equated to each other will be the key to solutions using logarithms. If the problem is stated in the form of an equation, the logarithm of each side will be taken, the two results will be connected by the sign of equality, and then the laws of logarithms will be applied to simplify the problem. If the problem is not stated as an equation, then let some letter, say N , equal the expression for which a numerical answer is desired. This gives an equation, and you proceed as described above. The following problems will illustrate the method and answer some of the questions that will arise.

Illustration 1. Using logarithms, find $\frac{(6.543)^2}{347.1}$.

Solution. Let $N = \frac{(6.543)^2}{347.1}$. Now take logarithms of both sides of this equation and equate the results.

$$\log N = \log \frac{(6.543)^2}{347.1}$$

The left-hand side of this equation cannot be simplified, but the laws of logarithms will simplify the right hand. Applying Law II and then Law III, we get

$$\begin{aligned}\log N &= \log (6.543)^2 - \log 347.1, \\ \log N &= 2 \log 6.543 - \log 347.1.\end{aligned}$$

It will be found convenient to make the following outline to carry out the operations indicated in the logarithmic equation. Using the tables to look up the logarithms, we get:

$$\begin{array}{rcl}2 \log 6.543 &= & 2(0.815777) = 1.631554 \\ \log 347.1 &= & \underline{2.540455}\end{array}$$

These two numbers are to be subtracted. If the subtraction is done as it now stands, the decimal part will be negative, whereas the table for mantissas is always for positive decimals. In order to keep the decimal part positive, add some whole number to the first expression and then subtract this same number by annexing minus the number on the right. Any whole number that will make the first expression greater than the second will be all right. It is customary to add and subtract either 10 or a multiple of 10.

Thus 1.631554 may be written $11.631554 - 10$, and the subtraction will be as follows:

$$\begin{array}{rcl}2 \log 6.543 &= & 11.631554 - 10 \\ \log 347.1 &= & \underline{2.540455} \\ \log N &= & 9.091099 - 10\end{array}$$

Since N , not $\log N$, was desired, the antilogarithm of each side must be taken. This gives, using interpolation,

$$N = .12334.$$

Illustration 2. Find x , if $x = \sqrt[3]{.056}$.

Solution.

$$\begin{aligned}x &= \sqrt[3]{.056} \\ \log x &= \log \sqrt[3]{.056} \\ \log x &= \frac{1}{3} \log .056 && \text{by Law III} \\ \log x &= \frac{1}{3} (8.748188 - 10) && \text{by tables}\end{aligned}$$

Now 3 must be divided into this entire number, that is, 10 must also be divided by 3. Since division of 10 by 3 will not be a whole number but will

contain a decimal part, the mantissa (decimal part) will now appear in two places and must be subtracted to get the mantissa to be used in finding the antilogarithm. Although this is not incorrect, it certainly is inconvenient. This can be avoided by changing the characteristic of $\log .056$ from 8. — 10 to, say, 7. — 9. That is to say, whenever a logarithm is to be divided by an integer, it is always desirable to have the negative part of the characteristic a number that is exactly divisible by this integer.

Thus

$$\log x = \frac{7.748188 - 9}{3},$$

$$\log x = 2.582729 - 3.$$

Taking antilogarithms of both sides,

$$x = .38259.$$

Illustration 3. Find $\frac{(34.25)^5 \sqrt{4725}}{52700}$.

Solution. Let

$$x = \frac{(34.25)^5 \sqrt{4725}}{52700},$$

$$\log x = \log \frac{(34.25)^5 \sqrt{4725}}{52700},$$

$$\log x = \log (34.25)^5 \sqrt{4725} - \log 52700,$$

$$\log x = \log (34.25)^5 + \log \sqrt{4725} - \log 52700,$$

$$\log x = 5 \log 34.25 + \frac{1}{2} \log 4725 - \log 52700.$$

This last equation tells just what to do with each logarithm. An outline should be made as in Illustration 1.

$5 \log 34.25$	$= 5(1.534661)$	$= 7.673305$
$\frac{1}{2} \log 4725$	$= \frac{1}{2}(3.674402)$	$= 1.837201$
(adding)		9.510506
$\log 52700$		$= 4.721811$
$\log x$		4.788695
x	$= 61475$	

Whenever an equation contains the unknown as an exponent, logarithms will be found very useful. The work may be arranged as in the following problem.

Illustration 4. Find x if $(1.02)^x = 15$.

Solution.

$$\begin{array}{rcl}
 (1.02)^x & = & 15 \\
 \log (1.02)^x & = & \log 15 \\
 x \log 1.02 & = & \log 15 \quad \text{by Law III} \\
 x (.008600) & = & 1.176091 \\
 x & = & \frac{1.176091}{.008600}
 \end{array}$$

Logarithms could be used to solve for x , but actual division will usually be shorter.

$$x = 136.75$$

Since x was to be found, and this last equation gives x , no further work is necessary.

Exercise 6

Compute, using logarithms.

- | | |
|---------------------------|--|
| 1. $(347)(56.43)$ | 10. $(.1754)^3$ |
| 2. $(18.75)(234)$ | 11. $\sqrt[3]{.0732}$ |
| 3. $(.8763)(.421)$ | 12. $\sqrt[4]{.175}$ |
| 4. $(17.32)(.9503)$ | 13. $\frac{(67400)(12.46)}{.032174}$ |
| 5. $\frac{57.23}{8.694}$ | 14. $\frac{17.476}{(3.475)(.6446)}$ |
| 6. $\frac{132.7}{98.4}$ | 15. $\frac{(476.3)^{\frac{1}{2}}(5.743)^2}{483}$ |
| 7. $\frac{.08472}{.7321}$ | 16. $\frac{(2.463)^3 \sqrt{17.47}}{.843}$ |
| 8. $\frac{1.725}{.5643}$ | 17. $[(46.21)(.36517)]^2$ |
| 9. $(4.732)^5$ | 18. $[(1.8423)^2(246)]^{\frac{1}{3}}$ |

Find x if—

- | | |
|-------------------------|---------------------------------|
| 19. $3^x = 11$ | 24. $(1.03)^{2x} = 1.754$ |
| 20. $5^x = 23$ | 25. $(1.03)^{-x} = .6421$ |
| 21. $x^7 = 18$ | 26. $(1.02)^{-x} = .4786$ |
| 22. $x^3 = 46$ | 27. $1,000 (1.01)^x = 1,197.40$ |
| 23. $(1.015)^x = 2.346$ | 28. $500 (1.02)^x = 584.37$ |

COMPOUND INTEREST

3.1 Compound Interest. For long-range transactions, compound interest is most commonly used. Here, the interest is computed at the end of each period and added to the principal for the beginning of that period. This gives a new principal for the next period, and the interest is then computed on this new principal. Interest for each period is computed on the original principal plus all the interest accumulated up to the beginning of that period. Thus interest is earned on each preceding interest as well as on the original principal. As an example, suppose \$1,000 is borrowed at a compound interest rate of 1 per cent per quarter. At the end of the first 3 months, an interest of $(.01)(1,000) = \$10$ will be due. For the second quarter the principal is now \$1,010 and interest for the second quarter will be $(.01)(1,010) = \$10.10$, giving a new principal of \$1,020.10.

It will be noted that the maturity value at the end of any period is the new principal for the next period. Thus \$1,020.10 is the maturity value at the end of the second quarter and is also the new principal for the third quarter.

The computation of this problem may be tabulated thus:

Original principal	=	\$1,000.00
Interest for first quarter = $(.01)(1,000)$	=	10.00
Principal for second quarter	=	<u>\$1,010.00</u>
Interest for second quarter = $(.01)(1,010)$	=	10.10
Principal for third quarter	=	<u>\$1,020.10</u>
Interest for third quarter = $(.01)(1,020.10)$	=	10.20
Principal for fourth quarter	=	<u>\$1,030.30</u>
Interest for fourth quarter = $(.01)(1,030.30)$	=	<u>10.30</u>

Principal for fifth quarter	=	\$1,040.60
Interest for fifth quarter = $(.01)(1,040.60)$	=	10.41
Principal for sixth quarter	=	\$1,051.01
Interest for sixth quarter = $(.01)(1,051.01)$	=	10.51
Principal for the seventh quarter or the maturity value at the end of the sixth quarter	=	\$1,061.52

Thus, the maturity value at the end of $1\frac{1}{2}$ years (6 quarters) will be \$1,061.52. Thus compound interest has resulted in a total interest charge of \$61.52 for a year and a half. Simple interest for the same time at .01 per quarter (4 per cent per year) would have given a total interest charge of \$60.00, or \$1.52 less than compound interest. This illustrates the fact that for the same numerical rate and for more than one period, the maturity value computed by compound interest is always greater than that computed by simple interest.

3.2 Compound Interest Formula. It is clear that the above method of arithmetically calculating compound interest is too tedious to be used over a long period of time, or when the interest rate is not as simple a number as .01. A formula will now be developed that will give this result without the tedious arithmetic.

As in the case of simple interest and bank discount, let P be the *present value* or *principal* and S the *final amount* or *maturity value* at the close of the n interest periods of the transaction. Let i denote the compound interest rate per interest period and n be the number of interest periods. Note that both i and n refer to interest periods. These interest periods may be of any length, say 1 month, 3 months, 6 months, etc. In simple interest and bank discount, 1 year was always used as the unit of time. Here, an interest period, whatever its length, is the basis of the calculation. Furthermore, n is to be taken as a whole number. (Fractional values for n will be discussed later.) For brevity, let S_1 be the maturity value at the end of the first interest period (and therefore the new principal for the second interest period), S_2 be the maturity value at the end of the second interest period, etc.

Since $(P)(i)$ is the interest for one period,

$$S_1 = P + Pi = P(1 + i).$$

Now the interest for the second period = $(S_1)(i) = P(1 + i)(i)$.

Thus

$$S_2 = P(1 + i) + P(1 + i)i = P(1 + i)(1 + i) = P(1 + i)^2.$$

Likewise

$$S_3 = P(1+i)^2 + P(1+i)^2i = P(1+i)^2(1+i) = P(1+i)^3,$$

$$S_4 = P(1+i)^3 + P(1+i)^3i = P(1+i)^3(1+i) = P(1+i)^4.$$

From this method it appears that, for example,

$$S_7 = P(1+i)^7,$$

$$S_{21} = P(1+i)^{21},$$

or in general

$$S_n = P(1+i)^n.$$

Letting S_n be S , this gives formula

$$(3) \quad S = P(1+i)^n.$$

That this formula actually follows from the above discussion can be rigorously established by use of mathematical induction. The complete proof will not be given here.

Illustration 1. This formula may be applied to the illustrative example above.

Solution. In this example, $P = 1,000$, $i = .01$ per quarter, and $n = 6$ quarters. From formula (3),

$$S = 1,000(1.01)^6.$$

The actual computation could be accomplished in several ways. $(1.01)^6$ can be computed by actual multiplication, by using the binomial theorem, or by using logarithms. Extensive tables have been computed which give the values of $(1+i)^n$ for the most common values of i and for integral values of n from 1 on to 200 or more in certain cases. Table 3 in the appendix gives values of both $(1+i)^n$ and $\log(1+i)^n$ for a sufficient number of values of i and n for use with the problems in this text. More extensive tables are available for applications in the business world. From this table for $(1+i)^n$, look in the column headed 1 per cent and read the figure directly across from $n = 6$. The table gives the value

$$(1.01)^6 = 1.06152015.$$

Thus

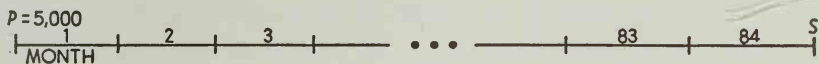
$$\begin{aligned} S &= 1,000(1.01)^6, \\ &= 1,000(1.06152015), \\ &= 1061.52 \text{ to the nearest cent.} \end{aligned}$$

This agrees with the previous answer.

Illustration 2. If \$5,000 is borrowed at a compound interest rate of $\frac{1}{2}$ per cent per month, how much should be paid back 7 years later?

Solution. First set up the time line. Since each period is 1 month, the total number of periods in 7 years will be 84. Thus

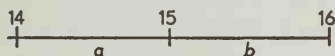
$$n = 84, P = \$5,000, \text{ and } i = .005 \text{ per month.}$$



$$\begin{aligned} S &= 5,000(1.005)^{84} \\ &= 5,000(1.52036964) \text{ by Appendix Table 3} \\ &= \$7,601.85 \text{ to the nearest cent} \end{aligned}$$

Several helpful suggestions may be noted in the construction of the time diagram above.

1. Each interest or conversion period contains the number of the period within the space, not at an end point. This tells precisely which period is associated with each number. If the diagram had been numbered thus:



it would not be clear whether the fifteenth period is the one labeled b or a until the entire line diagram had been examined carefully.

2. One of the interest periods contains a notation giving its length (in time). Thus, it is not necessary to refer back to the original problem to know what length periods are being used. In some later problems, it may be desirable to construct the period in the diagram of a different length from that of the interest period involved.

3.3 Terminology. In actual practice, the interest rate in the preceding illustration would not have been expressed as $\frac{1}{2}$ per cent per month but would have been stated: "Interest is at 6 per cent compounded (or converted) monthly." This 6 per cent per year is an interest rate in a technical sense only. It is called the *nominal rate*, that is, the named rate on a "year basis." Before it can be used in computation, it must be translated into a rate per interest period. The phrase "compounded monthly" tells us that the interest period is 1 month. Since there are 12 months in a year, the i is obtained by taking $i = \frac{.06}{12} = .005$ per month. Thus 6 per cent compounded monthly means $\frac{.06}{12} = .005$ per month. Likewise, 6 per cent compounded quarterly means $i = \frac{.06}{4} = .015$ per quarter.

To further shorten the notation, let j represent the named or nominal rate on a yearly basis and let m be the number of conversion (or interest) periods in a year. Then 6 per cent compounded monthly may be written 6 per cent, $m = 12$ (or .06, $m = 12$); and 6 per cent compounded quarterly may be written as 6 per cent, $m = 4$. For

$$j = .06, m = 12, \quad i = \frac{.06}{12} = .005 \text{ per month.}$$

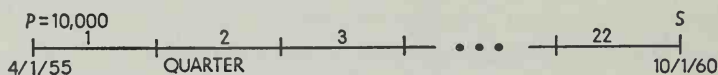
$$j = .06, m = 4, \quad i = \frac{.06}{4} = .015 \text{ per quarter.}$$

It is to be clearly noted that these two rates of interest are not the same rates. It can be verified by an example that they will not give the same maturity value if the same P and same time are used. A careful comparison of the equivalence of compound interest rates will be given in Section 3.5.

Whenever an interest rate is given without statement as to kind of interest or without statement as to the length of the interest period (or interval), it is commonly understood that the rate is a compound interest rate, and that the conversion or interest period is 1 year. Thus 6 per cent interest implies that the rate is 6 per cent compounded annually, or .06, $m = 1$. Thus, it is always very important to follow the interest rate by the number of conversion periods in a year (if different from one), or if the interest rate is for a period different from one year, to follow the rate by the length of the period.

Illustration. On April 1, 1955, Mr. Smith borrows \$10,000 at 5 per cent, $m = 4$. On October 1, 1960, this debt is paid. Find the amount (maturity value) Mr. Smith pays at that time.

Solution. $P = 10,000$, $i = \frac{.05}{4} = .0125$ per quarter. April 1, 1955, to October 1, 1960, is $5\frac{1}{2}$ years, or 22 quarters. Thus $n = 22$.



By formula (3),

$$S = 10,000(1.0125)^{22},$$

$$S = 10,000(1.31428848) \text{ by Appendix Table 3,}$$

$$S = \$13,142.88 \text{ to the nearest cent.}$$

3.4 Use of Logarithms in the Compound Interest Formula.
In many cases the numbers substituted in the compound interest

formula are such that the multiplication is rather tedious. In such cases, logarithms may be used. If logarithms are used in the preceding illustration, the work becomes:

$$\begin{aligned}
 S &= 10,000(1.0125)^{22} \\
 \log S &= \log 10,000 + 22 \log 1.0125 \\
 \log 10,000 &= 4.000000 \\
 22 \log (1.0125) &= 22(.005395) = .118690 \\
 \log S &= 4.118690 \\
 S &= \$13,143 \text{ to nearest dollar.}
 \end{aligned}$$

NOTE: Log 1.0125 may be found by interpolation in a six-place table of mantissas, Table 1 in the Appendix, or it can be read directly from a seven-place table of mantissas, Table 2 in the Appendix.

S agrees to the nearest dollar with the answer found by multiplication. The accuracy is limited here by the number of digits in the table of mantissas.

Extensive tables of $\log (1 + i)^n$ have been constructed. Portions of such a table are included in Table 3 in the Appendix. From this table $\log (1.0125)^{22}$ may be read directly. The work then becomes:

$$\begin{aligned}
 S &= 10,000(1.0125)^{22} \\
 \log S &= \log 10,000 + \log (1.0125)^{22} \\
 \log 10,000 &= 4.0000000 \\
 \log (1.0125)^{22} &= .1186907 \text{ by Appendix Table 3} \\
 \log S &= 4.1186907 \text{ or } 4.118691 \\
 S &= \$13,143.
 \end{aligned}$$

Exercise 7

1. Accumulate \$5,000 for 14 years at 6 per cent, $m = 4$.
2. Accumulate \$600 for 15 years at 5 per cent, $m = 2$.
3. Accumulate \$1,000 for $9\frac{3}{4}$ years at 4 per cent, $m = 4$.
4. Accumulate \$2,000 for $7\frac{5}{12}$ years at 6 per cent, $m = 12$.
5. Accumulate \$117.92 for 4 years and 9 months at 5 per cent, $m = 4$.
6. Accumulate \$593.47 for 8 years and 6 months at 3 per cent, $m = 4$.
7. Find the amount due at the end of 12 years if Mr. Jones borrows \$3,000 now and interest is at .055, $m = 2$.
8. What is the maturity value of a 10-year loan at 5 per cent, $m = 4$, if the present value is \$10,000?
9. On April 1, 1952, Mr. Smith borrows \$8,000. This is to be repaid on October 1, 1961, with interest at 3 per cent, $m = 4$. Find the amount due on October 1, 1961.
10. Mr. Jones deposits \$3,000 in a savings account on July 1, 1954. If the bank pays interest at .015, $m = 2$, how much is in the account on January 1, 1965?

11. Mr. Wayback borrows \$5,000 on October 1, 1956. He also borrows \$3,000 on April 1, 1958. If interest on both debts is at 5 per cent, $m = 4$, what single payment on July 1, 1965, will pay off both debts?

12. On April 1, 1954, Mr. John Smith III, bought a car for \$2,500 and signed a note bearing interest at 6 per cent, $m = 2$. On October 1, 1955, he purchased a second car for \$3,500 and signed another note bearing the same rate of interest. What must Mr. Smith pay on April 1, 1959, in order to pay off both notes?

13. On the day of the birth of his son, Mr. J. P. Oliver deposits \$5,000 in his son's name at the local bank. If the bank pays 2 per cent, $m = 2$, how much will the son have in his account on his sixteenth birthday?

14. On February 29, 1948, triplets were born to Mr. and Mrs. Campbell. Mr. Campbell deposited \$3,000 in a joint account for his three daughters. What would each girl's share be on February 29, 1968, if the interest is computed at 3 per cent, $m = 2$?

15. Find the maturity value of the following note:

<u>May 2</u> , 19 <u>56</u>
<u>24</u> months from today, I promise to pay to the order of
<u>Mr. S. L. Jones</u> , \$ <u>5,000.00</u> ,
and interest at <u>7</u> per cent compounded quarterly.
(Signed) <u>Bill Lane</u>

16. Find the amount due at the end of 20 years if \$10,000 is borrowed today and interest is computed at 4 per cent, $m = 2$, for the first 6 years, and at 5 per cent, $m = 4$, for the rest of the time.

17. On July 1, 1958, Mrs. Kelly borrows \$5,000. Interest is to be computed at 5 per cent, $m = 2$. On January 1, 1963, the interest rate is changed, from that date on, to 6 per cent, $m = 4$. Find the amount Mr. Kelly must pay on April 1, 1967.

18. \$7,000 is deposited in a bank now. Find the amount in the bank at the end of 8 years if interest is at 5 per cent, compounded annually, for the first 3 years, and at 6 per cent, compounded annually, for the last 5 years.

19. In Problem 18, interchange the interest rates but not the times. Find the amount due at the end of 8 years. Is this the same answer as in Problem 18? Explain.

20. Find the amount in an account at the end of 19 years if \$20,000 is invested at 4 per cent compounded semiannually.

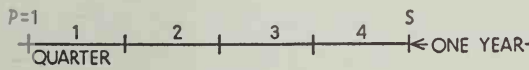
21. Find the amount in an account at the end of 47 years if \$100 is invested at 3.6 per cent, $m = 2$.

22. If \$400 is accumulated for 15 years at 2.7 per cent compounded annually, find this amount.

3.5 Equivalent Rates of Interest. In the comparison of simple interest and bank discount, it was seen that two such interest rates were equivalent, provided they gave the same results. Quite frequently in compound interest rates it is desired to find an interest rate compounded, for example, quarterly that would be equivalent to another rate compounded, say, monthly. That is, we wish a rate compounded quarterly that will give the same result or maturity value on a fixed principal for a fixed time as a given rate compounded monthly. As the following illustrations will show, neither the size of P nor the length of time involved will affect the answer.

Illustration 1. Find the nominal rate compounded quarterly that is equivalent to 6 per cent, $m = 12$.

Solution. The given rate is $i = \frac{.06}{12} = .005$ per month. An equivalent rate j , $m = 4$ (or i per 3 months), is to be found. To simplify computation, take a principal of \$1 and invest it at each rate for some time, say 1 year. Since the rates are equivalent, the S calculated in each case must be the same. The work may be arranged thus:

$j, m = 4$ Let $i =$ rate per quarter $\left(i = \frac{j}{4}\right)$ $n = 4$, since there are 4 quarters in a year	$.06, m = 12$ $i = \frac{.06}{12} = .005$ per month $n = 12$, since there are 12 months in a year
 $S = 1(1 + i)^4$	 $S = 1(1.005)^{12}$

Since these two rates are equivalent, the two values for S must be the same. Then

$$a) \quad (1 + i)^4 = (1.005)^{12}.$$

Next, extract the fourth root of each side of this equation.

$$(1 + i)^{\frac{4}{3}} = (1.005)^{\frac{12}{3}}$$

b)

$$1 + i = (1.005)^3$$

$$i = (1.005)^3 - 1$$

per quarter

$$i = 1.01507513 - 1$$

$$i = .01507513$$

$$i = .01508$$

per quarter

But

$$j = 4i = 4(.01508), \quad m = 4.$$

$$\therefore j = .0603, \quad m = 4.$$

Thus .06, $m = 12$, is equivalent to (gives the same result as) .0603, $m = 4$.

Unless something is said to the contrary, it is customary to give the answer correct to four decimal places.

In this problem P was taken as 1. If any other value (except 0) had been taken for P , say \$1,000, then equation (a) would have been

$$1,000(1 + i)^4 = 1,000(1.005)^{12}.$$

If both sides of this equation are divided by the value taken for P , here 1,000, equation (a) appears. This illustrates that the size of P does not affect the answer.

To show that the time interval does not affect the answer, let the time be t years, instead of 1 year ($t \neq 0$). Since there are $4t$ quarters in t years and $12t$ months, equation (a) would now read:

$$(1 + i)^{4t} = (1.005)^{12t}.$$

Extracting the $4t$ root of both sides,

$$(1 + i)^{\frac{4t}{4t}} = (1.005)^{\frac{12t}{4t}}.$$

This gives equation (b) and the same result as above.

Illustration 2. Find the rate compounded quarterly which is equivalent to 4 per cent, $m = 2$.

Solution.

$$j, m = 4$$

$$.04, m = 2$$

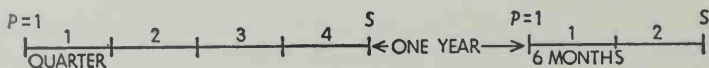
$$i = \text{rate per quarter}$$

$$i = \frac{.04}{2} = .02 \text{ per 6 months}$$

$$\left(\frac{j}{4} = i\right)$$

$$n = 2$$

$$n = 4$$



$$\begin{aligned}
 S &= (1+i)^4 & S &= (1.02)^2 \\
 (1+i)^4 &= (1.02)^2 \\
 1+i &= (1.02)^{\frac{1}{2}} \\
 i &= (1.02)^{\frac{1}{2}} - 1 && \text{per 3 months}
 \end{aligned}$$

Only integral values of n are given in Table 3 in the Appendix. Since fractional exponents were defined in algebra, the definition of $(1+i)^n$ can be extended to include fractional exponents. Here, only fractions with numerators 1 will be considered. Since the numerator is 1, let p , an integer, be the denominator and write $(1+i)^{\frac{1}{p}}$. Tables have been constructed for many values of i . Table 5 in the Appendix gives both $(1+i)^{\frac{1}{p}}$ and

$$\log (1+i)^{\frac{1}{p}}$$

for $p = 2, 3, 4, 6$, and 12 . To find $(1.02)^{\frac{1}{2}}$ in Table 5 look in the portion for 2 per cent and opposite $p = 2$. Thus

$$\begin{aligned}
 (1.02)^{\frac{1}{2}} &= 1.00995049, \\
 i &= .00995049, \\
 i &= .00995 \text{ per 3 months}, \\
 j = 4i &= .0398, \quad m = 4.
 \end{aligned}$$

The *nominal rate* was defined as the named rate on a year basis, regardless of the number of times it may be compounded. Now, if a nominal rate is compounded once a year, it is also called an *effective rate*. Thus an effective rate is compounded just once a year, while a nominal rate may be compounded once, twice, etc., per year.

Exercise 8

- Find the rate of interest compounded semiannually equivalent to (a) .06, $m = 12$; (b) .06, $m = 4$; and (c) .06, $m = 1$.
- Find the interest rate compounded quarterly which is equivalent to (a) .05, $m = 12$; (b) .05, $m = 2$; and (c) .05, $m = 1$.
- What rate of interest compounded monthly is equivalent to 4 per cent, $m = 4$?
- Find the rate of interest compounded quarterly equivalent to 7 per cent, $m = 12$.
- Find the rate of interest compounded semiannually that is equivalent to 3 per cent, $m = 4$.
- Find the effective rate corresponding to (a) 6 per cent, $m = 2$; (b) 6 per cent, $m = 4$; and (c) 6 per cent, $m = 12$.
- Find the effective rate corresponding to (a) 5 per cent, $m = 2$; (b) 5 per cent, $m = 4$; and (c) 5 per cent, $m = 12$.

8. What nominal rate compounded quarterly will be equivalent to the effective rate of 7 per cent?
9. Find the nominal rate compounded monthly that gives the same interest yield as an effective rate of 5 per cent.
10. Which is a better investment, a savings account paying $2\frac{1}{2}$ per cent, $m = 2$, or bonds yielding an interest rate of 2.6 per cent effective?
11. Mr. Able can borrow from the Savings Association at an interest rate of 5 per cent, $m = 12$, or he can secure the same loan from the E. T. Loan Company at a rate of 5.2 per cent compounded annually. From which concern would it be better for him to borrow?
12. What rate of interest compounded semiannually is equivalent to 6 per cent compounded quarterly?
13. Find the rate compounded monthly that is equivalent to 6 per cent compounded semiannually.
14. What rate of interest compounded quarterly is equivalent to 8.4 per cent compounded monthly?
15. What rate of interest compounded monthly is equivalent to 4.4 per cent compounded quarterly?
16. Find the rate compounded semiannually that is equivalent to an effective rate of 4.3 per cent.
17. Find the rate compounded semiannually that is equivalent to 5.6 per cent compounded quarterly.
18. Find the effective rate equivalent to 3.8 per cent compounded semiannually.
19. Mr. Jones can borrow money from a friend at an 8 per cent simple interest rate. If he wishes to borrow the money for 3 years, what rate of interest compounded quarterly would cost him just the same?
20. If Mr. Jones, in Problem 19, wishes to borrow the money for only 2 years, what rate of interest compounded quarterly would cost him just the same as the 8 per cent simple interest? Is this the same answer that is found in Problem 19?

3.6 Present Value. In many cases, the maturity value S is known and the principal P is to be calculated. Since $S = P(1 + i)^n$,

$$P = \frac{S}{(1 + i)^n}.$$

By using the definition of negative exponents, this last expression may be written

$$P = S(1 + i)^{-n}.$$

This is not a new formula but just a different form of formula (3), $S = P(1 + i)^n$. A table for $(1 + i)^{-n}$ is also available. Table 4 in the Appendix permits one to solve for P by multiplication rather than by division.

If logarithms are to be used, it is preferable to use the form

$$P = \frac{S}{(1+i)^n}$$

since no tables are given here for $\log(1+i)^{-n}$.

Finding P when S is given is called discounting at compound interest; $(1+i)^{-n}$ is called the discount factor.

Illustration 1. Find the present value of \$5,000 due 5 years from now if interest is computed at 6 per cent, $m = 12$.

Solution. Here $i = \frac{.06}{12} = .005$ per month; $n = 60$ since 5 years = 60 months.



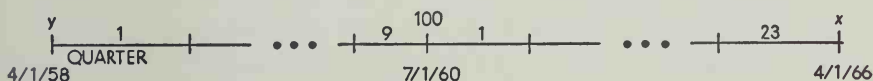
$$P = 5,000(1.005)^{-60} = 5,000(.74137220)$$

$$P = \$3,706.86$$

Note that the factor $(1+i)^n$ picks up a sum of money and moves it to the right, accumulating it or adding the interest to it in order to get S . But the factor $(1+i)^{-n}$ picks up a sum of money and moves it to the left, discounting it or deducting the proper interest charges in order to get P . Now it is customary in mathematics to assign the positive direction to the right and the negative direction to the left. Thus, mechanically, we can think of $(1+i)^{\pm n}$ as being the necessary factor to move a sum of money n interest periods to the right or left, according as a positive or negative exponent is used.

Illustration 2. A certain obligation has a value of \$100 on July 1, 1960. If money is worth 4 per cent, $m = 4$ (that is, interest is computed at .04, $m = 4$), find the value of this obligation on (a) April 1, 1966, and (b) April 1, 1958.

Solution. $i = \frac{.04}{4} = .01$ per 3 months. From April 1, 1958, to July 1, 1960, is 2 years and 3 months, or 9 quarters. The time from July 1, 1960, to April 1, 1966, is 5 years and 9 months, or 23 quarters. The time diagram may be constructed thus:



Part (a). Let x be the value on April 1, 1966. Since 100 lies to the left of x , $P = 100$ and $S = x$. By the compound interest formula, moving 100 to the right to get x ,

$$x = 100(1.01)^{23},$$

$$x = \$125.72.$$

Part (b). Let y be the value on April 1, 1958. Now y is to the left of 100. Then $P = y$ and $S = 100$. Moving 100 to the left,

$$y = 100(1.01)^{-9},$$

$$y = \$91.43.$$

The question arises, would we get the same value for x if we first moved 100 to the left to April 1, 1958, getting y , and then move y from April 1, 1958, to April 1, 1966? The answer is yes, for then

$$x = y(1.01)^{32} = [100(1.01)^{-9}](1.01)^{32} = 100(1.01)^{23},$$

which is the same answer as was gotten in part (a).

This is an illustration of the principle that if any sum of money is moved through any interest period in both directions, and at the same rate, then exactly the same amount of interest is added in moving to the right as is deducted in moving back to the left. This principle will be extremely useful in later problems in this chapter.

Exercise 9

1. Discount \$5,000 for 12 years at 4 per cent, $m = 2$.
2. Discount \$2,500 for 19 years at 5 per cent, $m = 4$.
3. Find the present value of \$10,000 due 20 years hence if money is worth 5 per cent, $m = 2$.
4. Find the present value of a loan of \$1,000 due 15 years from now if interest is computed at 4 per cent, $m = 4$.
5. Mr. Ozzie Little signs a note on May 5, 1952, promising to pay \$3,000 and interest at 5 per cent, $m = 2$, 10 years later. On November 5, 1956, this note is sold to the Farmer's Bank. If the bank purchases this note to yield interest at 6 per cent, $m = 4$, find the purchase price.
6. In order to have \$1,000 at the end of 10 years in an account paying interest at .015, $m = 2$, how much must Mr. Goldfish deposit now?
7. Mr. Small deposits a sum of money at the birth of his child so that the child may have \$5,000 on the day he becomes 15 years of age. If interest is at .035, $m = 2$, find the sum Mr. Small deposits.
8. Discount \$417.24 for 8 years at 4 per cent, $m = 4$.
9. Discount \$689.17 for 3 years at 6 per cent, $m = 12$.
10. Discount \$5,000 for 10 years at 4.2 per cent, $m = 2$.
11. Discount \$4,832 for 7 years at 6.4 per cent, $m = 4$.
12. Discount \$2,500 for 11 years at 5 per cent effective.
13. Find the present value of \$5,000 due at the end of 40 years if interest is at 6 per cent, $m = 4$.

$$\text{Hint: } (1.015)^{-160} = (1.015)^{-100} (1.015)^{-60}.$$

14. What is the present value of \$2,000 due at the end of 60 years if interest is at 4 per cent, $m = 4$?

15. Mr. I. M. Luke has \$8,000. He can invest his money at 3 per cent, $m = 2$. With this money he wishes to purchase a \$2,500 car 2 years from now and make a \$5,000 payment on his house 3 years from now. The money not needed for these two, he desires to give now to the United Fund. How much will the United Fund receive?

16. A note matures today for \$15,000. What was its value 7 years ago if interest was at 2.5 per cent, $m = 2$?

17. If interest is computed at 5 per cent compounded monthly, how much must be deposited now in order to have \$7,500 at the end of 10 years?

18. Discount \$2,500 for 14 years and 9 months at $5\frac{1}{2}$ per cent compounded quarterly.

3.7 Finding the Maturity Value for Terms Involving a Fraction of a Period. In all the problems so far, the term of the transaction has always contained a whole number of periods. If this is not the case and a fraction of a period is involved, some agreement must be reached as to how the interest is to be computed for this fraction of a period. In bank savings account, ordinarily no interest is allowed for such fractional periods. In other situations, compound interest may be defined for a fractional period by using a fractional exponent for n . This type of definition was already used in certain cases of finding equivalent interest rates. For example, if the time element is $17\frac{1}{4}$ periods at .01 per interest periods, the accumulating factor will be

$$(1.01)^{17\frac{1}{4}} = (1.01)^{17} (1.01)^{\frac{1}{4}}.$$

But this is not the method most commonly used in financial transactions involving fractional periods.

Unless otherwise stated, the procedure to be employed when accumulating by compound interest for a term involving a fraction of a period is as follows: Let n be the largest number of full interest periods in the term of the transaction. Accumulate P for the n periods, getting a sum x . Now for the remaining fractional period, this x is accumulated at simple interest. This gives the final amount S .

Illustration. On April 1, 1955, Mr. Smith borrows \$1,000. On May 1, 1957, this note is repaid. Find the maturity value if interest is computed at 6 per cent, $m = 4$.

Solution. From April 1, 1955, to May 1, 1957, is 2 years and 1 month, or 8 quarters plus $\frac{1}{3}$ of another quarter.



Now

$$x = 1,000(1.015)^8 = \$1,126.49.$$

This is the value of the obligation on April 1, 1957. This amount is to be accumulated for 1 month at simple interest. Since the interest rate is .015 per 3 months and the time is 1 month or $\frac{1}{3}$ of a period, the simple interest formula gives

$$\begin{aligned} S &= 1,126.49[1 + (.015)(\frac{1}{3})], \\ &= 1,126.49(1.005), \\ &= \$1,132.12. \end{aligned}$$

NOTE: Here i was used as .015 per 3 months with $t = \frac{1}{3}$ of 3 months. It is equally correct to use $r = .06$ per year and $t = \frac{1}{12}$ year since $(.015)(\frac{1}{3}) = (.06)(\frac{1}{12}) = .005$.

In case logarithms are to be used for the calculation, the work may be arranged thus:

$$\begin{aligned} x &= 1000(1.015)^8, \\ S &= x[1 + (.06)(\frac{1}{12})] = x(1.005), \\ S &= 1000(1.015)^8(1.005). \end{aligned}$$

Now logarithms should be taken of both sides of this equation. In doing this, $\log 1000$ and $\log (1.005)$ may be found in the regular table of logarithms, while $\log (1.015)^8$ can be read directly from the $\log (1 + i)^n$ table.

3.8 Finding the Present Value for Terms Involving a Fraction of a Period. The general procedure in solving for P , when a fraction of a period is involved, is as follows: First discount the final amount S for the smallest number n of whole periods that contain the entire term of the transaction. Call this result y . This method will discount the amount S back to a point earlier than (to the left of) the desired position of P . But y will precede P by only a fraction of a period. Next, accumulate y for this fraction of a period, using simple interest. This gives P .

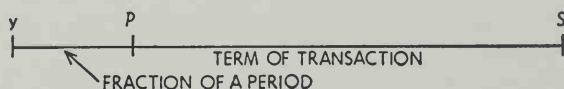
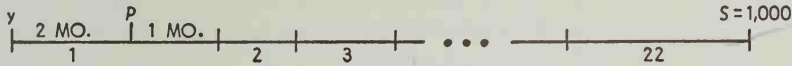


Illustration. Discount \$1,000 for $5\frac{1}{3}$ years at 6 per cent, $m = 4$.

Solution. $5\frac{1}{3}$ years = 5 years and 4 months = 21 quarters plus 1 month. Now the smallest whole number of quarters which contain $21\frac{1}{3}$ quarters is 22. Discount \$1,000 for 22 quarters at $i = \frac{.06}{4} = .015$ per quarter.



$$y = 1,000(1.015)^{-22}$$

Now y is 2 months before (to the left of) P . Then y must be accumulated for 2 months = $\frac{2}{3}$ of a period ($\frac{1}{6}$ of a year) at a simple interest rate of .015 per 3 months (.06 per year).

$$\begin{aligned} P &= y[1 + (.015)(\frac{2}{3})] = y(1.01) \\ P &= 1,000(1.015)^{-22}(1.01) \\ P &= \$727.89 \end{aligned}$$

This calculation may be done by actual multiplication, or by using logarithms.

NOTE: In solving for P or S , always accumulate when using the simple interest formula. In brief, carry the known amount to the nearest end point of an interest period such that this end point lies to the left of the place the final answer is to be. Then reach the final position by accumulating at simple interest for the fractional period remaining.

Exercise 10

1. Accumulate \$1,000 for 8 years and 2 months at 6 per cent, $m = 4$.
 - a) Use simple interest for the fractional period.
 - b) Use compound interest for the fractional period.
2. Discount \$1,000 for 8 years and 2 months at 6 per cent, $m = 4$.
 - a) Use simple interest method for the fractional period.
 - b) Use compound interest for the entire time.
3. If interest is at 8 per cent, $m = 4$, find the present value of \$10,000 due $9\frac{1}{3}$ years from now.
4. \$3,000 is invested at 5 per cent, $m = 4$. How much is in the account at the end of $10\frac{2}{3}$ years?
5. Discount \$3,000 for $8\frac{1}{4}$ years at 5 per cent effective.
6. Discount \$3,000 for $8\frac{1}{4}$ years at 5 per cent, $m = 4$.
7. On May 1, 1953, Mr. I. M. Broke signs a note promising to pay \$5,000 and interest at 5 per cent, $m = 4$, on October 1, 1965. Find the maturity value of this note.
8. The note of Problem 7 is sold on May 1, 1960, at an interest rate of 6 per cent, $m = 2$. Find the purchase price.

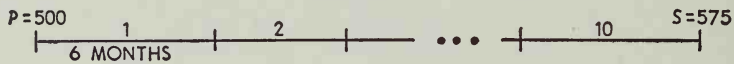
9. What sum of money invested now will accumulate to \$9,000 at the end of 24 years and 4 months if invested at 5 per cent effective?

10. If \$10,000 is invested in an account paying 4 per cent, $m = 2$, how much is in the account at the end of 25 years and 7 months?

3.9 Solving for i and n . In solving for either i or n , two methods are available: interpolation in the compound interest tables or use of logarithms. The following examples will illustrate both methods.

Illustration 1. At what rate of interest compounded semiannually will \$500 amount to \$575 in 5 years?

Solution. Let i = rate per 6 months. Thus $j = 2i$, $m = 2$, $n = 10$ interest periods.



$$500(1 + i)^{10} = 575$$

$$(1 + i)^{10} = \frac{575}{500}$$

By logarithms:

$$\log (1 + i)^{10} = \log \frac{575}{500},$$

$$10 \log (1 + i) = \log 575 - \log 500,$$

$$\log (1 + i) = \frac{\log 575 - \log 500}{10},$$

$$\log (1 + i) = \frac{2.759668 - 2.698970}{10},$$

$$\log (1 + i) = \frac{.060698}{10},$$

$$\log (1 + i) = .0060698.$$

Taking antilogarithms of both sides, using Appendix Table 2 of seven-place mantissas:

$$1 + i = 1.01407,$$

$$i = .01407 \text{ per 6 months,}$$

$$j = 2i = .0281, m = 2.$$

Unless otherwise directed, all interest in this text will be calculated correct to four decimal places. The antilogarithm was computed from Table 2 in the Appendix, a seven-place table of mantissas. This table is found immediately following the six-place table, Table 1, in the Appendix.

This same problem may be computed by interpolation in the compound interest tables.

$$(1 + i)^{10} = \frac{575}{500}$$

$$(1 + i)^{10} = 1.1500$$

In the table for $(1 + i)^n$ find the row opposite $n = 10$ and in this row look for the number 1.1500. Since it does not occur in this row, take the number just smaller and the one just larger than 1.1500 and interpolate between these two numbers. Since the answer is desired to only four-decimal accuracy, it can be shown that only four decimal places need be kept from the entries in the table. Be sure to round off the tabular entries to a four-decimal accuracy. Now 1.1500 lies between 1.14632740 and 1.16054083. Rounding these numbers off to four decimal places and noting that 1.1463 is in the $1\frac{3}{8}$ per cent ($= .01375$) column while 1.1605 is in the $1\frac{1}{2}$ per cent column, the interpolation may be tabulated thus:

$$.00125 \left[\begin{array}{cc} i & (1 + i)^n \\ c \left[\begin{array}{c} .01375 \\ ? \\ .01500 \end{array} \right. & \begin{array}{c} 1.1463 \\ 1.1500 \\ 1.1605 \end{array} \end{array} \right]^{37} 142$$

$$\frac{c}{.00125} = \frac{37}{142}$$

$$c = \frac{(37)(.00125)}{142} = .00033$$

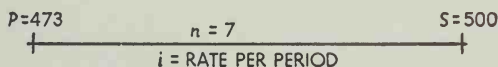
This correction $c = .00033$ must be made to the interest rate .01375. In order to keep the answer between the two interest rates listed, it must be added. Thus

$$i = .01408 \text{ per 6 months.}$$

This answer is very close to the one gotten by logarithms.

In the above interpolation, the decimals were ignored on the right-hand side of the tabulated form, but they were kept on the left-hand or interest rate side. Experience has shown this to be a wise procedure.

NOTE: In interpolation, either form of the compound interest formula may be used. In the above example, $S = P(1 + i)^n$ was used because the division of 575 by 500 was easier than the division of 500 by 575. If the time diagram of a problem were



it would be easier to write

$$473 = 500(1 + i)^{-7},$$

$$(1 + i)^{-7} = \frac{473}{500} = .9460.$$

The table for $(1 + i)^{-n}$ would then be used in the interpolation.

Both methods may also be used to solve for n . But the results have a different interpretation depending on which method is used. If logarithms are used, the interpretation is that compound interest was used not only for the whole periods but also for the fraction of a period involved. On the other hand, if interpolation is used, then compound interest was used for the whole period, but simple interest was used for the fraction of a period.

Illustration 2. How long will it take \$1,000 to amount to \$1,200 at 4 per cent, $m = 2$?

Solution. Let n = number of 6-month periods, $i = .02$ per 6 months, $P = 1,000$, and $S = 1,200$.



$$1,000(1.02)^n = 1,200$$

$$(1.02)^n = \frac{1,200}{1,000}$$

Taking logarithms of both sides,

$$n \log 1.02 = \log 1200 - \log 1000 ,$$

$$n = \frac{\log 1200 - \log 1000}{\log 1.02} ,$$

$$n = \frac{3.079181 - 3.000000}{.008600} ,$$

$$n = \frac{.079181}{.008600} ,$$

$$n = 9.207 \text{ 6-month periods.}$$

Thus the time is 4.6 years. Since logarithms were used, it is assumed that compound interest was used for the .2 period involved.

The solution by interpolation is left as an exercise for the reader.

Exercise 11

1. Solve Illustration 2 by interpolation.
2. At what rate of interest compounded quarterly will \$1,000 amount to \$1,297 in 5 years?
3. At what rate of interest compounded monthly will \$500 amount to \$650 in 6 years?
4. If \$497.32 is the present value of \$600 due at the end of 4 years, find the rate of interest compounded semiannually.
5. If \$517.43 invested for 8 years gives a maturity value \$700, find the effective rate of interest used.

6. How long will it take Mr. Smith to accumulate \$5,000 if he deposits \$3,000 in an account paying interest at 5 per cent, $m = 2$?

7. Mr. Jones deposits \$10,000 in an account paying interest at 4 per cent, $m = 4$. How long will it be until this account has increased to \$14,000?

8. How long will it take money to double itself at (a) 6 per cent, $m = 4$? (b) 5 per cent effective? and (c) 5 per cent simple interest?

9. When must a payment of \$7,000 be made to cancel a debt of \$6,000 which bears interest at 4 per cent, $m = 4$?

10. Find the effective rate of interest earned by an investment of \$750 which has a value of \$1,000 ten years later.

11. At what rate compounded monthly will money double itself in 15 years?

12. How long will it take \$1,000 to amount to \$1,275 at 7.2 per cent per year?

13. Mr. E. W. Smith has just received a gift of \$10,000. He can invest his money at 4 per cent compounded semiannually. As soon as he has accumulated \$15,000, he will purchase a house. How long will he have to wait?

14. At the birth of his son, Mr. Jones deposits \$5,000 in an account paying interest at 6 per cent, $m = 4$. His son is to be given this money when it amounts to \$7,500. How old will the son be then?

15. How long will it take \$750 to accumulate to \$3,000 at 4 per cent compounded semiannually?

16. How long will it take \$600 to accumulate to \$2,000 at 6 per cent compounded quarterly?

17. If Mr. Ashley deposits \$2,000 now and has \$3,200 at the end of 10 years, what rate of interest, compounded semiannually, has he earned?

18. \$2,000 now will accumulate to \$4,000 in 15 years. Find the rate, compounded quarterly, that is used.

19. If Mr. Jones deposits \$1,000 now and has \$1,500 at the end of 12 years, what rate of interest, compounded monthly, has he earned?

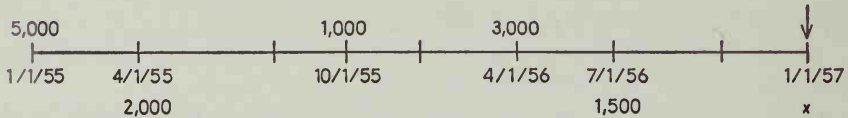
20. How long will it take \$100 to accumulate to \$10,000 at 3 per cent compounded semiannually?

3.10 Equations of Value. Money changes value with the passing of time. As time goes on, any sum of money could be invested (theoretically at least) at the current rate of interest and increases in size as the interest is earned and added to it. Thus, what is worth \$100 today was worth less, say, 5 years ago, because the 100 now includes the value 5 years ago and the interest earned in the meantime.

The following example will show how this idea is used in financial transactions.

Illustration 1. On January 1, 1955, Mr. Jones deposits \$5,000 in a bank paying interest at 4 per cent, $m = 4$. He makes deposits of \$1,000 on October 1, 1955, and \$3,000 on April 1, 1956. His withdrawals are as follows: \$2,000 on April 1, 1955, and \$1,500 on July 1, 1956. How much is in his account on January 1, 1957?

Solution. Let x be the amount left in the account on January 1, 1957. This problem may be diagramed as follows:



NOTE: In order to give a clear visual separation of the deposits and withdrawals, the deposits are placed on one side of the line, while the withdrawals are placed on the other side. Since x is the amount in the account on January 1, 1957, it is the size of the withdrawal that could be made at that time. Thus it is placed on the line diagram with the withdrawals.

You might be tempted to work this problem in the following manner. But this illustration should suffice to show that it is far too cumbersome. A second, much shorter and much more powerful, method will then be shown.

Since the \$5,000 accumulates interest for one period before the first withdrawal, the account will contain $5,000(1.01)^1 - 2,000$ just after the withdrawal on April 1, 1955.

This accumulates interest for two periods giving

$$[5,000(1.01)^1 - 2,000](1.01)^2.$$

To this is added a deposit of \$1,000. Thus on October 1, 1955, the account contains $[5,000(1.01)^1 - 2,000](1.01)^2 + 1,000$, and this accumulates interest for two periods. After the 3,000 is deposited on April 1, 1956, the account will contain $\{[5,000(1.01)^1 - 2,000](1.01)^2 + 1,000\}(1.01)^2 + 3,000$. To this is added interest for one period, and then 1,500 is subtracted. This gives, on July 1, 1956,

$$[\{[5,000(1.01)^1 - 2,000](1.01)^2 + 1,000\}(1.01)^2 + 3,000](1.01)^1 - 1,500.$$

Now this amount accumulates for two periods, and this gives the amount x in the bank on January 1, 1957.

$$x = \{ [\{ [5,000(1.01)^1 - 2,000](1.01)^2 + 1,000 \}(1.01)^2 + 3,000 \}(1.01)^1 - 1,500 \}(1.01)^2$$

The reader can verify that by removing the various forms of parentheses, this equation reduces to

$$x = 5,000(1.01)^8 - 2,000(1.01)^7 + 1,000(1.01)^5 + 3,000(1.01)^3 - 1,500(1.01)^2.$$

This equation may be rearranged thus:

$$x = 5,000(1.01)^8 + 1,000(1.01)^5 + 3,000(1.01)^3 - 2,000(1.01)^7 - 1,500(1.01)^2.$$

This discussion should certainly show clearly that this method is too awkward and tedious for any problem.

The same results may be obtained quickly and easily by the following method. Select any date as a *comparison date* or *focal date*. For example, suppose January 1, 1957, is selected. This date is marked by an arrow ↓ on the line diagram. Next move each item (with no exception) to this date by using the given rate of compound interest, here .01 per quarter. Any item that must be moved to the right to get to the comparison date is accumulated by the given compound interest rate for the number of periods it must be moved from where it is to the comparison date. If it must be moved to the left to get to the comparison date, it is discounted at the given compound interest rate for the number of interest periods through which it is moved. For the comparison date selected above, each item will be moved to the right, except x which does not have to be moved. The sum of the deposits evaluated on this date will be

$$5,000(1.01)^8 + 1,000(1.01)^5 + 3,000(1.01)^3.$$

Likewise, the sum of the withdrawals (counting x as a withdrawal) evaluated on this same date will be

$$x + 2,000(1.01)^7 + 1,500(1.01)^2.$$

Since the sum of the deposits should equal the sum of the withdrawals (including x), these two expressions may be equated.

$$x + 2,000(1.01)^7 + 1,500(1.01)^2 = 5,000(1.01)^8 + 1,000(1.01)^5 + 3,000(1.01)^3$$

Solving for x ,

$$x = 5,000(1.01)^8 + 1,000(1.01)^5 + 3,000(1.01)^3 - 2,000(1.01)^7 - 1,500(1.01)^2.$$

This is exactly the value found for x by the very cumbersome method used at first.

Such an equation as the one above is called an *equation of value*; that is, it shows the equality of the values of two sets of obligations on some specific date.

The date of comparison, called the *comparison* or *focal date*, may be selected completely at will. The selection of the date will have no effect on the answer. Precisely the same value will be found for x , no matter what comparison date is chosen. The reader should check this statement by choosing some other comparison date, say Janu-

ary 1, 1955, and solving for x . Of course, less arithmetic may be needed in the calculations for one comparison date than would be required for another such date. But the answer will be precisely the same.

Actually, this equation of value idea is the fundamental basis of the entire work in financial mathematics. A check should convince you that it has really been used in all the previous problems in this book, and that it will be used in all the remaining parts of this study.

The equation of value procedure can be summed up as follows: If two sets of obligations are to be equivalent, place each item in its proper place on a line diagram. In order to clearly distinguish one set of obligations from the other, place one set above the line and the other set below the line. (Using different colors for writing each set would achieve the same results.) Next select a comparison date you consider most convenient. No matter what comparison date you choose, the solution for unknowns will give the same result. Now move every item—do not omit a single one—to this comparison date, using the interest rate agreed upon. Then equate the sum of the one set on this date to the sum of the values of the other set on this same date. This gives the equation of value from which the value of the unknown can be found.

NOTE: If any of the items are interest-bearing obligations, the maturity value of this item must be placed at its maturity date on the line diagram, and this maturity value must be moved to the comparison date by the same interest rate used to move each of the other items there.

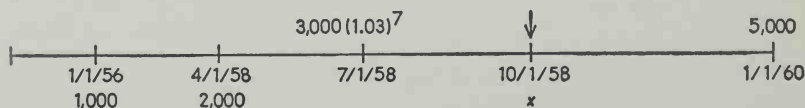
Illustration 2. Mr. Jones owes these two debts: (a) \$5,000 due on January 1, 1960; and (b) \$3,000 due on July 1, 1958, with interest at 6 per cent, $m = 2$, from January 1, 1955.

He wishes to replace these debts by the following set of payments: \$1,000 on January 1, 1956; \$2,000 on April 1, 1958; and the balance on October 1, 1958. Find this balance if money is worth 4 per cent, $m = 4$.

Solution. Let x be the balance due on October 1, 1958. Now, the \$3,000 debt is an interest-bearing debt, and its maturity value will be

$$\$3,000(1.03)^7$$

on July 1, 1958. This is the figure to be put on the diagram. This diagram becomes:



The statement that money is worth 4 per cent, $m = 4$, means that this is the interest rate that will be used to move each item to the focal or comparison date. Suppose October 1, 1958, is taken as the comparison date. It is marked by the arrow on the diagram. Then each item is to be moved to this date at $i = \frac{.04}{4} = .01$ per 3 months. Some items will have to be moved forward (to the right), while others will have to be moved to the left. Moving these items to the focal date, and equating the sum of those of one set (above the line) to the sum of those of the other set (below the line), we have

$$x + 2,000(1.01)^2 + 1,000(1.01)^{11} = 3,000(1.03)^7(1.01)^1 + 5,000(1.01)^{-5}.$$

Solving for x ,

$$x = 3,000(1.03)^7(1.01) + 5,000(1.01)^{-5} - 1,000(1.01)^{11} - 2,000(1.01)^2.$$

This arithmetic computation may be done, using the tables for $(1 + i)^n$ and $(1 + i)^{-n}$. This gives:

$$x = \$5,327.98.$$

The reader should choose another comparison date and calculate x , thus verifying that the same value for x is obtained by using this new focal date as was obtained in the above illustration.

Exercise 12

1. Complete the solution of Illustration 1 by computing the value of x as given in the last equation in this illustration.

2. By choosing July 1, 1958, as a focal date, solve Illustration 2. Which choice of a focal date gives an easier arithmetic computation?

3. Mr. Green owes these two debts: (a) \$4,000 due on October 1, 1962, with interest from April 1, 1956, at 6 per cent, $m = 2$; and (b) \$5,000 due on April 1, 1964.

Mr. Green wishes to replace these debts by the following set of payments: \$2,000 on October 1, 1958; \$4,000 on April 1, 1962; and the balance on April 1, 1963. Find this balance if money is worth 8 per cent, $m = 4$.

4. On January 1, 1957, Mr. A. B. Smith borrows \$10,000 which is to be paid back on July 1, 1964, with interest at 6 per cent, $m = 4$. Instead of waiting until that date to pay, he agrees to make three equal payments, the first on October 1, 1958, the second on July 1, 1960, and the third on January 1, 1963. Find the size of these payments if money is worth 4 per cent, $m = 4$.

5. In order to have \$5,000 in an account at the end of 10 years, Mr. Smith deposits \$2,000 now, \$1,000 at the end of 3 years, and two equal deposits, one at the end of 6 years, the other at the end of 8 years. If the account pays interest at $1\frac{1}{2}$ per cent per annum, find the size of these equal deposits.

6. Mr. Upjohn wishes to refinance his debts. He owes (a) \$2,000 due 5 years from now; and (b) \$8,000 due 8 years from now with interest from today at 7 per cent, $m = 2$.

Mr. Upjohn agrees to pay in this manner: \$1,000 now, \$3,000 at the end of 6 years, and the balance to be paid 7 years from now. He agrees that interest will be computed at 5 per cent, $m = 4$. Find the size of the unknown payment.

7. Mr. Irvin Willoby purchases a motorboat which is priced at \$500 cash, or \$200 down and 3 easy equal monthly payments. The first payment is due 1 month after the down payment. Find the size of these payments if interest is charged at 12 per cent, $m = 12$.

8. Mr. Willie Simpson had three children, aged 3, 7, 9, respectively, at the time of his death. In his will, he left \$50,000 to be invested in a trust fund for these three children, with the stipulation that they should receive equal amounts as they reach the age of 21. If the trust fund earns interest at .025, $m = 2$, what does each child receive at the age of 21?

9. Mr. Jones purchases a \$5,000 automobile and pays \$2,000 cash. He agrees to pay the balance in two installments, one a year later and the other 2 years from now. If interest is at 6 per cent, $m = 2$, and the second payment is to be twice the size of the first payment, find the size of each payment.

10. Mr. Riley has \$2,000 in a savings account that pays interest at 2 per cent, $m = 4$. If he wishes to have \$4,000 in his account at the end of one more year, what equal deposits must be made at the end of each quarter?

11. Mr. Ozzie Nelson deposits \$5,000 in an account today. Three years later he deposits \$3,000. One year from now he withdraws \$2,000, and 4 years from now he withdraws \$1,500. If the account pays interest at 4 per cent, $m = 4$, how much is left in the account 5 years from now?

12. Mr. W. K. Wilson owes these debts: (a) \$15,000 due on April 1, 1965; and (b) \$8,000 due on October 1, 1960, with interest from April 1, 1952, at 6 per cent, $m = 2$.

Mr. Wilson desires to pay these debts by paying \$5,000 on July 1, 1955, and two equal payments, the first on January 1, 1958, and the second on July 1, 1963. Find the size of these payments if money is worth 4 per cent, $m = 4$.

13. Mr. Jones wishes to make two payments to repay a debt of \$5,000 due now. The first payment is to be made 1 year from now. The second payment is to be \$500 larger than the first payment, and it is to be made 2 years from now. Find the size of these payments if interest is at 6 per cent, $m = 2$.

14. Mr. Nelson makes three annual deposits of \$500 each into a fund earning interest at 4 per cent, $m = 4$. Find the amount in the fund just after the third deposit.

15. Mr. A. Q. Smith has just inherited \$10,000. This sum is to be invested in a fund earning 4 per cent, $m = 2$. Mr. Smith is to receive \$1,000 at the end of the first year and 3 equal payments. These equal payments are to be made at the end of the second, third, and fourth years. Find the size of these payments.

16. Mr. O. K. Peterson deposited \$3,000 in a savings account on July 1, 1954. On October 1, 1955, he withdraws \$1,000. He deposits \$2,500 on January 1, 1957, and withdraws \$1,500 on April 1, 1958. If the account pays 2 per cent, $m = 4$, find the balance in the savings account on July 1, 1960.

17. Mr. Nelson owes the following debts: (a) \$10,000 due 3 years from now, without interest if paid at that time; and (b) \$6,000 due at the end of 6 years with interest from today at 4 per cent compounded semiannually.

To pay these debts, Mr. Nelson wishes to make a \$2,000 payment 1 year from now and 2 equal payments, one made 2 years from now and the other made 3 years from now. Find the size of these equal payments if money is worth 6 per cent compounded quarterly.

18. You owe the following two debts: (a) a note for \$2,000 due 5 years from now; and (b) a mortgage for \$5,000 due 7 years from now with interest at 5 per cent compounded annually.

Your bank agrees to take over these two debts if you will pay them \$1,000 now and 2 equal payments, the first due 3 years from now and the second 6 years from now. If money is worth 6 per cent, $m = 2$, find the size of these equal payments.

4.1 Definitions. In many applications, a series of equal payments, equally spaced in time, arise and are to be evaluated at some specific date, using a given interest rate. The equation of value discussed in Chapter 3 would work any such problem, but shorter methods will be developed here.

Originally, the word *annuity* meant a set of annual payments. It has now enlarged its meaning to include any set of equal payments made at equal intervals, regardless of the length of the payment interval. Thus we have the definition: An *annuity* is a set of equal payments made at equal intervals of time. The time interval between successive payments is called the *payment interval*. If there is a finite fixed number of payments, the set is called an *annuity certain*, i.e., a certain definite number of payments. If the number of payments is not definitely fixed in advance but depends upon conditions that develop during the time of the annuity payments, the annuity is called a *contingent annuity*. An example of such would be the number of life insurance premiums that are to be paid on a policy. The number is not known in advance but is contingent upon either the death of the insured or else the termination of the maximum paying period, whichever occurs first. A third type of an annuity consists of a set of payments that continue for ever, that is, the payments never stop. This is called a *perpetuity*.

In this text annuities certain and perpetuities will be considered in Chapters 4 through 7. Contingent annuities involving the mathematical theory of probability and life insurance will be studied briefly in Chapters 8 and 9.

In a given problem, if the interest period equals the payment period in length, the annuity is called an *ordinary* or *simple annuity*.

If the length of the interest period is different from the length of the payment period, it is called a *general annuity*. Ordinary annuities will be considered first. General annuities will be studied in Section 4.9.

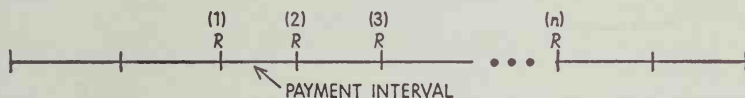
4.2 Amount of Ordinary Annuities Certain. In order to develop formulas for handling ordinary annuities, the following symbols will be used.

R = size of each payment.

n = number of payments.

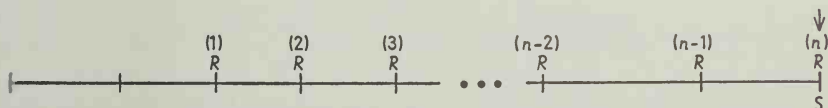
i = the interest rate per payment period.

On a time diagram, these may be recorded:



It is possible to develop a formula that will collect these n payments of R each and carry them to any specified date, using the rate i per payment period. Practice has shown that two dates are used so frequently that it is desirable to have a formula for each of these dates. If the value is desired on any other date, the compound interest formula may be used with these formulas to get the desired value.

The time of the last (or n th) payment will be chosen for one of these formulas. Let S equal the value (including interest) of all n payments at the time of the last payment, and including the last payment. S will be called the *amount* of an annuity.



S may be found by setting up an equation of value, taking the time of S as a comparison date.

Thus

$$S = R + R(1 + i) + R(1 + i)^2 + R(1 + i)^3 + \dots$$

continuing until all n payments R have been moved to the date on which S is. Thus, there would be n terms on the right-hand side of the equation.

The right side of this equation is the sum of a geometric series of n terms, with the first term $a = R$ and the ratio $r = 1 + i$. It is known

from the study of elementary algebra that the sum S of a geometric series

$$a, ar, ar^2, ar^3, \dots \quad (n \text{ terms})$$

is given by the formula

$$S = \frac{ar^n - a}{r - 1}.$$

Substituting in this formula, the right member of the equation of value gives

$$S = \frac{R(1+i)^n - R}{1+i-1} = R \frac{(1+i)^n - 1}{i}.$$

Thus

$$(4) \quad S = R \frac{(1+i)^n - 1}{i}$$

is the formula for the amount of an annuity.

In many cases, it will not be necessary to write out in full the fraction

$$\frac{(1+i)^n - 1}{i}.$$

Instead, an abbreviation or symbol may be used for it. This symbol is written as $s_{\overline{n}|i}$ and reads as "s angle n at i ." Thus

$$s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}$$

is a definition of the symbol $s_{\overline{n}|i}$.

Formula (4) may now be written

$$(4) \quad S = Rs_{\overline{n}|i} \quad \text{or} \quad S = R \frac{(1+i)^n - 1}{i}.$$

$s_{\overline{n}|i}$ could be calculated by using the compound interest tables. But tables have been collected that give directly the value of $s_{\overline{n}|i}$ for many values of i and n . This table will be used whenever it contains the desired values of i and n . Tables are also available for $\log s_{\overline{n}|i}$, and these may be used to eliminate tedious arithmetic computation. Table 6 in the Appendix gives values for both $s_{\overline{n}|i}$ and $\log s_{\overline{n}|i}$ for many values of i and n . $s_{\overline{n}|i}$ is often called the amount of an annuity of \$1 per period.

Illustration 1. \$50 is deposited on the fifteenth of each month in a fund earning interest at 6 per cent, $m = 12$. Find the amount in the fund just after the forty-seventh deposit.

Solution. Here, $R = 50$, $n = 47$, and $i = .005$ per month. The payment period is also 1 month. The time diagram will be:



Since the result desired is at the time formula (4) gives the value S , we have

$$S = 50 \frac{(1.005)^{47} - 1}{.005},$$

$$S = 50s_{\overline{47}|.005},$$

$$S = 50(52.83366390),$$

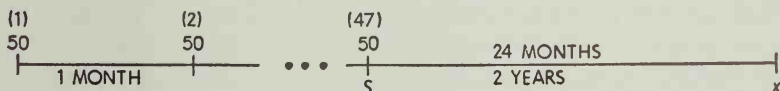
$$S = \$2,641.68.$$

Table 6 in the Appendix was used here to find the value of $s_{\overline{47}|.005}$.

NOTE: For this problem the symbol $s_{\overline{n}|i}$ was sufficient, but in certain later problems it will be necessary to write the formula out in full. Thus, both the symbol $s_{\overline{n}|i}$ and the fraction for which it stands should be learned thoroughly.

Illustration 2. In the problem of Illustration 1, suppose no deposits are made after the forty-seventh deposit but the money is left in the fund. How much would be in the fund 2 years after the last (forty-seventh) deposit?

Solution. Let x be the value in the fund 2 years after the forty-seventh deposit. The time diagram will now be:



Formula (4) will not give the value x , but it will collect the set of 47 payments and deposit them at the point indicated by S on the time line. In Illustration 1 it was found that

$$S = 50s_{\overline{47}|.005}.$$

Now this entire sum S will accumulate interest at .005 per month for 2 years, or 24 interest periods. Using the compound interest formula with $S = P$ and $n = 24$ interest periods,

$$x = (50s_{\overline{47}|.005})(1.005)^{24},$$

$$x = 50(52.83366390)(1.12715978),$$

$$x = (2,641.683195)(1.12715978),$$

$$x = \$2,977.60 \text{ to the nearest cent.}$$

Here, the tables for $s_{\overline{n}|i}$ and $(1+i)^n$ were used.

The tedious multiplication could have been avoided by using logarithms. Taking logarithms of the equation,

$$\begin{aligned}\log x &= \log 50 + \log s_{\overline{47}|.005} + \log (1.005)^{24} \\ \log 50 &= 1.698970 \\ \log s_{\overline{47}|.005} &= 1.722911 \\ \log (1.005)^{24} &= .051986 \\ \log x &= \underline{3.473867} \\ x &= \$2,977.60 \text{ to the nearest 10 cents.}\end{aligned}$$

Exercise 13

1. On the tenth day of each month, Mr. I. M. Sharpe deposits \$25 in an account which pays interest at 4 per cent, $m = 12$. Find the amount in the account just after the forty-fifth deposit.

2. On January 1, 1953, Mr. O. P. Able deposits \$20 in a savings account paying 6 per cent, $m = 12$. On the first day of each month thereafter he makes the same size deposit. Just after his deposit on September 1, 1955, how much is in his account?

3. Mr. Goforth deposits \$100 at the beginning of each quarter into a fund paying interest at 2 per cent, $m = 4$. He makes his first quarterly deposit on April 1, 1954, and his last deposit on July 1, 1959. If no further deposits are made and no withdrawals have been made from the fund, find the amount in this fund on January 1, 1963.

4. Mr. Wilson invests \$500 each 6 months. His investments earn interest at 4 per cent, $m = 2$. After making 32 such deposits, Mr. Wilson stops the deposits but leaves his money invested. Find the amount invested (a) just after the last deposit, and (b) 3 years after the last deposit.

5. On January 1, 1953, Mr. A. B. Smith deposits \$8,000 into an account earning 4 per cent, $m = 4$. On April 1, 1954, and each 3 months thereafter, he deposits \$200. He makes a final deposit of \$200 on July 1, 1959. Find the amount in his account on October 1, 1963.

6. In order to provide money to replace a machine, the A B C Corporation deposits \$1,500 at the end of each 6 months for 12 years. If the fund earns interest at 3 per cent, $m = 2$, find the amount in the fund (a) just after the eighth deposit, and (b) just after the final deposit.

7. To provide for his son's education, a man deposits \$500 at the birth of his son and makes a \$250 deposit on each birthday thereafter. How much does this fund contain just after the deposit made on the seventeenth birthday? Interest is paid at 5 per cent effective.

8. At the birth of his son, Mr. Smith established a fund by depositing \$1,000. This fund was to earn interest at 4 per cent per year and was to be given to the son on his sixteenth birthday. On each birthday anniversary, Mr. Smith deposited \$500. During the year after he had made his ninth \$500 deposit, and before the tenth deposit had been made, Mr. Smith died, and no further deposits were made. How much did the son receive on his sixteenth birthday?

9. Mr. Wilson makes 25 quarterly deposits of \$250 each into a fund paying 6 per cent, $m = 4$. If the first deposit is made on October 1, 1955, and the money is left in the account after the last deposit, find the balance to Mr. Wilson's credit on July 1, 1965.

10. Mr. Jones deposits \$5,000 on April 1, 1962. He makes deposits of \$100 at the end of each 3 months thereafter, until October 1, 1965. He makes a final deposit of \$100 on October 1, 1965. If money is worth 4 per cent, $m = 4$, how much is in his account on July 1, 1968?

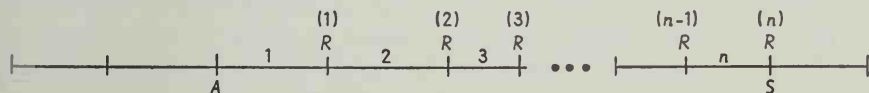
11. Mr. Oscar Smith deposits in a savings account \$100 at the end of each quarter for 5 years. He then deposits \$50 at the end of each quarter for the next 4 years. If the bank pays 2 per cent compounded quarterly, how much does he have in his account at the end of 9 years?

12. Willie Jones deposits \$300 at the end of each 6 months for 4 years. During the next 5 years he makes no deposits. He again makes deposits of \$300 at the end of each 6 months for the next 6 years following that. If the fund earns interest at 3 per cent, $m = 2$, how much is in his account at the end of 15 years?

13. Beginning on July 1, 1962, and on each 3 months thereafter, Mr. Wilson deposits \$75 in an account paying interest at 3 per cent, $m = 4$. If he makes his final deposit on April 1, 1968, find how much is in his account on (a) April 1, 1968; (b) October 1, 1970; and (c) January 1, 1975.

14. A deposit of \$50 is made every month into an account earning 6 per cent, $m = 12$. If the first deposit is made on June 15, 1961, and the last deposit is made on November 15, 1964, find the amount in this account (a) on November 15, 1964; (b) on April 15, 1967; and (c) on September 15, 1968.

4.3 Present Value of Ordinary Annuities. Although formula (4) and the compound interest formula (3) are sufficient for working problems involving annuities, it is very convenient to have another formula which collects the n payments and deposits them at another date. This other date will be taken as one payment period before the first payment. Let A equal the value of all n payments (with interest deducted) one payment period before the first payment.



A formula for A may be calculated by writing an equation of value using the time of A as a comparison date. The side of the equation containing the R 's will again be a sum of a geometric progression, and the work can be completed as in the development of the formula for S .

But since formula (4) for S collects all n of the R 's and deposits

them at the time of S , A can be gotten by simply discounting S for the proper number of interest periods. If each payment is associated with the payment period preceding it, the periods may be numbered as in the diagram. Thus, it is seen that there are n interest periods from the time of S to the time of A .

Therefore,

$$A = S(1 + i)^{-n},$$

$$A = R s_{\overline{n}|i} (1 + i)^{-n},$$

$$A = R \frac{(1 + i)^n - 1}{i} (1 + i)^{-n},$$

$$A = R \frac{1 - (1 + i)^{-n}}{i}.$$

Let

$$a_{\overline{n}|i} = \frac{1 - (1 + i)^{-n}}{i}.$$

This gives formula

$$(5) \quad A = R a_{\overline{n}|i}, \quad \text{or} \quad A = R \frac{1 - (1 + i)^{-n}}{i}.$$

A is called the *present value* of an annuity. The time from A to S is called the *term* of an *ordinary annuity*. It may be noted that the term of an annuity contains exactly as many payment periods as there are payments.

Table 7 in the Appendix gives values for both $a_{\overline{n}|i}$ and $\log a_{\overline{n}|i}$.

Two annuity formulas are now available, each of which will collect the n payments and deposit them at a fixed place. S gives the value at the time of, and including, the last payment, while A gives the value one payment period before the first payment. If the value of these n payments is desired at any other time, first, find either A or S , and then move it by compound interest the necessary interest periods to carry it to the desired place.

Illustration 1. How much money must be deposited now in a bank paying 2 per cent, $m = 2$, in order to make 20 semiannual withdrawals of \$500 each? The first withdrawal is to be made 6 months after the deposit.

Solution. $R = 500$, $i = .01$ per 6 months, $n = 20$. By formula (5),

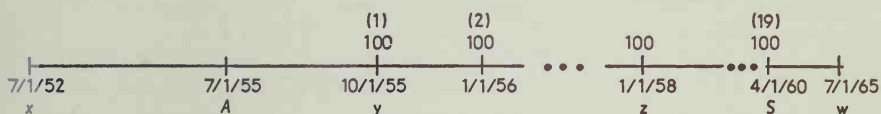
$$A = 500 a_{\overline{20} | .01}, \quad \text{or} \quad A = 500 \frac{1 - (1.01)^{-20}}{.01},$$

$$A = 500(18.04555297),$$

$$A = \$9,022.78.$$

Illustration 2. Mr. Jones has agreed to make quarterly payments of \$100 each. He is to make the first payment on October 1, 1955, and the last payment on April 1, 1960. Instead of making these payments, Mr. Jones desires to make a single payment that will cancel the entire debt. If money is worth 6 per cent, $m = 4$, find the size of this single payment if it is made (a) July 1, 1952; (b) October 1, 1955; (c) January 1, 1958; and (d) July 1, 1965.

Solution. It is necessary to know how many payments there will be. One sure way of getting this number is to count the payments in the "end years," that is, in 1955 and in 1960, and add to this 4 payments for each year between these two. Here, only 1 payment is due in 1955 (on October 1, 1955). In 1960, 2 payments are due. There are 4 payments in each of the intervening years. Thus $n = 1 + 16 + 2 = 19$, $R = 100$, and $i = .015$ per quarter.



Here, neither A nor S is on any of the dates for which a value is desired. But

$$A = 100a_{\overline{19}|.015} \text{ on July 1, 1955;}$$

$$S = 100s_{\overline{19}|.015} \text{ on April 1, 1960.}$$

Let x be the value on July 1, 1952. x can be calculated by discounting either A or S to July 1, 1952. There are 12 interest periods from July 1, 1952, to July 1, 1955, and 31 interest periods from July 1, 1952, to April 1, 1960. Thus

$$x = 100a_{\overline{19}|.015}(1.015)^{-12},$$

or

$$x = 100s_{\overline{19}|.015}(1.015)^{-31}.$$

Now x may be calculated either by direct multiplication of the values found in the tables for $a_{\overline{n}|i}$ (or $s_{\overline{n}|i}$) and the tables for $(1+i)^{-n}$ or by using logarithms. Note that

$$\log(1+i)^{-n} = -\log(1+i)^n.$$

By logarithms, using the first expression for x ,

$$\begin{aligned} \log x &= \log 100 + \log a_{\overline{19}|.015} - \log (1.015)^{12}, \\ x &= \$1373.90 \text{ to the nearest 10 cents.} \end{aligned}$$

In a similar manner, if y is the value on October 1, 1955, then

$$\begin{aligned} y &= 100a_{\overline{19}|.015}(1.015)^1, \\ y &= 100s_{\overline{19}|.015}(1.015)^{-18}. \end{aligned}$$

or

If z is the value on January 1, 1958, then

$$\begin{aligned} z &= 100a_{\overline{19}|.015}(1.015)^{10}, \\ \text{or} \quad z &= 100s_{\overline{19}|.015}(1.015)^{-9}. \end{aligned}$$

Finally, if w is the value on July 1, 1965, then

$$\begin{aligned} w &= 100a_{\overline{19}|.015}(1.015)^{40}, \\ \text{or} \quad w &= 100s_{\overline{19}|.015}(1.015)^{21}. \end{aligned}$$

The arithmetic computation is left as an exercise for the reader.

This last illustration shows how to find the value of an annuity at any time. All that is needed is a combination of either the $a_{\overline{m}|i}$ or the $s_{\overline{m}|i}$ formula with the compound interest formula.

Exercise 14

1. Find the present value of an annuity of 25 quarterly payments of \$100 each. The first payment is made 3 months from now, and interest is at 3 per cent, $m = 4$.

2. Mr. I. O. Yen borrows a sum of money on September 1, 1957. He promises to repay it by making monthly payments of \$20 each. The first payment is to be made on October 1, 1957, and the last payment will be made on February 1, 1962. If interest is charged at 6 per cent, $m = 12$, find the sum Mr. Yen borrowed.

3. How much must be deposited now in order to withdraw \$50 3 months from now and \$50 each 3 months thereafter until 35 withdrawals have been made? Money is worth 2 per cent, $m = 4$.

4. Find the present value of an annuity of 40 monthly payments of \$100 each if money is worth 3 per cent, $m = 12$.

5. Find the value on January 1, 1956, of a set of quarterly payments of \$200 each if the first payment is made on July 1, 1957, and the last payment is made on October 1, 1962. Money is worth 5 per cent, $m = 4$.

6. Mr. Osborn purchases a house, making a down payment of \$4,000 and agreeing to pay \$250 at the end of each quarter for the next 15 years. Find the cash price of the house if interest is computed at 6 per cent, $m = 4$.

7. Find the cash price of a boat which can be bought for \$200 down and 18 monthly payments of \$25 each. The first monthly payment is due 1 month after the down payment, and interest is 9 per cent, $m = 12$.

8. Find the value on May 1, 1962, of a set of monthly payments of \$50 each. The first payment is due on July 1, 1964, and the last payment is to be made on November 1, 1968. Money is worth 4 per cent, $m = 12$.

9. Mr. Jones owes a debt calling for quarterly payments of \$300 each. The first payment is due on October 1, 1956, and the final payment is due on July 1, 1962. Instead of making these payments in this manner, he wished to make a single payment that will cancel the entire debt. If money is worth 5 per cent, $m = 4$, find the size of this single payment

if it is made on (a) January 1, 1955; (b) October 1, 1956; (c) April 1, 1960; or (d) July 1, 1965.

10. Mr. Billy Groate wishes to refinance his debt to the Acme Loan Company by making a single payment that will cancel his entire debt. His debt to the Loan Company calls for semiannual payments of \$500 each, the first due on May 5, 1957, and the last due on May 5, 1965. If Mr. Groate pays interest at 7 per cent, $m = 2$, find the size of this single payment if it is made on (a) November 5, 1955; (b) May 5, 1957; (c) November 5, 1960; or (d) May 5, 1970.

11. Find how much must be deposited now in an account paying 2 per cent, $m = 2$, in order that 12 semiannual withdrawals of \$500 each may be made. The first withdrawal will be made 9 years after the deposit.

12. Find the value now at 6 per cent, $m = 4$, of 25 quarterly payments of \$100 each, the first payment due 11 years from now.

13. Mr. Apex rents a house, paying rent of \$100 a month in advance, with the option of buying it at the end of 3 years. If he buys the house, the rent payments will be considered as payments on the house. If the house has a cash price of \$20,000 at the time Mr. Apex first rents it, how much would he have to pay at the end of 3 years to complete the purchase? Interest is to be computed at 6 per cent, $m = 12$.

14. Mr. Jones, on his forty-fifth birthday, wished to purchase an annuity that will pay \$300 a quarter to him or his estate for 15 years. The first payment is to be made on his sixty-fifth birthday. If money is worth 3 per cent, $m = 4$, find the price Mr. Jones must pay for this annuity.

15. Mr. Beetle will deposit \$20 at the end of each month for the next 3 years, and he will deposit \$30 at the end of each month thereafter for the following 2 years. If the fund earns interest at 3 per cent, $m = 12$, how much is in the fund at the end of the 5 years?

16. Mr. Overby has signed a contract to make a set of quarterly payments of \$500 each, with the first due on October 1, 1965, and the last due on January 1, 1968. If money is worth 4 per cent, $m = 4$, find the single payment that will discharge this entire debt if this single payment is made on (a) July 1, 1963; (b) October 1, 1965; (c) January 1, 1967; or (d) April 1, 1970.

17. You have been promised 12 annual payments of \$1,000 each with the first due on January 1, 1960. If money is worth 3 per cent, $m = 1$, what single payment would be equivalent to the entire set of 12 payments if the single payment is made on (a) January 1, 1956? (b) January 1, 1960? (c) January 1, 1970? and (d) January 1, 1975?

18. You have been promised 10 annual payments of \$1,000 each, with the first due 4 years from now. If money is worth 2 per cent compounded annually, what single payment would be equivalent to the entire set of 10 payments if that single payment were made (a) now? (b) 4 years from now? or (c) 13 years from now?

19. Find the cash price of a house that can be bought for \$5,000 down and \$300 at the end of each 3 months for 6 years. Money is worth 6 per cent, $m = 4$.

20. The purchaser of a farm will pay \$1,500 at the end of each 6 months for 6 years. If money is worth 6 per cent, $m = 2$, find the equivalent cash price of this farm.

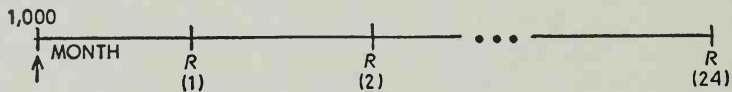
21. Mr. Mayo has promised to pay \$500 at the end of each 6 months for 7 years and \$700 at the end of each 6 months thereafter for the next 5 years. Mr. Mayo wishes to pay this entire debt now (6 months before the first \$500 payment is due). Find the size of this single payment if money is worth 4 per cent, $m = 2$.

22. Mr. Frank is offered a piece of property for \$8,000 cash. He agrees to make a cash payment and to retire the remaining obligation by making 12 semiannual payments of \$500 each, with the first \$500 payment being made $3\frac{1}{2}$ years from now. What should the cash payment be if money is worth 6 per cent, $m = 2$?

4.4 Finding the Periodic Payment R . It is often necessary to find the size of the equal payments R such that an annuity will have a given value on a specified date. In this case, the problem is set up just as in the preceding section, with R as the unknown. The resulting equation is solved for R .

Illustration 1. After making his down payment on a purchase, Mr. Smith still owes \$1,000. To pay this, he agrees to make equal monthly payments, beginning 1 month after the date of the purchase, for 24 months. If interest is charged at 12 per cent, $m = 12$, find the size of each of these payments.

Solution. Let R equal the size of each payment, $n = 24$ equals the number of payments, and $i = .01$ per month. The payment period and the interest period are the same.



If the time of the purchase is taken as the comparison date, then

$$Ra_{\overline{24}|.01} = 1,000,$$

$$R = \frac{1,000}{a_{\overline{24}|.01}}.$$

$a_{\overline{24}|.01}$ can be read from Table 7 in the Appendix and divided into 1,000 to get the value of R . But tables have been constructed, the use of which makes possible a multiplication rather than a division. Let the above equation be written

$$R = 1,000 \frac{1}{a_{\overline{24}|.01}}.$$

Table 8 in the Appendix gives values for $\frac{1}{a_{\overline{n}|i}}$. Thus $\frac{1}{a_{\overline{24}|.01}}$ may be read directly from this table and then multiplied by \$1,000.

$$R = 1,000 \frac{1}{a_{\overline{24}|.01}} = 1,000(.04707347)$$

$$R = \$47.07$$

In some problems, when solving for R , the symbol $\frac{1}{s_{\overline{n}|i}}$ also appears. Tables for $\frac{1}{s_{\overline{n}|i}}$ exist, but they are not necessary. A simple relationship between $\frac{1}{a_{\overline{n}|i}}$ and $\frac{1}{s_{\overline{n}|i}}$ exists, and this relationship makes it so that either can be obtained easily from the table of the other. This relationship may be developed as follows:

Since $s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}$, then $\frac{1}{s_{\overline{n}|i}} = \frac{i}{(1+i)^n - 1}$ and $a_{\overline{n}|i} = \frac{1 - (1+i)^{-n}}{i}$, then $\frac{1}{a_{\overline{n}|i}} = \frac{i}{1 - (1+i)^{-n}}$, it follows that

$$\frac{1}{a_{\overline{n}|i}} - \frac{1}{s_{\overline{n}|i}} = \frac{i}{1 - (1+i)^{-n}} - \frac{i}{(1+i)^n - 1}.$$

If the first fraction on the right side of this equation has its numerator and denominator multiplied by $(1+i)^n$, the two fractions will have a common denominator and the subtraction may be performed.

$$\begin{aligned} \frac{1}{a_{\overline{n}|i}} - \frac{1}{s_{\overline{n}|i}} &= \frac{i(1+i)^n}{[1 - (1+i)^{-n}](1+i)^n} - \frac{i}{(1+i)^n - 1} \\ &= \frac{i(1+i)^n}{(1+i)^n - 1} - \frac{i}{(1+i)^n - 1} \\ &= \frac{i(1+i)^n - i}{(1+i)^n - 1} \\ &= \frac{i[(1+i)^n - 1]}{(1+i)^n - 1} \end{aligned}$$

Therefore

$$\frac{1}{a_{\overline{n}|i}} - \frac{1}{s_{\overline{n}|i}} = i.$$

By transposing, this gives equation

$$(6) \quad \frac{1}{s_{\overline{n}|i}} = \frac{1}{a_{\overline{n}|i}} - i.$$

This says that for each interest rate i , $\frac{1}{s_{\overline{n}|i}}$ can be gotten by merely sub-

tracting this rate i from the corresponding value for $\frac{1}{a_{\overline{n}|i}}$. For most interest rates, this subtraction may be performed mentally as the number is read from the table.

Illustration 2. How much money must be deposited quarterly into a bank paying 2 per cent, $m = 4$, in order to have \$1,000 just after the twelfth deposit?

Solution. Let R equal the size of the deposit, $i = .005$ per month, and $n = 12$ payments.



$$Rs_{\overline{12}|.005} = 1,000$$

$$R = \frac{1,000}{s_{\overline{12}|.005}} = 1,000 \cdot \frac{1}{s_{\overline{12}|.005}}$$

Now $\frac{1}{a_{\overline{12}|.005}} = .08606643.$

Therefore $\frac{1}{s_{\overline{12}|.005}} = .08606643 - .005 = .08106643,$

$$R = 1,000(.08106643),$$

$$R = \$81.07.$$

NOTE: Tables for $\log \frac{1}{a_{\overline{n}|i}}$ and $\log \frac{1}{s_{\overline{n}|i}}$ are not necessary, since division by use of logarithms is just as easy as multiplication. Thus the tables for $\log a_{\overline{n}|i}$ and $\log s_{\overline{n}|i}$ may be used. Thus in the last illustration, the logarithmic computation could be accomplished by using

$$\log R = \log 1000 - \log s_{\overline{12}|.005}.$$

Exercise 15

1. Mr. Smith purchases a car whose cash price is \$3,500. He pays \$1,500 cash and agrees to pay the balance in 8 equal quarterly payments. If the first quarterly payment is made 3 months after the down payment and interest is charged at 8 per cent, $m = 4$, find the size of the quarterly payment.

2. Bill Jones was given \$4,000 to provide for 4 years of college education. If he deposits this money in an account paying 6 per cent, $m = 12$, how much can be withdrawn from this account at the end of each month for the entire 4 years? The first withdrawal is to be made 1 month after the deposit, and there is to be no money left in the account at the end of 4 years.

3. In order to have \$10,000 in a savings account at the end of 15 years, how much must be deposited at the end of each 6 months? Interest is at 3 per cent, $m = 2$.

4. Bill Jones has a note for \$5,000 which matures 8 years from now. In order to have the \$5,000 available to pay this note, he makes equal monthly deposits into a fund earning interest at 6 per cent, $m = 12$. If the deposits are made at the end of each month, find the size of each deposit.

5. In order to pay a debt of \$8,000 due now, Bob Smith agrees to pay \$1,000 now, \$2,000 at the end of a year, and the balance in 5 equal annual payments. The first of these equal payments will be made 2 years from now. If money is worth 7 per cent effective, find the size of these payments.

6. A deposit of \$3,000 is made now and a deposit of \$2,000 is made 6 months from now into a fund earning 4 per cent, $m = 2$. At the end of each 6 months thereafter a fixed sum is deposited. After 6 of these equal deposits, the account has a balance of \$10,000. Find the size of these equal deposits.

7. The cash price of a house is \$15,000, and money is worth 7 per cent, $m = 2$. Mr. Holmes wishes to pay for this house with 40 equal semiannual payments, the first due at the time of purchase. Find the size of these payments.

8. Mr. Joe Edmunds retires at the age of 65. At this time he has \$12,000 to his credit in the retirement fund. He elects to leave this money in the fund and to have equal payments made to him or to his estate for the next 15 years. If the fund pays interest at 3 per cent, $m = 1$, find the size of these payments. The first payment is to be made 1 year after he retires. They are to be made annually.

9. A fund which contains \$20,000 and pays interest at 3 per cent, $m = 2$, is to provide for 50 equal semiannual payments. Find the size of these payments if the first payment is made (a) now, (b) 6 months from now, and (c) 4 years from now.

10. Bill Akers wishes to purchase a motorboat priced at \$800. Instead of paying cash, he desires to make 18 equal monthly payments, the first due now. Find the size of these payments if interest is charged at 12 per cent, $m = 12$.

11. Mr. Akers purchases the boat on the installment plan as described in Problem 10. When the tenth payment comes due, he desires to pay this payment and all the balance of the debt. Find the total payment he would have to make at this time.

12. Mr. Apex wishes to provide \$300 each quarter for 4 years toward the education of his son. He desires that his son will receive the first \$300 on his sixteenth birthday and the remaining payments at the end of each 3 months thereafter until he has received all sixteen. In order to accumulate this money, Mr. Apex deposits a sum of money at the time of the birth of his son and continues to deposit the same amount at the end of each 3 months thereafter until the boy reaches the age of 12. He makes his final

deposit on that birthday. If money is worth 4 per cent, $m = 4$, find the size of Mr. Apex's deposits.

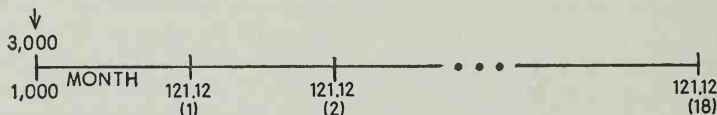
13. In order to save enough to purchase a \$2,500 car 3 years from now, Mr. Overstreet makes equal monthly deposits into an account paying 6 per cent, $m = 12$. If the first deposit is made 1 month from now and the last at the end of 3 years, find the size of these deposits.

14. A house is priced at \$18,000 cash. You agree to pay \$6,000 cash and to make 24 equal semiannual payments, with the first of these equal payments being made 6 months from now. Find the size of these payments if money is worth 6 per cent, $m = 2$.

4.5 Finding an Unknown Interest Rate. If the size and number of the payments of an annuity and either the present value or the amount of this annuity are known, the interest rate used may be found by interpolation, using either the $a_{\overline{m}|i}$ or the $s_{\overline{m}|i}$ table. This is the same method as was used in Chapter 3 to find i from the compound interest tables. In using interpolation in any of these tables, the tabular entries may be rounded off to four decimal places and still give reasonable accuracy, i.e., accuracy to four decimal places in the value of i .

Illustration. Bill Smith buys a \$3,000 automobile, pays \$1,000 down, and agrees to pay \$121.12 at the end of each month thereafter for 18 months. What rate of interest, compounded monthly, is he paying?

Solution. Let i = rate per month.



The 18 payments of \$121.12 form an annuity whose present value A at an interest rate i per month is given by

$$A = 121.12a_{\overline{18}|i}.$$

This value is on the date \$1,000 is paid. Taking this date as a comparison date,

$$1,000 + 121.12a_{\overline{18}|i} = 3,000,$$

$$121.12a_{\overline{18}|i} = 2,000,$$

$$a_{\overline{18}|i} = \frac{2,000}{121.12},$$

$$a_{\overline{18}|i} = 16.5125.$$

In the table for $a_{\overline{n}|i}$, find 18 in the row headed by n . In this row look for 16.5125. It lies between 16.5872 in the $\frac{7}{8}$ per cent column and 16.3983 in the 1 per cent column. Thus i lies between $\frac{7}{8}$ per cent and 1 per cent per month. Since $\frac{7}{8}$ per cent equals .00875, the interpolation can be arranged thus:

$$\begin{array}{cc}
 \text{Rate} & a_{\overline{18}|i} \\
 .00125 \left[\begin{array}{c} \left[\begin{array}{c} .00875 \\ ? \\ .01 \end{array} \right. & \begin{array}{c} 16.5872 \\ 16.5125 \\ 16.3983 \end{array} \end{array} \right]^{747} & 1,889 \\
 \frac{c}{.00125} = \frac{747}{1,889}, & \\
 c = \frac{(747)(.00125)}{1,889} = .00049. &
 \end{array}$$

This correction must be added to .00875. This gives:

$$\begin{aligned}
 i &= .00924 \text{ per month,} \\
 j = 12i &= .1109, \quad m = 12.
 \end{aligned}$$

Thus Mr. Smith is paying 11.09 per cent, $m = 12$, interest for the financing of his car.

NOTE: The tedious division, $\frac{2,000}{121.12}$, could have been avoided here by inverting both sides of the equation, giving

$$\frac{1}{a_{\overline{18}|i}} = \frac{121.12}{2,000} = .06056.$$

By interpolation ,

$$\begin{array}{cc}
 \text{Rate} & \frac{1}{a_{\overline{18}|i}} \\
 .00125 \left[\begin{array}{c} \left[\begin{array}{c} .00875 \\ ? \\ .01 \end{array} \right. & \begin{array}{c} .06029 \\ .06056 \\ .06098 \end{array} \end{array} \right]^{27} & 69 \\
 \frac{c}{.00125} = \frac{27}{69}, &
 \end{array}$$

$$c = \frac{27(.00125)}{69} = \frac{.03375}{69} = .00049,$$

$$\begin{aligned}
 i &= .00875 + .00049 = .00924 \text{ per month,} \\
 j = 12i &= .1109, \quad m = 12.
 \end{aligned}$$

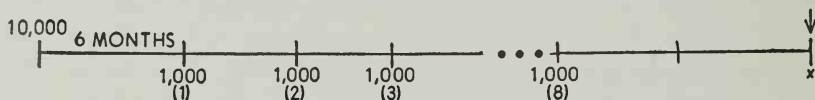
This is the same answer, and the calculations were not as tedious.

If more decimal places are desired in the interest rate, more decimal places must be used in the interpolation.

4.6 Finding the Size of a Final Payment. A direct application of the equation of value will give the size of a final payment. The following illustration will show the procedure.

Illustration. Mr. Abel borrows \$10,000. At the end of each 6 months thereafter he pays \$1,000 until he has made 8 such payments. One year after the eighth payment, he pays the balance of the debt. Find the size of this final payment if interest is computed at 6 per cent, $m = 2$.

Solution. Let x be the size of the final payment. The line diagram will be:

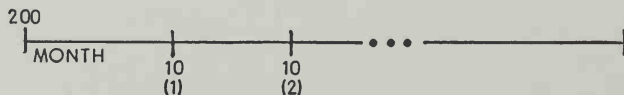


Taking the time of the final payment as the comparison date, the equation of value is:

$$\begin{aligned} x + 1,000 s_{\overline{8}|.03} (1.03)^2 &= 10,000(1.03)^{10}, \\ x &= 10,000(1.03)^{10} - 1,000 s_{\overline{8}|.03} (1.03)^2, \\ x &= 13,439.16 - 9,433.88, \\ x &= \$4,005.28. \end{aligned}$$

This final payment was made 1 year, or two regular payment periods after the last payment of the annuity. Therefore, the value S of the annuity at the time of the last payment had to be moved forward two periods by compound interest.

4.7 Finding the Number of Payments. If \$200 is borrowed at 6 per cent, $m = 12$, and is repaid by monthly payments of \$10 each, it is very unlikely that an exact multiple of \$10 payments would repay the principal and interest. What is likely to happen is that a final payment of a different size must be made. For suppose n = number of payments necessary to repay this debt. Then,



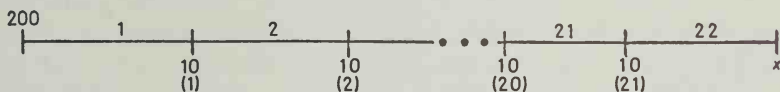
$$\begin{aligned} 10a_{\overline{n}|.005} &= 200 \\ a_{\overline{n}|.005} &= 20 \end{aligned}$$

From the table for $a_{\overline{n}|i}$,

$$a_{\overline{21}|.005} = 19.88797925,$$

$$a_{\overline{22}|.005} = 20.78405896.$$

This says that 21 payments of \$10 each are not enough to settle the debt, but 22 payments of \$10 each will be too much. Thus 21 payments of \$10 plus a final payment (which in this case will be smaller than \$10) will be necessary to settle this obligation. It is customary to agree that this final smaller payment will be made one full payment period after the last full payment. Unless something is said to the contrary, that will be the assumption in this book. The size of this final payment may be calculated as in the preceding section. Thus, if x is the size of the final payment,



$$x + 10s_{\overline{21}|.005}(1.005) = 200(1.005)^{22}$$

$$x = 200(1.005)^{22} - 10s_{\overline{21}|.005}(1.005)^1$$

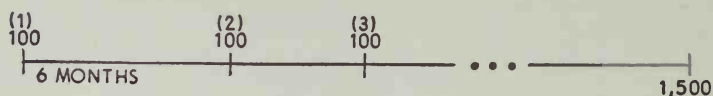
$$x = 223.19 - 221.94$$

$$x = \$1.25$$

The same procedure may be followed in calculating n and the size of a final payment when a fund is being accumulated and $s_{\overline{n}|i}$ is being used. In this case, it sometimes happens that when n is found to lie between, say 17 and 18, it will be found that the 17 deposits will be so close to the desired amount that interest alone will build up the fund to the desired amount, or even beyond that amount, without having to make a final deposit. But the algebra will take care of this situation, even if it is not detected ahead of time. For in this case, the size of the final payment will come out to be either 0 or negative in size. This says that the interest alone has just completed the fund, or else the interest alone has built the fund up beyond the desired amount. In this last case, the negative answer tells how much could be withdrawn (not deposited) in order to have the exact desired amount in the fund.

Illustration. To accumulate \$1,500, Mr. Wilson deposits \$100 each 6 months into a savings account paying 2 per cent, $m = 2$. Find the number of deposits necessary and the size of the final deposit to be made 1 month after the last \$100 deposit.

Solution. Let n be the number of deposits.

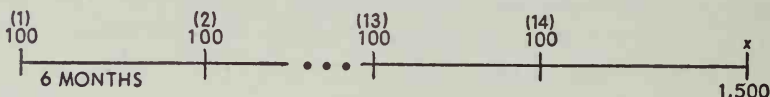


$$100s_{\overline{n}|.01} = 1,500$$

$$s_{\overline{n}|.01} = 15$$

By the tables, $14 < n < 15$.

This answer suggests that 14 payments of \$100 are not enough, and that a fifteenth smaller payment would be made. To calculate this final payment:



$$x + 100s_{\overline{14}|.01}(1.01) = 1,500$$

$$x = 1,500 - 100s_{\overline{14}|.01}(1.01)$$

$$x = 1,500 - 1,509.69$$

$$x = -\$9.69$$

This says that it is not necessary to make any payment 6 months after the fourteenth payment because the account already has \$9.69 more in it than the \$1,500 desired. That is to say, you could withdraw \$9.69 and still have the desired \$1,500.

In other cases, a final deposit may be necessary, and the above method will find such a payment.

Exercise 16

1. On June 25, 1957, Mr. I. N. Hair borrows \$250 from his local bank. To repay this debt, Mr. Hair makes 12 monthly payments of \$22.25 each, with the first payment being made on July 25, 1957. What rate of interest, compounded monthly, is Mr. Hair paying for the use of this money?

2. The Easy Credit Company makes a \$200 loan which is repaid by 8 monthly payments of \$26.25. The first monthly payment was due 1 month after the loan was made. Find the rate of interest, compounded monthly, that is being charged.

3. Mr. Wilson purchases an automobile and after making the down payment still owes \$2,000. He wishes to finance this by making equal payments at the end of each month after the purchase for 18 months. He is told that the payments will be \$126.11. What interest rate, $m = 12$, is he being charged?

4. A \$500 frozen food locker may be bought for \$100 down and \$36 at the end of each month for a year. What rate of interest, compounded monthly, is being charged?

5. Mr. Smith purchases an \$18,000 house and makes a down payment

of \$6,000. He agrees to pay the balance by paying \$450 at the end of each 6 months for 20 years. At what rate, compounded semiannually, is the interest computed?

6. Mr. Goforth, after making the down payment, still owes \$10,000 on his house. He pays the Building and Loan Association \$80 at the end of each month for 15 years in order to pay off this debt. At what rate of interest, compounded monthly, is he being charged?

7. Mr. Smith purchases a \$300 television set, pays \$50 down, and agrees to pay \$30 at the end of each month as long as necessary. If interest is being charged at 12 per cent, $m = 12$, find (a) the number of payments he will make; and (b) the size of the final payment, assuming it will be made 1 month after the last full payment.

8. Mr. Ajax borrows \$10,000 and agrees to pay \$2,000 at the end of each year. If interest is being charged at 6 per cent effective, find (a) the number of payments he must make; (b) the size of the final payment made 1 year after the last full payment; and (c) how much he still owes just after he has made his fourth payment of \$2,000.

9. Upon entering college, John Smith borrows \$2,500. He agrees to repay this debt by making quarterly payments of \$200 each, the first due 5 years after the money is borrowed. If interest is charged at 4 per cent, $m = 4$, find (a) the number of payments he will have to make; and (b) the size of the final payment made 3 months after the last \$200 payment.

10. Mr. Puckett desires to purchase a small farm as soon as he has saved \$5,000. If he can deposit \$100 at the end of each month into a fund earning interest at 4 per cent, $m = 12$, find (a) the number of deposits; and (b) the size of the final deposit made 1 month after the last \$100 deposit.

11. To accumulate \$6,000, Mr. Goforth deposits \$200 at the end of each quarter in a fund earning interest at 5 per cent, $m = 4$. Find (a) the number of deposits he must make; and (b) the size of the final smaller deposit necessary to bring the fund to exactly \$6,000.

12. To accumulate \$2,100, Mr. Smith deposits \$100 at the end of each 3 months in an account paying 2 per cent, $m = 4$. Find (a) the number of deposits he must make; and (b) the size of the final smaller deposit necessary to bring the fund to exactly \$2,100.

4.8 Special Terminology. The *term* of an *ordinary annuity* has already been defined as the time between the points where the A formula and the S formula give the values of an annuity. That is to say, the term of an ordinary annuity begins one payment period before the first payment is due and ends at the time of the last payment. Since an entire payment period elapses before the first payment, it is sometimes said that the payments are made at the ends of the respective payment periods.

At times, as has been seen in the problems already discussed, the value of an annuity is desired at the time of the first payment. Illus-

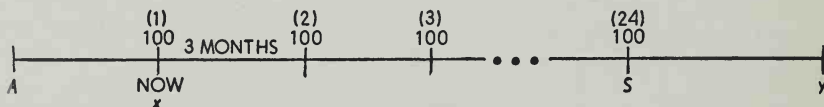
tration 2 (b) of Section 4.3 shows how the regular annuity formula coupled with the compound interest formula will give this result. In such a case, the payments can be thought of as coming at the beginning of each payment period and the value desired at the beginning of the first payment period. If the payments are made at the *beginning* of each payment interval, the annuity may be called an *annuity due*. The *term* of an *annuity due* is the time from the first payment to one payment period after the last payment. The *present value* of an *annuity due* is the value at the time of the first payment, while the *amount* of an *annuity due* is the value one payment period after the last payment. Neither A nor S give the present value nor the amount of an annuity due. Each will have to be moved by compound interest to get the desired result. This is in contrast to the *present value* of an *ordinary annuity* (one payment period before the first payment) and the *amount* of an *ordinary annuity* (the value at the time of the last payment). Here, A and S give the present value and amount, respectively, of an ordinary annuity.

Illustration 1. (a) Find the present value of an annuity due of \$100 per quarter for 24 quarters if interest is at 4 per cent, $m = 4$. (b) Find the amount of this annuity due.

This same problem could be stated thus:

Mr. Smith holds a contract calling for payments of \$100 at the beginning of each quarter for the next 6 years. If money is worth 4 per cent, $m = 4$, find (a) the value of this contract now, (b) the value of this contract 6 years from now.

Solution. $R = 100$, $n = 24$, and $i = .01$ per quarter. Let x be the value now (present value of the annuity due) and y be the value 6 years from now (the amount of the annuity due). The line diagram will be:



$$x = A(1.01)^1 = 100a_{\overline{24}|.01}(1.01) = \$2,145.58$$

$$y = S(1.01)^1 = 100s_{\overline{24}|.01}(1.01) = \$2,724.32$$

A second method of working this problem would be this: In order to use the $a_{\overline{n}|}$ formula directly in finding x , separate the set of 24 payments into two sets of payments. In one set, put the first payment only. In the second set put the remaining 23 payments. Now take the time of x as the comparison date. On that date, the first payment of \$100 has a value of 100, x has a value of x , and the set of the last 23 payments of \$100 each has a value of $100a_{\overline{23}|.01}$. Thus

$$x = 100 + 100a_{\overline{23}|.01} = \$2,145.58.$$

In order to find y , assume that an extra \$100 payment is made at the time of y . This gives 25 payments. Then subtract a \$100 payment on this same date. Taking the time of y as a comparison date, the value of the 25 payments will be $100s_{\overline{25}|.01}$ at that time, the value of y will be y , and the value of the 100 subtracted will be 100. Thus

$$y = 100s_{\overline{25}|.01} - 100 = \$2,724.32.$$

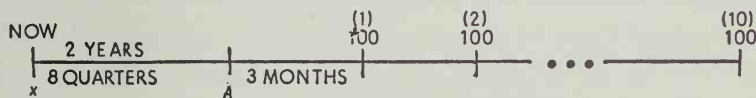
A *deferred annuity* is an annuity whose payments begin at some future time. The *period of deferment* is the time from now until the beginning of the term of the annuity, that is, the time from now until the place where A gives the value of the annuity. If an annuity of quarterly payments is deferred 2 years, this means the beginning of the term of the annuity is 2 years, or 8 quarters, from now. Actually, the first payment will not be made until 3 more months pass, i.e., the first payment will be made $2\frac{1}{4}$ years, or 9 quarters, from now.

Illustration 2. Find the value now of an annuity of 10 quarterly payments of \$100 each deferred for 2 years. Interest is at 6 per cent, $m = 4$.

This problem could be stated thus:

Find the value now of an annuity of 10 quarterly payments of \$100 each if the first payment is made $2\frac{1}{4}$ years from now. Interest is at 6 per cent, $m = 4$.

Solution. $R = 100$, $n = 10$, $i = .015$ per quarter. Let x be the value now.

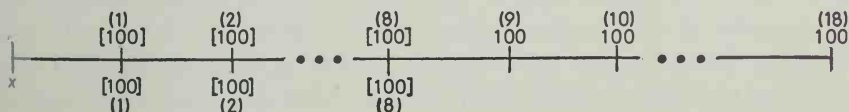


$$x = A(1.015)^{-8} = 100a_{\overline{10}|.015}(1.015)^{-8}$$

$$x = \$818.66$$

A second method of solution is as follows:

Assume that a \$100 payment is made at the end of each quarter, beginning 3 months from now, and continuing until the time for the first regular \$100 payment. This means that 8 extra payments will be made. Now assume that a \$100 will be subtracted on each of these dates on which an extra \$100 was paid. This will be eight \$100 payments to subtract. The line diagram will now look like this:



If the present is taken as the comparison date, then

$$\begin{aligned}x + 100a_{\overline{8}|.015} &= 100a_{\overline{18}|.015}, \\x &= 100a_{\overline{18}|.015} - 100a_{\overline{8}|.015}, \\x &= \$818.66.\end{aligned}$$

Exercise 17

1. Find the present value of an annuity due of \$500 per quarter for 9 years. Interest is at 3 per cent, $m = 4$.
2. Mr. Jones deposits \$100 on the first of each month for 7 years. Find the amount in the account at the end of 7 years if interest is at 6 per cent, $m = 12$.
3. What is the cash price of a house that can be bought for 15 semiannual payments of \$1,000 each, the first due now? Money is worth 7 per cent, $m = 2$.
4. Find the amount of an annuity due of 20 payments of \$1,000 each if money is worth 2 per cent per payment period.
5. Mr. Smith wishes to deposit a sum of money at the birth of his son, so that at the age of 17 and each 6 months thereafter the son can withdraw \$750. The last withdrawal will be made on the twentieth birthday of the son. Find the size of the deposit if money is worth 4 per cent, $m = 2$.
6. Find the cash price of a house which can be purchased for \$5,000 down and 10 semiannual payments of \$800 each, with the first \$800 payment being made 3 years after the down payment. Money is worth 5 per cent, $m = 2$.
7. Find the present value of an annuity of 11 annual payments of \$1,000 each, deferred 25 years. Interest is computed at 3 per cent effective.

4.9 General Case of Annuities. In every problem so far in this chapter, the payment period of the annuity and the interest period of the given rate have been the same in length. In such a case, the annuity is called a *simple annuity*. This restriction will now be removed, and the payment interval and the interest or conversion period need not be the same. If they are different in length, the annuity is called a *general annuity*.

The present value and amount of a general annuity may be found by working with an equivalent simple annuity. No new formulas will be needed, but the method for finding equivalent interest rates (discussed in Section 3.5) will be used; it should be reviewed at this time.

Note that the only distinguishing characteristic that tells whether an annuity is a simple case or the general case is the relationship of the lengths of the interest period and the payment period. In any problem, the length of each should be noticed immediately. If the length of the interest period equals the length of the payment inter-

val, it is a simple annuity. If the length of the interest period is unequal to the length of the payment period, it is a general annuity.

The following example will illustrate the method used in solving the general case of an annuity.

Illustration 1. Find the value 1 payment period before the first payment of 18 quarterly payments of \$100 each if money is worth 6 per cent, $m = 12$.

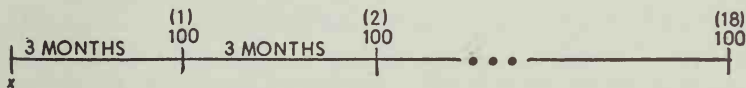
Solution. Here, the interest period equals 1 month, while the payment period is 3 months in length. Thus it is a general annuity.

Let i equal the interest rate per 3 months (the length of the payment period) that is equivalent to .005 per month (the given interest rate). Then by the procedure of Section 3.5,

$$\begin{aligned}(1 + i)^4 &= (1.005)^{12}, \\ 1 + i &= (1.005)^3, \\ i &= (1.005)^3 - 1 \text{ per 3 months.}\end{aligned}$$

Do not look this up in a table and calculate a numerical value for i . This would complicate your calculations.

Now, with a rate i per 3 months, $R = 100$, and $n = 18$ payments, this is a simple case and the ordinary annuity formulas hold.



$$x = 100a_{\overline{18}|i} = 100 \frac{1 - (1 + i)^{-18}}{i}$$

Next, substitute for $1 + i$ and i from the equivalent rate calculations above. Then,

$$x = 100 \frac{1 - [(1.005)^3]^{-18}}{(1.005)^3 - 1},$$

$$x = 100 \frac{1 - (1.005)^{-54}}{(1.005)^3 - 1}.$$

x could be computed here by using the compound interest tables, but a simple operation will save much of this arithmetic. Since the interest rate involved in both the numerator and denominator is .005, the fraction on the right side of the equation will be multiplied by

$$\frac{.005}{.005},$$

and the resulting fraction broken up into two fractions as follows:

$$x = 100 \frac{1 - (1.005)^{-54}}{(1.005)^3 - 1} \cdot \frac{.005}{.005},$$

$$x = 100 \frac{1 - (1.005)^{-54}}{.005} \cdot \frac{.005}{(1.005)^3 - 1}.$$

It will be observed that the first fraction on the right side of the equation is simply $a_{\overline{54}|.005}$, while the second fraction is $\frac{1}{s_{\overline{3}|.005}}$.

Therefore,

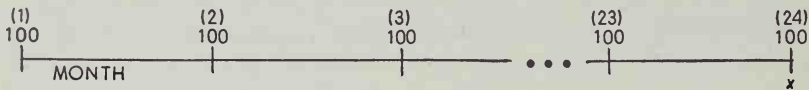
$$x = 100a_{\overline{54}|.005} \frac{1}{s_{\overline{3}|.005}}.$$

The computation may now be done by using the tables for $a_{\overline{n}|i}$ and $\frac{1}{s_{\overline{n}|i}}$, or by using the tables for $\log a_{\overline{n}|i}$ and $\log s_{\overline{n}|i}$.

Illustration 2. Find the amount in an account just after the twenty-fourth monthly deposit of \$100 if money is worth 4 per cent, $m = 4$.

Solution. Here the interest period equals 3 months, while the payment period is 1 month. Thus, it is a general case of an annuity. Proceeding exactly as in the previous illustration, let i = rate per month then

$$\begin{aligned}(1+i)^{12} &= (1.01)^4, \\ 1+i &= (1.01)^{\frac{1}{3}}, \\ i &= (1.01)^{\frac{1}{3}} - 1 \text{ per month.}\end{aligned}$$



Now use this rate i and substitute from the above equations:

$$\begin{aligned}x &= 100s_{\overline{24}|i} = 100 \frac{(1+i)^{24} - 1}{i}, \\ x &= 100 \frac{[(1.01)^{\frac{1}{3}}]^{24} - 1}{(1.01)^{\frac{1}{3}} - 1} = 100 \frac{(1.01)^8 - 1}{(1.01)^{\frac{1}{3}} - 1}, \\ x &= 100 \frac{(1.01)^8 - 1}{(1.01)^{\frac{1}{3}} - 1} \cdot \frac{.01}{.01}, \\ x &= 100 \frac{(1.01)^8 - 1}{.01} \cdot \frac{.01}{(1.01)^{\frac{1}{3}} - 1}.\end{aligned}$$

Now the first fraction on the right side of this last equation is $s_{\overline{8}|.01}$ and the second looks like $\frac{1}{s_{\overline{n}|i}}$ except $s_{\overline{n}|i}$ was only defined for whole numbers for n . But in compound interest, the formula for $(1+i)^n$, n a whole number, was extended to the case where n was a fraction of the form $\frac{1}{p}$. Likewise, it is logical to extend the symbol $\frac{1}{s_{\overline{n}|i}}$ to the case where n is a fraction. Only fractions with 1 as a numerator will be needed here. Therefore, we define

$$s_{\overline{1}|i}^{\frac{1}{p}} = \frac{(1+i)^{\frac{1}{p}} - 1}{i}.$$

Table 9 in the Appendix gives both $\frac{1}{s_{\overline{1}|i}^{\frac{1}{p}}}$ and $\log \frac{1}{s_{\overline{1}|i}^{\frac{1}{p}}}$.

Now

$$x = 100s_{\overline{8}|.01} \frac{1}{s_{\overline{3}|.01}^{\frac{1}{3}}},$$

$$x = 100(8.28567056)(3.00997789),$$

$$x = \$2,493.97.$$

These two illustrations cover both possible cases: the interest period shorter than the payment period, and the interest period longer than the payment period. The same technique was applied to each problem. In one case the factor $\frac{1}{s_{\overline{n}|i}}$ appeared in the solution; in the other case $\frac{1}{s_{\overline{p}|i}^{\frac{1}{p}}}$ appeared. But it was not necessary to know in ad-

vance which would appear. The simple procedure automatically produced the proper one.

If the value of a general annuity is desired on a date other than the ones on which the A and the S will take the deposits, either A or S may be calculated and moved to the desired place, using the compound interest formula and counting the interest periods from A or S to the place where the value is wanted. Note that in all annuity problems, A is the value *one payment period* before the first payment, while S is the value at the time of and including the last payment.

Occasionally, but rather rarely, in solving a general case problem, an expression like the following one appears.

$$\frac{[(1.02)^{\frac{1}{4}}]^{11} - 1}{(1.02)^{\frac{1}{4}} - 1}$$

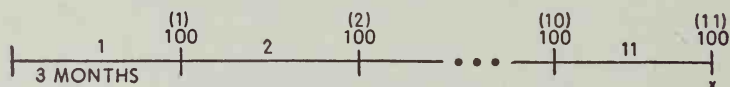
In such a case, this complicated expression may be avoided. The following illustration will show how this case is handled.

Illustration 3. Mr. Jones deposits \$100 at the end of each quarter in a fund earning interest at 4 per cent, $m = 2$. Find the amount in the fund just after the eleventh payment.

Solution. Since the interest period is 6 months and the payment period is 3 months, this is a general annuity. Let i be the rate per 3 months equivalent to 2 per cent per 6 months. Then,

$$\begin{aligned}(1+i)^4 &= (1.02)^2, \\ 1+i &= (1.02)^{\frac{1}{2}}, \\ i &= (1.02)^{\frac{1}{2}} - 1 \text{ per 3 months.}\end{aligned}$$

Here, $n = 11$, $R = 100$, and S is to be found.



$$x = 100s_{\overline{11}|i} = 100 \frac{(1+i)^{11} - 1}{i}$$

$$x = 100 \frac{[(1.02)^{\frac{1}{2}}]^{11} - 1}{(1.02)^{\frac{1}{2}} - 1} = 100 \frac{(1.02)^{\frac{11}{2}} - 1}{(1.02)^{\frac{1}{2}} - 1}$$

There are no tables to handle the numerator of this last fraction when the exponent on $1+i$ is an improper fraction.

To avoid this, select the largest number n out of the total number of payments such that this n times the fraction involved in the equivalent rate gives a whole number. Move this set of n payments by the annuity formula, and move the other payments by compound interest. Here, 10 is the largest number, less than 11, such that $\frac{1}{2}(10)$ is a whole number. Select the time of x as a comparison date and then move the last 10 payments by the annuity formula. The last 10 were selected here because the $s_{\overline{n}|i}$ formula will deposit these at the desired point. In general, select the first or last group, whichever will be more convenient. Now bring the remaining payment (or payments) to the comparison date by the compound interest formula. Here the first payment must be moved 10 quarters or 5 half-years at .02 per 6 months. Thus

$$x = 100s_{\overline{10}|i} + 100(1.02)^5,$$

$$x = 100 \frac{(1+i)^{10} - 1}{i} + 100(1.02)^5,$$

$$x = 100 \frac{(1.02)^5 - 1}{(1.02)^{\frac{1}{2}} - 1} \cdot \frac{.02}{.02} + 100(1.02)^5,$$

$$x = 100s_{\overline{5} | .02} \frac{1}{s_{\overline{1} | .02}} + 100(1.02)^5.$$

The reader may complete the arithmetic computation.

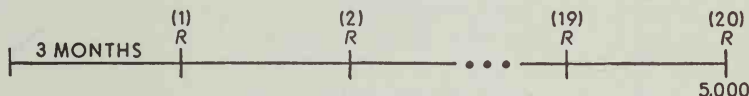
When solving for R in a general case, it is best to do the simplification as in Illustrations 1 and 2 before solving for R .

Illustration 4. In order to accumulate \$5,000 at the end of 5 years, deposits of R dollars each are made at the end of each quarter into a fund paying 6 per cent, $m = 12$. Find the size of the deposit.

Solution. Since the payments are made quarterly and the interest is paid monthly, this is a general case. Let i be the rate per 3 months equivalent to .005 per month. Then

$$\begin{aligned}(1 + i)^4 &= (1.005)^{12}, \\ (1 + i) &= (1.005)^3, \\ i &= (1.005)^3 - 1 \text{ per quarter.}\end{aligned}$$

Now, $n = 20$ payments, and $S = 5,000$.



$$Rs_{\overline{20}|i} = 5,000$$

Do not solve for R here but simplify the left side of the equation first.

$$\begin{aligned}R \frac{(1 + i)^{20} - 1}{i} &= 5,000 \\ R \frac{[(1.005)^3]^{20} - 1}{(1.005)^3 - 1} &= 5,000 \\ R \frac{(1.005)^{60} - 1}{.005} \cdot \frac{.005}{(1.005)^3 - 1} &= 5,000 \\ Rs_{\overline{60} | .005} \frac{1}{s_{\overline{3} | .005}} &= 5,000\end{aligned}$$

Now solve for R .

$$R = 5,000 \frac{1}{s_{\overline{60} | .005} s_{\overline{3} | .005}}$$

The computation may now be done either by direct multiplication or by using logarithms. The actual computation will be left to the reader.

In finding an interest rate in the general case, it is better to first calculate the numerical value of the rate i per payment period (thus using the simple case). Then find the rate for the desired interest period equivalent to that found for the payment period. Logarithms will be necessary here.

Illustration 5. Bill Smith buys a \$3,000 automobile, pays \$1,000 down, and agrees to pay \$121.12 at the end of each month thereafter for 18 months. What rate of interest compounded quarterly is he paying?

Solution. Since the rate is to be found, first solve the problem for a rate per month, that is, for a rate per payment period. This reduces the problem to the simple case. This new problem is the same as Illustration 1 of Section 4.5. There, the rate per month was found to be .00924 per month.

Let i be the rate per quarter equivalent to this .00924 per month. Then

$$\begin{aligned}(1 + i)^4 &= (1.00924)^{12}, \\ 1 + i &= (1.00924)^3.\end{aligned}$$

Since this rate is not in the compound interest tables, solve by taking logarithms of both sides.

$$\begin{aligned}\log(1 + i) &= 3 \log 1.00924 \\ \log(1 + i) &= 3(.0039944) \\ \log(1 + i) &= .0119832 \\ 1 + i &= 1.02798 \\ i &= .02798 \text{ per 3 months} \\ j = 4i &= .01192 = .0119, m = 4\end{aligned}$$

Exercise 18

1. Find the present value (3 months before the first payment) of a set of 24 quarterly payments of \$200 each if money is worth 6 per cent, $m = 12$.

2. Work Problem 1 if the interest rate is 6 per cent, $m = 2$.

3. Find the amount of an annuity of 36 semiannual payments of \$500 each if interest is at 4 per cent, $m = 4$.

4. Solve Problem 3 if the interest rate is 4 per cent effective.

5. On April 1, 1959, Mr. Whaples deposits \$250 in an account paying 2 per cent, $m = 2$. He deposits \$250 at the end of each 3 months thereafter until July 1, 1967, making a final deposit at that time. Find the amount in the account on July 1, 1967.

6. On July 1, 1954, and at the end of each 3 months thereafter, Mr. Jones invests \$500 in a fund earning interest at 6 per cent, $m = 12$. If his final deposit is made on October 1, 1961, find the amount in the account on April 1, 1963.

7. Mr. Abercombe wishes to deposit a sum of money now, so that he can make 48 monthly withdrawals of \$100 each, the first withdrawal being made 7 months from now. Find the size of this deposit if the fund earns interest at 4 per cent, $m = 4$.

8. Solve Problem 6 if the interest rate is 6 per cent, $m = 2$.

9. In order to pay off a debt of \$5,000 due now, Mr. Jones wishes to make 24 equal quarterly payments, the first due 3 months from now. Find the size of each payment if money is worth 6 per cent, $m = 12$.

10. Solve Problem 9 if interest is at 6 per cent, $m = 2$.

11. In order to pay a debt of \$2,000 due now, a man pays \$126.11 at the end of each month for 18 months. What interest rate, compounded semi-annually, is he paying?

12. Twelve monthly payments of \$36 each will pay a debt of \$400 provided the first payment is made 1 month after the debt is contracted. Find the interest rate, compounded quarterly, that is being paid.

13. Mr. Weston purchases a boat that is priced at \$300 down and \$30 at the end of each month for 2 years. If money is worth 5 per cent, $m = 2$, find the cash price of this boat.

14. Mr. Willoby deposited \$1,000 in an account paying 2 per cent, $m = 2$. Three months later, he deposits \$50. At the end of each month thereafter, he makes a \$50 deposit. Find the amount in the account just after the forty-eighth deposit of \$50.

15. Mr. A. B. Kinard makes a deposit of \$3,000 in an account paying interest at 4 per cent, $m = 4$. At the end of each 6 months thereafter, he deposits \$500 in this account. Three months after the original deposit of \$3,000, Mrs. Kinard withdraws \$250. At the end of each 3 months thereafter, Mrs. Kinard withdraws \$250. Mr. Kinard stops his deposits after making twenty-four of the \$500 deposits. How much is left in the account just after Mrs. Kinard makes her fiftieth withdrawal?

16. Under the conditions of Problem 15, find out how many withdrawals of \$250 each can be made by Mrs. Kinard before the account is depleted. Find the size of the smaller withdrawal (made 3 months after the last \$250 withdrawal) that will close the account.

17. Mr. Smith purchases an automobile for \$1,000 in cash and 18 monthly payments of \$90 each. Find the cash price if money is worth (a) 6 per cent, $m = 12$; and (b) 6 per cent, $m = 4$.

18. Mr. Jones has a contract calling for payments to him of \$400 at the end of each 6 months for the next 12 years. After 5 years of payments have been made, Mr. Jones sells this contract to Mr. A. B. Carter. If money is worth 6 per cent, $m = 4$, find the price Mr. Carter pays for this contract.

19. Find the present value of 24 quarterly payments of \$200 each. The first payment is due 3 months from now, and money is worth 6 per cent compounded semiannually.

20. Find the amount in a fund just after the twelfth deposit of \$500. Deposits are made quarterly, and money is worth 6 per cent compounded monthly.

21. For a house costing \$25,000, Mr. Overton pays \$5,000 cash and agrees to pay the balance, with interest at 7 per cent compounded annually, by a set of equal semiannual payments, the first due $7\frac{1}{2}$ years from now and the last due 15 years from now. Find the size of these payments.

22. A man pays \$1,000 cash on a truck, and then a sequence of 12 monthly payments of \$75 each, the first due 1 month from the date of purchase. Find the total cash price of the truck if interest is figured at 6 per cent, $m = 2$.

23. What sum of money must be deposited every month in a bank, paying interest at 3 per cent compounded quarterly, in order to have \$2,000 at the end of 10 years? The first deposit is to be made at the end of 1 month.

24. The purchaser of a farm will pay \$1,200 at the end of each 6 months

for 7 years. If money is worth 6 per cent compounded monthly, find the equivalent cash price.

25. Robert J. James had promised to repay a debt by making 20 annual \$1,000 payments. He fails to make the first 6 payments. When the time for the seventh payment comes, what single payment will discharge this entire debt? Money is worth 4 per cent, $m = 2$.

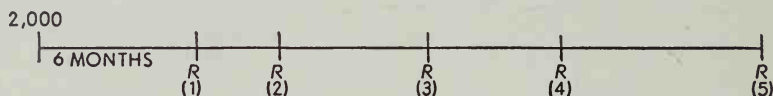
AMORTIZATION AND SINKING FUNDS

5.1 Amortization. Originally, amortization meant the paying of a debt by any plan. But in present usage, *amortization* means the paying of a debt, both principal and interest, by a set of periodic payments that are usually equal in size. This means that a part of each periodic payment is for the repayment of principal. The remaining part of the periodic payment pays all the interest due on the debt for the preceding period. This means that the investor gets his interest when it is due, and he also gets a small part of his principal returned at the end of each period. Since the principal is being reduced at the end of each period, the interest for each succeeding period is smaller than the interest in the preceding period. If the periodic payments are equal, this means that a greater amount of the principal is being paid by each periodic payment than by the preceding payment. One of the problems here will be to separate each periodic payment into its two parts, interest and principal.

Two cases usually arise. In one case, the debt is to be paid by a fixed number of payments and the size of each payment is to be calculated. In the other case, the size of the periodic payment will be known and the number of payments is to be determined. Usually, in this latter case, the final payment will be of a different size from that of the other payments. Both of these situations have already been met in Chapter 4. (See Illustration 1 of Section 4.4 and Problem 7 of Exercise 16.) The following illustrations will recall the methods used there.

Illustration 1. Mr. W. O. Smith borrows \$2,000 with interest at 4 per cent, $m = 2$. He is to amortize this debt by 5 equal semiannual payments, the first due 6 months after he borrowed the \$2,000. Find the size of each payment.

Solution. In this problem, the \$2,000 is the size of the debt 6 months or 1 payment period before the first payment. Thus this is the simple case of an annuity. Now $i = .02$ per 6 months and $n = 5$. R is to be found.



By the annuity formula (5):

$$Ra_{\overline{5}|.02} = 2,000,$$

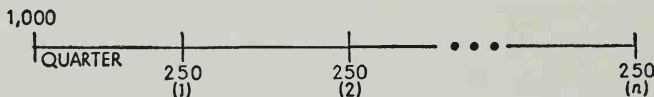
$$R = \frac{1}{a_{\overline{5}|.02}} 2,000,$$

$$R = 2,000(.21215839),$$

$$R = \$424.32 \text{ to the nearest cent.}$$

Illustration 2. Mr. Olson borrows \$1,000 at 4 per cent, $m = 4$. He wishes to repay principal and interest by making payments of \$250 at the end of each 3 months as long as necessary. Find the number of full \$250 payments and the size of the smaller payment (made 3 months after the last \$250 payment) that will repay this debt.

Solution. Again, the interest period and the payment period are of the same length, 3 months. This is a simple annuity, with $A = 1,000$, $R = 250$, and $i = .01$ per quarter. Let n be the number of full-size payments:

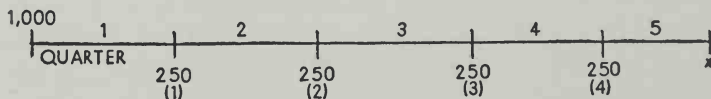


$$250a_{\overline{n}|.01} = 1,000$$

$$a_{\overline{n}|.01} = 4$$

$$4 < n < 5$$

Thus, there will be 4 full payments of \$250 each and a smaller payment 3 months later. Let x be the size of this smaller payment.



Using the date on which x is to be paid as the focal date,

$$\begin{aligned}
 x + 250s_{\overline{4}|.01}(1.01) &= 1,000(1.01)^5, \\
 x &= 1,000(1.01)^5 - 250s_{\overline{4}|.01}(1.01), \\
 x &= 1,000(1.05101005) - 250(4.06040100)(1.01), \\
 x &= \$25.76.
 \end{aligned}$$

5.2 Amortization Schedule. As was indicated in the preceding section, a portion of each amortization payment is for interest while the rest is for repayment of the principal. In order to make the proper entries in the set of books of either the lender or the borrower, the exact portion, in dollars and cents, of each payment that goes for interest as well as the portion for repayment of principal, must be calculated. An amortization schedule may be set up as in the following illustrations to show precisely how much of each payment goes for interest and how much goes toward the repayment of the principal.

Illustration 1. Construct the amortization schedule of Illustration 1 of Section 5.1.

Solution. In this example, the original debt is \$2,000 and interest is to be computed at 2 per cent each 6 months. At the end of the first 6-month period, the amount of interest due is $(.02)(\$2,000) = \40 . The periodic payment is \$424.32. If the interest of \$40 is subtracted from this, \$384.32 is left to be paid on the principal of the debt. This leaves $\$2,000 - \$384.32 = \$1,615.68$ still due on the principal at the end of the first period, or, what amounts to the same thing, at the beginning of the second period. These figures have been recorded in the amortization schedule below. The interest for the second period is $(.02)(1,615.68) = \$32.31$. Subtracting this from the payment of \$424.32 leaves \$392.01 to be paid on the principal. $\$1,615.68 - \$392.01 = \$1,223.67$ as the outstanding principal at the beginning of the third period. This process is continued until the debt is completely paid off. The amortization schedule will now appear as follows:

AMORTIZATION SCHEDULE

<i>Period</i>	<i>Outstanding Principal at Beginning of Period</i>	<i>Interest Due at .02</i>	<i>Periodic Payment</i>	<i>Principal Repaid</i>
1	\$2,000.00	\$ 40.00	\$ 424.32	\$ 384.32
2	1,615.68	32.31	424.32	392.01
3	1,223.67	24.47	424.32	399.85
4	823.82	16.48	424.32	407.84
5	415.98	8.32	424.32	416.00
Totals		\$121.58	\$2,121.60	\$2,000.02

It will be noted in the last line (line 5) preceding the totals that the principal repaid is 2 cents greater than the outstanding principal for that period. This error is due to the fact that all computation was carried only to two decimal places. Actually, the periodic payment was only \$424.317, and this was rounded off to the nearest cent, giving \$424.32. Likewise each interest calculation was rounded off to the nearest cent. The lack of balance between the fractions of cents counted as whole cents and the total fractions discarded brought about this slight error. Thus, one may expect a few cents error in the computation of the final line of any schedule. (It is possible no error will appear.) To compensate for this error, whenever it appears, the final payment should be changed (increased or decreased as the case may be) so as to make the final principal repaid agree exactly with the outstanding principal at that time. (Note: If the contract insists that all the periodic payments should be of exactly the same size, allowing for no adjustment in the final one, the schedule can be balanced by making the appropriate change in the interest due at the end of the last period. That is, the interest paid for the last period may be slightly more or less than that designated by the stated interest rate.) The amortization schedule will then appear:

AMORTIZATION SCHEDULE

<i>Period</i>	<i>Outstanding Principal at Beginning of Period</i>	<i>Interest Due at .02</i>	<i>Periodic Payment</i>	<i>Principal Repaid</i>
1	\$2,000.00	\$ 40.00	\$ 424.32	\$ 384.32
2	1,615.68	32.31	424.32	392.01
3	1,223.67	24.47	424.32	399.85
4	823.82	16.48	424.32	407.84
5	415.98	8.32	424.30	415.98
Totals		\$121.58	\$2,121.58	\$2,000.00

The "Totals" line has no significance except as a check on the work. The total of the principal repaid should always be the amount of the original debt. The difference between the totals of the periodic payment and interest due columns should always equal the total of the principal repaid column.

Illustration 2. Construct the amortization schedule for Illustration 2 of Section 5.1.

Solution. This is a case in which there is an odd-sized final payment. The construction of the schedule is precisely the same as in the preceding

problem except the final entry in the Periodic Payment column should be changed from the \$250 to \$25.76. Again, in this schedule, the rounding off to two decimal places causes a slight error, here 1 cent. Thus, the final payment will be recorded as \$25.77 instead of \$25.76. In fact, the schedule could have been used to calculate the size of this final payment. But, unless the number of payments is small, the method already used to calculate the payment size is shorter. The schedule will be:

AMORTIZATION SCHEDULE

<i>Period</i>	<i>Outstanding Principal at Beginning of Period</i>	<i>Interest Due at .01</i>	<i>Periodic Payment</i>	<i>Principal Repaid</i>
1	\$1,000.00	\$10.00	\$ 250.00	\$ 240.00
2	760.00	7.60	250.00	242.40
3	517.60	5.18	250.00	244.82
4	272.78	2.73	250.00	247.27
5	25.51	.26	25.77	25.51
Totals		\$25.77	\$1,025.77	\$1,000.00

5.3 Finding the Outstanding Principal without a Schedule.

In an amortization problem, the interest is kept paid. Thus, the *outstanding principal* just after any payment has been made is the amount still owed on the debt. The debtor would probably refer to this as his *outstanding liability*. Precisely, *outstanding principal* means the amount of the principal still owed, while *outstanding liability* means the total amount, principal and interest, still owed. Just after a payment has been made in an amortization problem, no interest is due. Therefore, at that time, the outstanding principal and the outstanding liability are exactly the same. If this debt has been contracted by the purchase of an asset, the outstanding principal is the *seller's equity* in the asset. That is to say, the seller's equity is the portion, in dollars and cents, of the asset that the seller is still entitled to collect. The difference between the original value of the asset and the outstanding principal is the *buyer's equity*. The buyer's equity is that portion, in dollars and cents, of the asset the buyer has actually paid for at any given time.

The outstanding principal at the end (or beginning) of any period may be read from the amortization schedule. It may also be calculated directly without reference to a schedule. If the schedule is long, this direct calculation will be much shorter. There are two methods for doing this calculation directly. What is desired is the amount still owed. Thus, if the number and size of each of the remaining pay-

ments are known, the outstanding principal can be computed by getting the sum of the present values of all these remaining payments.

If either the number of remaining payments or the size of some of these payments is unknown, the outstanding principal may be calculated by subtracting the accumulated value of all the payments already made from the value of the original debt accumulated to this time. This second method may also be used when the computation is too tedious by the first method.

The following examples will illustrate both methods.

Illustration 1. Mr. Smith owes \$1,000. He agrees to amortize this debt by making 24 equal monthly payments, beginning 1 month after the debt is incurred. If interest is charged at 12 per cent, $m = 12$, find (a) the size of each payment, and (b) the outstanding principal just after the seventeenth payment has been made.

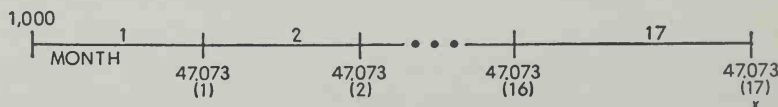
Solution. In Illustration 1 of Section 4.4, the size of this payment was calculated to be \$47.07. This answers part (a). Now 47.07 is accurate to only two decimal places. In computing part (b), this number will have to be multiplied by other numbers. To get an answer to part (b) accurate to two decimal places, the payment 47.07 should be accurate to three or more decimal places. Reference to the solution in Section 4.4 shows that $R = 47.073$ correct to three decimal places. This value will be used here. Part (b) will now be solved by each of the two methods described above. Since 17 payments have been made, there remain 7 payments to be made. The outstanding principal will be the present value of these payments. Let x be the outstanding principal. The line diagram will be:



This is a simple case of an annuity with $R = 47.073$, $n = 7$, $i = .01$ per month, and $A = x$.

$$\begin{aligned} x &= 47.073a_{\overline{7}|\cdot 01} \\ x &= 47.073(6.72819453) \\ x &= 316.72 \end{aligned}$$

To calculate x , by the second method, it is noted that x is the size of an extra payment made just after the seventeenth that would cancel the debt. Thus the line diagram would be:

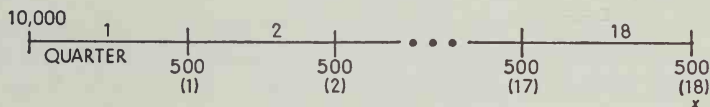


Taking the time of the seventeenth payment as the focal date,

$$\begin{aligned}x + 47.073s_{\overline{17}|.01} &= 1,000(1.01)^{17}, \\x &= 1,000(1.01)^{17} - 47.073s_{\overline{17}|.01}, \\x &= 1,000(1.18430443) - 47.073(18.43044314), \\x &= 1,184.30 - 867.58, \\x &= 316.72.\end{aligned}$$

Illustration 2. In order to amortize a debt of \$10,000 with interest at 6 per cent, $m = 4$, Mr. Jones pays \$500 at the end of each quarter. What is the outstanding principal just after the eighteenth payment?

Solution. Since neither the number of payments nor the size of the final payment is known, the second method described above will be the better method. Let x be the outstanding principal; $i = .015$ per quarter.



$$\begin{aligned}x + 500s_{\overline{18}|.015} &= 10,000(1.015)^{18} \\x &= 10,000(1.015)^{18} - 500s_{\overline{18}|.015} \\x &= 10,000(1.30734064) - 500(20.48937572) \\x &= 13,073.41 - 10,244.69 \\x &= \$2,828.72\end{aligned}$$

5.4 Finding the Amount of Interest or Principal Included in Any Payment. The amortization schedule separates each payment into two parts: the interest payment and the repayment on the principal. This same separation can be done directly without constructing a schedule. If it is desired to separate a certain payment into interest and principal parts, proceed as follows:

1. Calculate the outstanding principal just after the preceding payment.
2. Compute the interest on this outstanding principal for one period. This is the amount of the interest due at the time of the next payment.
3. To obtain the amount of principal repaid by this particular payment, subtract this interest from the size of the periodic payment.

Illustration. In Illustration 1 of Section 5.3, find the part of the eighteenth payment that (a) goes to pay interest, and (b) goes toward the repayment of the principal.

Solution. First, it will be necessary to calculate the outstanding principal just after the preceding, here the seventeenth, payment. This

was done in Section 5.3. This outstanding principal was found to be \$316.72. Now for part (a) the interest due at the time of the eighteenth payment is $(.01)(316.72) = \$3.17$. Since the periodic payment is \$47.07, the part of the eighteenth payment that is used to repay principal is $47.07 - 3.17 = \$43.90$.

Exercise 19

1. The Acme Corporation borrows \$10,000 at 6 per cent, $m = 2$. They agree to repay this debt, principal and interest, by making equal payments at the end of each 6 months for 10 years. Find (a) the size of each payment; (b) the outstanding principal just after the sixteenth payment; and (c) the portion of the seventeenth payment that is used to repay principal.

2. A debt of \$5,000 with interest at 4 per cent, $m = 2$, is to be amortized by equal semiannual payments for 4 years. Construct the amortization schedule.

3. A debt of \$3,000 with interest at 4 per cent, $m = 4$, is to be amortized by making payments of \$500 at the end of each quarter as long as necessary. Construct the amortization schedule.

4. Mr. Owens purchases a house for \$15,000. After making a down payment of \$6,000, he agrees to pay the remaining principal and interest at 5 per cent, $m = 4$, by making equal payments at the end of each quarter for the next 15 years.

a) Find the size of the quarterly payment.

b) Just after making his twenty-fifth payment, Mr. Owens decides to pay the entire balance of his debt. Find this balance.

5. a) Just after the twenty-fifth payment of Problem 4, what is Mr. Owens's equity in the house?

b) What is the seller's equity in this house at the same time?

6. Mr. Austel purchases a car and, after making the down payment, still owes \$2,000. He agrees to pay this by making payments of \$100 at the end of each month. If interest is computed at 6 per cent, $m = 12$, find the number of \$100 payments he must make and the size of the final smaller payment to be made 1 month after the last \$100 payment.

7. The A. B. Jones Company borrows \$50,000 at 6 per cent compounded annually. They agree to repay this debt by making 10 equal annual payments, the first due at the end of 8 years. Find the size of each payment.

8. Find the outstanding liability just after the seventh payment has been made in Problem 7.

9. What part of the eighth payment of Problem 7 is for interest due for the preceding year?

10. Mr. Wilson purchases a \$30,000 house and pays \$10,000 down, \$5,000 at the end of a year and \$2,500 at the end of each year thereafter as long as necessary. Interest is to be computed at 5 per cent effective. Find the outstanding principal just after the payment is made at the end of the sixth year.

11. In Problem 10, how much interest is paid when the payment is made at the end of the seventh year? How much of this payment goes to repay the principal?

12. When John Jones died, he left an insurance policy for \$50,000. His will directed that \$5,000 should be paid to his wife immediately and the rest of this money be invested in a fund earning 4 per cent, $m = 2$. At the end of each 6 months thereafter as long as his wife lived (or until the fund was exhausted) she would receive \$2,000. If the wife died before the fund was exhausted, all the money in the fund, at the time the next payment would have been due her, would be given to the local orphanage. If Mrs. Jones dies $7\frac{3}{4}$ years later, how much does the orphanage receive?

13. A debt of \$10,000 is to be amortized by making equal payments at the end of each year for the next 4 years. Interest is at 6 per cent, $m = 4$. Find the size of the annual payment.

14. Construct the amortization schedule for Problem 13.

15. Mr. Owens borrows \$5,000 with interest at 6 per cent, $m = 2$. He will amortize this debt by making payments at the end of each month for 48 months. Find the size of the monthly payments.

16. Construct the first four lines of the amortization schedule for Problem 15. Use compound interest for the fractional periods.

17. Find the outstanding principal of the debt in Problem 15 just after the twelfth payment has been made.

18. A debt of \$10,000, bearing interest at 6 per cent, $m = 12$, is to be amortized by equal payments at the end of each 6 months for 20 years. Find the size of the semiannual payments.

19. Construct the first four lines of the amortization schedule for Problem 18.

20. Find the size of the quarterly payments that will amortize a debt of \$10,000 in 10 years. Interest is to be computed at 5 per cent, $m = 2$.

21. Construct the first three lines of the amortization schedule for Problem 20. Use compound interest for the fractional periods.

22. Find the size of the outstanding principal just after the sixteenth payment in Problem 20.

23. How much of the seventeenth payment in Problem 20 goes to pay interest? Use compound interest for the fractional period.

5.5 Amortization of a Bonded Debt. Frequently, an organization borrows money by selling bonds. Bonds will be discussed in greater detail in Chapter 6. Here, it suffices to note that a bond is a written contract to repay a fixed sum at some future date and to make periodic interest payments until the bond is retired (paid off). It is often desired to repay some of the bonds each period, and to do this in such a manner that the total payments each period (that is,

the value of the bonds retired that period plus the interest paid for the same period) are as nearly equal as possible. If the principal could be repaid in any amount desired for any period, this would be an ordinary amortization problem, and the total payments made each period would be exactly equal. Since it is necessary to retire an entire bond at one time, this is not the case, and an adjusted amortization schedule must be constructed. The following illustration will show the method that is to be used.

Illustration. The city of Columbia issues \$100,000 in \$500 bonds bearing annual interest at 4 per cent. These bonds are to be retired during the next 10 years in such a way as to make the total yearly payments as nearly equal as possible. Construct the amortization schedule.

Solution. As a guide to determining the size of the total payment to be made each year, the size of the annual amortization payment is calculated. Let this payment be R . Then

$$Ra_{10|.04} = 100,000,$$

$$R = 100,000 \frac{1}{a_{10|.04}} = \$12,329.09.$$

Each payment will be made as near \$12,329.09 as can be done by repaying the principal in multiples of \$500. During the first year, $.04(100,000) = \$4,000$ interest is due. This will leave $\$12,329.09 - \$4,000 = \$8,329.09$ as a guide to the amount of principal to be repaid. Since the multiple of \$500 nearest this figure is \$8,500, seventeen bonds will be retired. This makes the first annual payment amount to \$12,500, or slightly more than the figure \$12,329.09.

This leaves an outstanding principal of $\$100,000 - \$8,500 = \$91,500$ at the beginning of the second year. For the second year the interest due will be $(.04)(91,500) = \$3,660.00$. Since $12,329.09 - 3,660.00 = \$8,669.09$, the nearest multiple of \$500 is again \$8,500. Thus 17 bonds will be retired this year too, making the total actual payment $\$3,660.00 + \$8,500.00 = \$12,160.00$. This is slightly less than the guide figure.

Each line in the schedule will be computed in this manner. In the final year, all the remaining bonds must be retired, even if this year's payments are not in line with the others.

The completed schedule is shown on page 97.

Exercise 20

1. The United Railway sells \$250,000 in \$1,000 bonds bearing interest at 3 per cent effective. The bonds are to be retired over a 6-year period. Construct the amortization schedule so as to make the total annual payments as nearly equal as possible.

2. The City of Murdock Falls issues \$50,000 in \$5,000 bonds which

AMORTIZATION SCHEDULE OF A BONDED DEBT

<i>Period</i>	<i>Outstanding Principal at Beginning of Period</i>	<i>Interest Due at End of Period</i>	<i>Value of Bonds Retired</i>	<i>Total Periodic Payment</i>
1	\$100,000	\$ 4,000	\$ 8,500	\$ 12,500
2	91,500	3,660	8,500	12,160
3	83,000	3,320	9,000	12,320
4	74,000	2,960	9,500	12,460
5	64,500	2,580	9,500	12,080
6	55,000	2,200	10,000	12,200
7	45,000	1,800	10,500	12,300
8	34,500	1,380	11,000	12,380
9	23,500	940	11,500	12,440
10	12,000	480	12,000	12,480
Totals		\$23,320	\$100,000	\$123,320

pay interest annually at 2 per cent. Construct the schedule to retire these bonds over a 5-year period.

3. Ye Olde Iron Works issues \$100,000 in bonds paying 5 per cent effective. If 5 of these bonds are \$10,000 each, 30 are \$1,000 each, 30 are \$500 each, and 50 are \$100 each, construct a schedule for retiring these bonds over a 7-year period, making each yearly payment as nearly equal as possible.

4. Construct a schedule for the retiring of \$100,000 in \$1,000 bonds over a 10-year period. These bonds bear interest at 3 per cent effective.

5.6 Sinking Fund Method for Retiring a Debt. The amortization plan has the disadvantage (to the lender) that the principal is repaid in relatively small amounts over a long period of time. Each of these repayments of principal must be reinvested in order to keep the lending company's money earning interest. Therefore, in many cases, the investor prefers his entire principal repaid in one single payment at the maturity of the debt. Sometimes the total interest is also paid on this maturity date. Such cases were considered in Chapter 3. Frequently, the lender desires that the interest be paid as it comes due. This leaves only the principal (in addition to the interest for the last period) to be repaid on the maturity date. In order to have the funds to repay this principal, the borrower may invest periodically sums of money in a *sinking fund* which earns interest on the money invested in it. The sinking fund rate of interest is usually different from the interest rate paid on the debt. In fact, the sinking fund rate is usually smaller than the debt rate.

In many cases, the establishing of a sinking fund held by some

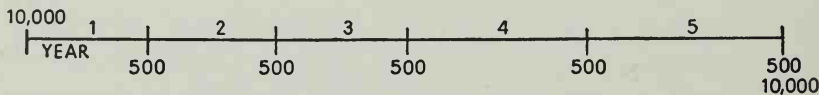
outside party is required by law. This is particularly true in the case of governmental units such as cities, counties, etc. Even when not required by law, many concerns prefer an outside organization to hold the sinking fund. But, in many cases, the concern prefers to invest the sinking fund in its own business. Then, the sinking fund process is merely a set of bookkeeping entries. Such accounting procedure is needful in computing profit and loss statements and in figuring income tax returns, etc.

Under the sinking fund plan, the borrower has two outlays of money (either actual or as bookkeeping entries) each period. He makes the periodic interest payment and he deposits the desired amount into the sinking fund. The sum of these two payments is called the total *periodic expense* of retiring the debt.

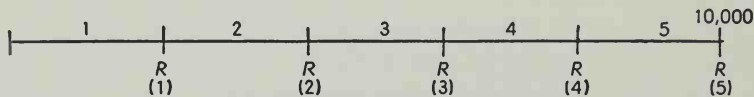
Illustration. The Crescent Hill Packing Company borrows \$10,000 and agrees to pay interest annually at a rate of 5 per cent. To repay the principal at the end of 5 years, equal deposits are made into a sinking fund earning interest at 3 per cent, $m = 1$.

- a) Find the size of the annual deposit in the sinking fund.
- b) Find the total annual expense of this debt.

Solution. In order to get a clear picture, the debt diagram will first be constructed. The amount of money borrowed is \$10,000. At the end of each year $(.05)(10,000) = \$500$ interest is paid. After this interest is paid, the company still owes exactly \$10,000 so the interest for the next year will be the same. This will be true for each year. On the maturity date, after the \$500 interest has been paid, \$10,000 (the original principal) is still owed and must be paid then. The debt diagram will be:



In order to have this \$10,000 at the end of the 5 years, a sinking fund is established by making equal payments into the fund at the end of each year. Let R be the size of each deposit, $n = 5$, and $i = .03$ per year. The sinking fund diagram will then be:



Taking the end of 5 years (the maturity date) as the focal date,

$$\begin{aligned}
 R s_{\overline{5}|.03} &= 10,000, \\
 R &= 10,000 \frac{1}{s_{\overline{5}|.03}}, \\
 R &= \$1,883.55.
 \end{aligned}$$

This answers part (a) and says that if \$1,883.55 is deposited at the end of each year, for 5 years, the sinking fund will contain \$10,000 on the maturity date.

For part (b), the annual interest cost has already been calculated to be \$500. Adding this to the yearly deposit gives the

$$\text{yearly expense} = \$1,883.55 + \$500 = \$2,383.55.$$

5.7 Net Indebtedness. Under the sinking fund plan, the amount of the debt owed (just after each interest payment has been made) remains precisely the same, that is the original principal. But as time elapses, the amount set aside in the sinking fund increases in size. The difference between the original principal and the amount of money accumulated in the sinking fund is called the *net indebtedness* just after any interest payment has been made. That is to say, the *net indebtedness* is the additional money that would be needed over and above what is in the sinking fund in order to pay off the debt at that time.

Illustration. Find the net indebtedness at the end of 3 years of the Crescent Hill Packing Company's loan described in the illustration of the preceding section.

Solution. At the end of 3 years, the third interest payment has been made and the company owes \$10,000. But three deposits of \$1,883.55 have been made into the sinking fund. This fund will now contain

$$1,883.55s_{\overline{3}|.03} = \$5,821.86.$$

Therefore,

$$\text{net indebtedness} = \$10,000 - \$5,821.86 = \$4,178.14.$$

5.8 Growth of the Sinking Fund. It may be noted that the growth of the sinking fund is brought about by two means: (1) the periodic deposit, and (2) the interest earned by and added to the fund at the end of each period (except the first). This growth may be tabulated in the form of a schedule, as in the following illustration.

Illustration. Construct the sinking fund growth schedule for the illustrative problem of Section 5.6.

Solution. The interest received on the fund is calculated at 3 per cent. The total increase in the fund is the sum of the interest received column and the payment column. The last payment in the fund (line 5) was decreased 3 cents to \$1,883.52 in order to make the amount in the fund

at the beginning of the sixth period (end of the fifth period) exactly \$10,000. Again, this 3-cent error was due to the "rounding off" of all the numbers to two decimal places.

The totals serve only as a check on the arithmetic.

The completed schedule will appear as follows:

SINKING FUND GROWTH SCHEDULE

<i>Period</i>	<i>In Fund at Beginning of Period</i>	<i>Interest Received on Fund</i>	<i>Payment into Fund</i>	<i>Total Increase in Fund</i>
1	0	0	\$1,883.55	\$ 1,883.55
2	\$ 1,883.55	\$ 56.51	1,883.55	1,940.06
3	3,823.61	114.71	1,883.55	1,998.26
4	5,821.87	174.66	1,883.55	2,058.21
5	7,880.08	236.40	1,883.52	2,119.92
6	10,000.00			
Totals		\$582.28	\$9,417.72	\$10,000.00

Exercise 21

1. The Acme Corporation borrows \$10,000 and agrees to pay interest when due at 6 per cent, $m = 2$. The principal is to be repaid at the end of 10 years. In order to provide for this principal at the end of 10 years, a sinking fund is set up paying interest at 4 per cent, $m = 2$. If equal deposits are made into this fund at the end of each 6 months, find (a) the size of these deposits, and (b) the total semiannual expense of this debt. Compare this answer with the answer to Problem 1 (a) of Exercise 19.

2. In Problem 1, find (a) the amount in the sinking fund just after the eighth deposit, and (b) the net indebtedness at the end of 4 years.

3. Construct the first five lines of the sinking fund schedule for Problem 1.

4. If the Acme Corporation had been able to set up a sinking fund bearing interest at 6 per cent, $m = 2$, in order to repay the principal of the debt in Problem 1, find (a) the size of the semiannual deposit into the sinking fund, and (b) the total yearly expense. Compare this answer with the answer to Problem 1 (b) above and Problem 1 (a) of Exercise 19.

5. The Atlas Company borrows \$100,000 and agrees to pay interest at 4 per cent, $m = 4$, as it comes due. A sinking fund, earning interest at 2 per cent, $m = 4$, is set up to provide for the repayment of the principal at the end of 20 years. If equal deposits are made quarterly into this fund, find (a) the size of each deposit, (b) the total quarterly expense of the debt, and (c) the net indebtedness at the end of the twelfth year.

6. The Overland Construction Company borrows \$50,000 and agrees to pay interest quarterly at the rate 6 per cent, $m = 4$. In order to repay the principal at the end of 10 years, equal semiannual deposits are made into a sinking fund paying interest at 4 per cent, $m = 12$. Find (a) the

size of the semiannual deposit into the sinking fund, and (b) the total semiannual expense of the debt.

7. In Problem 6, find the net indebtedness at (a) the end of 4 years, and (b) the end of 4 years and 3 months.

8. Mr. Hill borrows \$10,000 and agrees to repay principal and interest at 5 per cent, $m = 2$, in one sum at the end of 15 years. In order to have this money at that time, Mr. Hill makes equal quarterly payments into a sinking fund paying interest at 4 per cent, $m = 4$.

a) Find the size of this quarterly deposit.

b) What is the net indebtedness at the end of 4 years?

9. During a period of financial difficulties, the Wedrivem Truck Corporation borrows \$50,000 and agrees to pay interest annually at 8 per cent. The principal is to be repaid at the end of 15 years, with the option that the principal can be repaid at the end of any year prior to the fifteenth, if the corporation so desires. A sinking fund, earning interest at 3 per cent effective and consisting of annual deposits, is set up to repay the principal at the end of 15 years. Five years later, the financial situation has cleared so that the corporation can borrow, from another source, at 5 per cent, $m = 1$. They borrow enough at this rate so that what they borrow plus what is in the sinking fund will pay off the high interest rate debt. Interest on the new loan is to be paid annually, and a sinking fund, at the same rate as before, is established so as to repay the principal of this new loan at the end of 10 years. Find the size of the deposit into this sinking fund and the total annual saving for each of the last 10 years which resulted from paying off the high interest rate debt.

10. The City Bank has loaned Mr. E. P. Jones \$10,000. Mr. Jones is to make annual interest payments at 5 per cent. In addition, he is to repay the principal of the loan just as soon as he can accumulate the \$10,000 in a sinking fund by making annual deposits of \$2,000. The sinking fund is to earn interest at 4 per cent compounded annually.

a) Find the total annual expense of this debt.

b) Determine the number of \$2,000 deposits that must be made into this sinking fund.

c) Find the size of the final deposit that is to be made 1 year after the last \$2,000 deposit.

11. Mr. Goforth borrows \$20,000 and agrees to pay interest annually at 6 per cent. To repay the principal at the end of 15 years, he sets up a sinking fund which earns interest at 3 per cent, $m = 1$.

a) Find the size of the annual payment into the sinking fund.

b) Find the total annual expense of the debt.

c) Find the net indebtedness at the end of 11 years.

12. A man borrows \$10,000. He agrees to retire the debt by the sinking fund plan, paying interest annually, and making annual deposits into the fund at the end of each year for 15 years. If the sinking fund earns interest at 4 per cent effective and the total annual expense of the debt is \$1,200, find (a) the interest rate on the debt; and (b) the net indebtedness at the end of 6 years.

13. Mr. Krenshaw borrows \$10,000 from Mr. Simpson and agrees to pay interest when due at the rate of 6 per cent annually. In order to accumulate the \$10,000 to repay the principal at the end of 20 years, Mr. Krenshaw makes equal annual payments into a sinking fund earning interest at 2 per cent compounded annually.

- a) Find the size of the annual payment.
- b) Find the total annual expense of the debt.
- c) Find the net indebtedness at the end of 17 years.

14. To repay the principal of a debt of \$50,000, equal payments are made at the end of each 6 months into a sinking fund which earns interest at 2 per cent compounded semiannually. If the principal must be repaid at the end of 15 years, find (a) the size of each semiannual payment into the sinking fund; (b) the amount in the sinking fund at the end of 7 years; and (c) the total semiannual expense of the debt if the debt bears interest at 5 per cent compounded semiannually.

5.9 Comparison of the Amortization and Sinking Fund Methods.

In borrowing money, the question arises as to which is less expensive, the amortization plan or the sinking fund method. A few examples will convince you that if the debt rate and the sinking fund rate are the same, then amortization at the debt rate is precisely the same as the sinking fund plan. If the sinking fund rate is less than the debt rate, then amortization is cheaper than the sinking fund procedure. But if the sinking fund rate is greater than the debt rate, then amortization is the more expensive.

Illustration 1. \$10,000 is borrowed for 10 years at an interest rate of 5 per cent effective.

- a) If this debt is amortized, find the yearly payment.
- b) If this debt is repaid by the sinking fund plan, find the total yearly expense if the sinking fund earns interest at (1) 3 per cent effective, (2) 5 per cent effective, and (3) 6 per cent effective.

Solution. If the debt is amortized, let R be the size of each payment. Then

$$Ra_{\overline{10}|.05} = \$10,000 ,$$

$$R = 10,000 \frac{1}{a_{\overline{10}|.05}} ,$$

$$R = \$1,295.05 .$$

If this debt is repaid by the sinking fund plan, the yearly interest payment will be $(.05)(10,000) = \$500$. Let x be the size of the deposit into the sinking fund. Then—

At 3 per cent:

$$\begin{aligned}
 xs_{\overline{10}|.03} &= 10,000, \\
 x &= 10,000 \frac{1}{s_{\overline{10}|.03}}, \\
 x &= \$872.31.
 \end{aligned}$$

At 5 per cent:

$$\begin{aligned}
 xs_{\overline{10}|.05} &= 10,000, \\
 x &= 10,000 \frac{1}{s_{\overline{10}|.05}}, \\
 x &= \$795.05.
 \end{aligned}$$

At 6 per cent:

$$\begin{aligned}
 xs_{\overline{10}|.06} &= 10,000, \\
 x &= 10,000 \frac{1}{s_{\overline{10}|.06}}, \\
 x &= \$758.68.
 \end{aligned}$$

Therefore the total yearly expense (found by adding \$500 to each of these) will be:

At 3 per cent	\$1,372.31
At 5 per cent	\$1,295.05
At 6 per cent	\$1,258.68

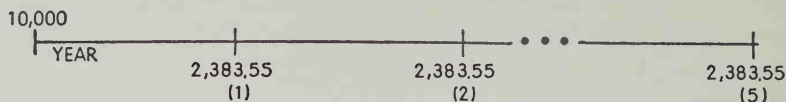
Comparing these with the amortization cost of \$1,295.05, it is seen that amortization is the same as the sinking fund plan at 5 per cent. Amortization at the specified rate of 5 per cent is cheaper than the 3 per cent sinking fund plan, but it is more expensive than the 6 per cent sinking fund plan.

The question now arises, if the sinking fund method is used, just what interest rate is the borrower actually paying on the retirement of his debt? The following problem will show how this interest rate may be computed.

Illustration 2. Find the rate of interest the Crescent Hill Packing Company is actually paying for the retirement of the debt described in the illustrative example of Section 5.6.

Solution. In this problem, the total annual expense of this debt was calculated to be \$2,383.55. This means that each year (for 5 years) the

company spends \$2,383.55 in order to retire a debt of \$10,000. The line diagram will be:



Let i be the rate per year and take the time the \$10,000 was borrowed as the comparison date.

$$2,383.55a_{\overline{5}|i} = 10,000$$

$$a_{\overline{5}|i} = \frac{10,000}{2,383.55}$$

The tedious division may be avoided by inverting both sides of the equation.

$$\frac{1}{a_{\overline{5}|i}} = \frac{2,383.55}{10,000}$$

$$\frac{1}{a_{\overline{5}|i}} = .238355$$

Or using only four decimal places,

$$\frac{1}{a_{\overline{5}|i}} = .2384$$

Interpolating for i from the $\frac{1}{a_{\overline{m}|i}}$ table,

$$i = .0616 \text{ per year.}$$

Exercise 22

1. In Problem 1 of Exercise 21, find the interest rate, compounded semiannually, that the Acme Corporation is actually paying to retire this debt.

2. In Problem 5 of Exercise 21, find the interest rate, compounded quarterly, that the Atlas Company is actually paying to retire this debt.

3. In Problem 5 of Exercise 21, suppose the Atlas Corporation had been allowed to amortize this debt at 4 per cent, $m = 4$. Find the quarterly amortization payment. How much, each quarter, would they save by amortization over the sinking fund plan?

4. Mr. Jones wishes to borrow \$100,000. One bank will permit him to amortize the debt by semiannual payments for 10 years at 5 per cent, $m = 2$. The other bank requires interest to be paid semiannually at .045, $m = 2$, and the principal to be repaid at the end of 10 years. He can set up a sinking fund by making semiannual payments, with the fund paying interest at .035, $m = 2$. Which plan is more economical and how much will Mr. Jones save each 6 months by taking the more economical loan?

5. A debt of \$100,000 will be repaid at the end of 40 years, and interest at 4 per cent, $m = 2$, will be paid semiannually. A sinking fund can be set up by making equal semiannual deposits invested at 3 per cent, $m = 2$. Find the rate, compounded semiannually, at which this debt could be amortized with the same semiannual expense.

6. A debt of \$10,000 will be repaid at the end of 25 years, and interest at 4 per cent, $m = 4$, will be paid quarterly. In order to provide the \$10,000 at the end of the 25 years, equal quarterly deposits are made into a sinking fund earning interest at 3 per cent, $m = 4$. At what rate of interest, compounded semiannually, could this debt be amortized (by equal quarterly payments) with the same quarterly expense?

7. In Problem 8 of Exercise 21, find the interest rate, compounded quarterly, that Mr. Hill is actually paying to retire this debt.

8. In Problem 11 of Exercise 21, find the interest rate, compounded annually, that Mr. Goforth is actually paying to retire his debt.

9. In Problem 12 of Exercise 21, find the interest rate, compounded annually, that this man is actually paying for the retirement of his debt.

10. Find the rate of interest, compounded annually, that Mr. Krenshaw, in Problem 13 of Exercise 21, is paying for the retirement of his debt.

11. Find the rate of interest, compounded semiannually, that is actually being paid to retire the debt of Problem 14, Exercise 21.

12. Mr. Holcombe borrows \$20,000 and agrees to pay interest annually at 6 per cent. In order to repay the principal at the end of 15 years, he sets up a sinking fund that earns interest at 3 per cent compounded annually. At what rate of interest, compounded annually, could this debt be amortized with the same annual expense?

13. Mr. A. Thomas borrows \$5,000 from Oscar Peabody and pays him 6 per cent annually as interest on this debt. In order to repay the principal at the end of 22 years, Mr. Thomas makes annual deposits into a sinking fund earning interest at 4 per cent, $m = 1$. Find the rate of interest that Mr. Thomas is paying for the retirement of this debt.

14. Mr. Underwood borrows \$25,000 and agrees to pay the interest as it comes due at 5 per cent compounded semiannually. In order to provide for the repayment of the principal at the end of 18 years, he sets up a sinking fund earning interest at 6 per cent, $m = 2$. Find the rate of interest that Mr. Underwood is actually paying for the retirement of his debt.

15. Solve Problem 14 if the sinking fund rate of interest is 5 per cent compounded semiannually.

16. In Problem 10 of Exercise 21, find the interest rate, compounded annually, that Mr. Jones is actually paying to retire his debt. Hint: Set up the equation of value and then estimate different rates of interest that you think will give the correct value. After constructing a small table in this manner, interpolate for the correct interest rate.

6.1 Introduction. In many cases, when an organization, either public or private, needs a large amount of money, this money is not borrowed from a single source. Instead, bonds are sold to the general public in order to raise the desired funds.

A *bond* is a written contract to pay a fixed sum of money at a future date and to make periodic payments of smaller amounts until this date. These smaller payments are called *interest payments*, while the larger fixed sum is called the *redemption value* of the bond. In order to secure a large number of investors, bonds are usually issued in relatively small denominations. The denomination, the amount mentioned in the contract, is called the *face* of the bond. The most common size of bond is one with a face value of \$1,000, although any other face value might be used. In almost all cases the face value is a multiple of \$100; it may range from \$100 to \$10,000 or even higher.

The face value of a bond is used for two, and only two, purposes. From the face the size of the periodic bond *interest* payment is calculated, and from the face the *redemption value* is determined. For all other calculations involving bonds, these two items, the bond interest payments and the redemption value, are used.

Each bond contains a *bond interest rate*. From this rate and the face of the bond, the size and frequency of the bond interest payments are determined. The frequency of the compounding of the interest rate gives the frequency of the paying of the bond interest payments. For example, if the bond rate is compounded quarterly, the bond interest will be paid quarterly. The actual size of the interest payment is found by multiplying the bond rate per period by the face of the bond.

Many writers refer to this bond interest payment as a "dividend"

payment. But, according to correct usage, a dividend is a payment on stocks, not bonds. So dividend payment will not be used here. The reader should be careful not to confuse the bond interest rate and interest payment with the investor's rate, that is, the rate of interest actually earned by the purchaser of a bond. This investor's rate will be discussed later.

In giving the bond interest rate, the frequency of conversion and the date of the interest payments are often indicated by the initial letters of the months involved. For example:

1. 4 per cent, $m = 2$, payable on January 1 and July 1 would be expressed as 4 per cent, *JJ*.
2. 5 per cent, $m = 2$, payable on April 1 and October 1, would be stated as 5 per cent, *AO*.
3. 4 per cent, $m = 4$, payable on January 1, April 1, July 1, and October 1 would be stated as 4 per cent, *JAJO*.

In many bonds the redemption value is never mentioned. In that event the redemption value is always to be taken as the face value of the bond. If the redemption value and the face value are the same, the bond is said to be redeemed *at par*. If the redemption value is different from the face value, the redemption value is usually expressed as a percentage of the face value and the word "per cent" is omitted. For example:

A \$1,000 bond redeemable at 110 has a redemption value equal to 110 per cent of the face of \$1,000, i.e., the redemption value is \$1,100. (If the face value and redemption value differ, almost always, the redemption value is greater than the face value.)

Unless something is said to the contrary, it will be assumed that a bond matures on an interest payment date.

Bond interest is usually collected in one of two ways. In the case in which the owner's name is registered in the home office of the company issuing the bond, the interest payment check is sent to the owner each period. More often, the bond is a *coupon* bond. Attached to such a bond are coupons with the amount of each interest payment and the date it is due stated on a coupon. On each interest date, the owner "clips" the appropriate coupon and presents it for payment.

Bonds may be bought and sold many times in the interval from the date of their issue to their redemption date. Naturally, the owner will sell the bond to the buyer who offers the highest price. As a result, a bond is rarely sold for exactly its face value. In fact, even when a bond is first issued, the selling price is seldom the same as the

face of the bond. Thus, the sale and resale of bonds leads to two questions:

1. What price will the investor have to pay for a bond if he wishes to receive a specific rate of interest on the money invested?
2. If the investor pays a certain price for a bond, how much interest is he actually receiving on his investment?

These questions will be answered in this chapter.

6.2 Price of a Bond Bought on an Interest Date. If an investor pays exactly the face value, on an interest date, for a bond redeemable at par, then he will earn interest on his investment at precisely the bond interest rate. Since bonds are usually long-term contracts, the current investment rates of interest change during the life of a bond. Therefore, the purchaser would wish to pay a price for a bond that gives him an interest return on his investment in keeping with the current rate of interest at the time of purchase. Since the bond rate of interest never changes, the purchaser would have to pay a price different from the face value if he desired to earn an interest rate different from the bond rate. This interest rate earned by the buyer on his investment is called the *investor's rate* or *yield rate* of interest.

Each time a bond is bought and sold, this yield rate may change. The bond rate, and therefore the bond interest payment, and the redemption value of the bond remain the same, no matter how many times the bond is sold.

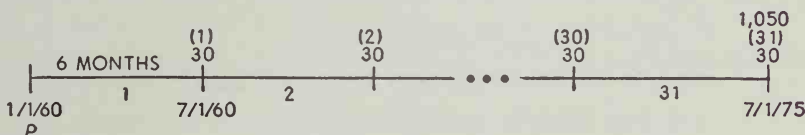
If a bond is bought on a bond interest date, the seller, who has had the bond up to that date, receives the bond interest payment due on that date. The buyer has a claim to the next bond interest payment, which is due one full payment period later, and to all future bond interest payments. Since these interest payments are equal in size and equally spaced in time, they form an annuity. Thus the buyer purchases this annuity of bond interest payments. In addition, he purchases a claim to the redemption value of the bond. Now these two, the annuity of interest payments and the redemption value, must return the purchaser's original investment and, at the same time, pay him interest on his investment at his yield rate. Thus, the *purchase price* of a bond is the present value of the redemption value of the bond plus the present value of the annuity of bond interest payments, both computed at the investor's or yield rate.

Illustration 1. A \$1,000, 6 per cent, *JJ* bond redeemable at 105 on July 1, 1975, is bought to yield 4 per cent, $m = 2$, on January 1, 1960. Find the purchase price.

Solution. The face of the bond is \$1,000. The bond rate is 3 per cent per 6 months. Therefore the bond interest payment will be $(.03)(1,000) = \$30$ each 6 months. These interest payments are made on January 1 and July 1. The redemption value is 105 per cent of the face, that is,

$$(1.05)(1,000) = \$1,050.$$

The seller receives the \$30 due on the purchase date, January 1, 1960. Therefore the buyer will receive his first \$30 payment on July 1, 1960, and he will receive such a payment each 6 months thereafter until and including July 1, 1975. Thus, he will receive 31 bond interest payments of \$30 each. He will also receive \$1,050 fifteen and a half years (or 31 half-years) after the purchase date. Let P be the purchase price. The line diagram will be:



The yield rate is 4 per cent, $m = 2$, or 2 per cent per 6 months. Taking the purchase date as the comparison date and using the yield rate,

$$P = 30a_{\overline{31}|.02} + 1,050(1.02)^{-31},$$

$$P = 30(22.93770152) + 1,050(.54124597),$$

$$P = 688.1310 + 568.3083,$$

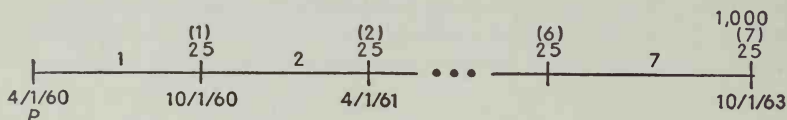
$$P = \$1,256.44.$$

As will be illustrated in a later section, this purchase price of \$1,256.44 is of such a size that (if the buyer does not resell the bond) the 31 payments of \$30 each and the redemption value of \$1,050 will repay the purchase price and also pay the buyer 2 per cent interest on all the money he has invested each period.

Illustration 2. A \$1,000, 5 per cent, *AO* bond is redeemable on October 1, 1963. If this bond is purchased on April 1, 1960, find the purchase price to yield the investor (a) 4 per cent, $m = 2$; (b) 5 per cent, $m = 2$; and (c) 6 per cent, $m = 2$.

Solution. Since no mention is made of the redemption value, it is assumed that the redemption value is the same as the face value, i.e., the bond is redeemed at par. Here, the redemption value will be \$1,000 and the redemption date is $3\frac{1}{2}$ years, or seven periods, after the purchase date.

The bond interest payment will be $(.025)(1,000) = \$25$ every 6 months. The buyer will receive the first \$25 on October 1, 1960, and the last on October 1, 1963. Thus he will receive 7 of these interest payments. Let P be the purchase price on April 1, 1960.



Part (a). Here the yield rate is 2 per cent per 6 months. Thus,

$$\begin{aligned} P &= 25a_{\overline{7}|.02} + 1,000(1.02)^{-7}, \\ P &= 25(6.47199107) + 1,000(.87056018), \\ P &= 161.7998 + 870.5602, \\ P &= \$1,032.36. \end{aligned}$$

Part (b). In this case, the yield rate is .025 per 6 months. Thus,

$$\begin{aligned} P &= 25a_{\overline{7}|.025} + 1,000(1.025)^{-7}, \\ P &= 25(6.34939060) + 1,000(.84126524), \\ P &= 158.7348 + 841.2652, \\ P &= \$1,000.00. \end{aligned}$$

Part (c). Here the yield rate is 3 per cent per 6 months:

$$\begin{aligned} P &= 25a_{\overline{7}|.03} + 1,000(1.03)^{-7}, \\ P &= 25(6.23028296) + 1,000(.81309151), \\ P &= 155.7571 + 813.0915, \\ P &= \$968.85. \end{aligned}$$

This problem illustrates the following facts. If a bond is redeemable at par and is bought on an interest date, and—

1. If the yield rate is the same as the bond rate, the purchase price will be the same as the face (redemption value) of the bond.
2. If the yield rate is greater than the bond rate, the purchase price will be smaller than the face of the bond.
3. If the yield rate is less than the bond rate, the purchase price will be greater than the face of the bond.

While the above statements are not quite accurate when the bond is not redeemable at par, the following general statement always holds. On a given date, and for a given bond, as the yield rate increases, the bond purchase price decreases. Likewise, as the yield rate decreases, the bond purchase price increases.

These facts will not only be helpful as a check on calculations here but will also be useful in some later work with bonds.

In all the illustrative examples above both the yield rate and the bond rate were for the same length periods. If the yield rate is for a period different in length from the period of the bond rate, the procedure will be the same, except the interest payments will now form a general case annuity.

Exercise 23

1. Find the purchase price on April 1, 1958, of a \$1,000, 6 per cent, JAJO bond redeemable on October 1, 1970. The bond is bought to yield 4 per cent, $m = 4$.

2. Find the purchase price of the bond in Problem 1 if the yield rate is 8 per cent, $m = 4$.

3. Find the purchase price of the bond in Problem 1 if the yield rate is 5 per cent, $m = 2$.

4. A \$1,000, 4 per cent bond, interest payable semiannually and redeemable at 108, is purchased 12 years before the redemption date. Find the purchase price if the yield rate is 5 per cent, $m = 2$.

5. Solve Problem 4 if the yield rate is 4 per cent, $m = 2$.

6. Find the price of the bond in Problem 4 if the yield rate is 3 per cent, $m = 2$.

7. Solve Problem 4 if the yield rate is 4 per cent, $m = 4$.

8. A \$500, 4 per cent bond with quarterly bond interest payments and redeemable at par is bought $11\frac{1}{2}$ years before its redemption date to yield 3 per cent, $m = 4$. Find the purchase price.

9. Find the purchase price of a \$10,000, 3 per cent bond with annual interest payments if bought 14 years before redemption date to yield 3 per cent, $m = 2$.

10. A \$2,000, 6 per cent, JJ bond is redeemable at par on January 1, 1990. If this bond is bought on January 1, 1965, to yield 4 per cent, $m = 2$, find its purchase price.

11. A \$1,000, 5 per cent bond, paying interest on February 1 and August 1, is redeemable at par on August 1, 1970. If this bond is purchased on February 1, 1965, to yield the investor 6 per cent compounded semiannually, find the purchase price.

12. A \$10,000, 6 per cent bond, paying interest on June 1 and December 1, is redeemable at 105 on June 1, 1978. Find the purchase price on December 1, 1966, to yield the purchaser 5 per cent compounded semiannually.

13. A \$100, 3 per cent, United States Treasury Bond, with interest dates June 1 and December 1, is redeemable on June 1, 1978. Find the price on June 1, 1963, to yield the purchaser 2 per cent compounded semiannually.

14. A \$10,000, 4 per cent, JAJO bond is redeemable at par on October 1, 1980. Find the purchase price on April 1, 1968, to yield the purchaser 3 per cent compounded semiannually.

15. On January 1, 1967, Mr. Smith purchases a \$5,000, 4 per cent bond with annual interest payments on January 1. This bond is redeemable on

January 1, 1999. If Mr. Smith gets a yield rate of 3 per cent compounded quarterly, find the purchase price.

16. A \$1,000, 4 per cent, JJ bond is redeemable at 115 on July 1, 1985. Find the purchase price of this bond on January 1, 1960, to yield the purchaser 4 per cent compounded quarterly.

6.3 The Premium-Discount Method. If a bond is redeemable at par, the purchase price minus the face value of the bond is called the *premium* paid on the bond. The subtraction is always taken in this order. This sometimes gives a negative premium. A *negative premium* is called a *discount*.

In Illustration 2 of Section 6.2, the bond in part (a) was bought at a premium of $1,032.36 - 1,000 = \$32.36$. In part (c) the premium was $968.85 - 1,000 = -\$31.15$. Therefore, this bond was bought at a discount of \$31.15. In part (b), the premium was $1,000 - 1,000 = 0$. Such a bond is said to be bought at par.

The above definitions of premium and discount are usually extended to bonds that are not redeemable at par. But in dealing with such bonds, certain ambiguities arise. Therefore, in this text the terms *premium* and *discount* will be applied only to bonds redeemable at par.

The method given in the preceding section for finding the purchase price on an interest date will work on any and every bond problem. But if the bond is redeemable at par, there is an arithmetically easier way to calculate the premium. Thus the price can be found by adding (algebraically) the premium (which may be positive or negative) to the face value.

The method of finding the premium directly is as follows:

1. Find the *expected interest* on the investment, assuming the price to be equal to the face value of the bond. That is, multiply the face value by the yield rate per period to obtain the *expected interest*.
2. Subtract the *expected interest* from the *bond interest* (always in that order) to get the *periodic difference*. This periodic difference may be either positive or negative. Keep the sign with it and find the present value, at the yield rate, of an annuity whose periodic payments are these *periodic differences*. The present value of this annuity is the premium (discount if it is negative).

Adding this signed premium to the face of the bond gives the purchase price.

The above method of finding the price of a bond is called the *premium-discount* method, and it is given here without proof.

Illustration. Find the purchase price of the bond in Illustration 2 of Section 6.2 by the premium-discount method.

Solution. In part (a) the yield rate is 2 per cent per 6 months. The expected interest is $(.02)(1,000) = \$20$. The bond interest is \$25; $\$25 - 20 = \5 , the periodic difference. Thus,

$$\begin{aligned}\text{premium} &= 5a_{\overline{7}|.02} = \$32.36, \\ P &= 1,000 + 32.36 = \$1,032.36.\end{aligned}$$

In part (b) the yield rate is .025 per 6 months and the expected interest is $(.025)(1,000) = \$25$. The bond interest is also \$25. The difference is 0. Thus the premium is 0, and

$$P = 1,000 + 0 = \$1,000.$$

In part (c), the yield rate is 3 per cent per 6 months and the expected interest is $(.03)(1,000) = \$30$. Since the bond interest is \$25, the periodic difference is $25 - 30 = -5$.

$$\begin{aligned}\text{Premium} &= -5a_{\overline{7}|.03} = -31.15 \\ P &= 1,000 + (-31.15) \\ P &= 1,000 - 31.15 = \$968.85\end{aligned}$$

Exercise 24

1. Use the premium-discount method to find the price of the bond in Problem 1 of Exercise 23.
2. Use the premium-discount method to find the price of the bond in Problem 2 of Exercise 23.
3. Find the purchase price of a \$5,000, 3 per cent, JJ bond redeemable at par and bought 12 years before the redemption date to yield 4 per cent, $m = 2$.
4. A \$10,000, 6 per cent, MS bond is redeemable at par on September 1, 1978. It is purchased on September 1, 1962, to yield 4 per cent, $m = 2$. Find the purchase price.
5. A \$5,000, 4 per cent bond, with interest payable semiannually, will be redeemed at the end of 30 years. Find the purchase price to yield 6 per cent, $m = 2$.
6. A \$500, 5 per cent bond, with interest payable quarterly, will be redeemed at the end of 20 years. If it is bought to yield 6 per cent, $m = 4$, find the purchase price.
7. If the yield rate on the bond in Problem 6 is 4 per cent, $m = 4$, find the purchase price.

6.4 Amortization of the Difference between the Purchase Price and the Redemption Value. When a bond is purchased on an in-

terest date, the purchase price is the *book value* of the bond at that time, i.e., the book value is the amount of capital invested in the bond at any given time.

If a bond, redeemable at par, is bought on an interest date at a yield rate equal to the bond rate, the purchase price (and book value) will be the same as the face value. Each bond interest payment will be precisely the interest expected at the yield rate on the capital invested, and the redemption value will return the original capital invested.

But if the redemption value is different from the face value or the yield rate differs from the bond rate, this is generally not the case. The question arises as to how the purchase price may be adjusted (or amortized) to the redemption value. The following examples will show how this can be accomplished.

Illustration 1. A \$1,000, 6 per cent, JJ bond is redeemable at 110 on July 1, 1965. If this bond is bought on July 1, 1962, to yield 4 per cent, $m = 2$, construct the investment schedule.

Solution. Since this bond is redeemable at 110, the redemption value is \$1,100. The bond interest payment is \$30, and there are 6 of these payments still to be made. Thus the price P is

$$P = 30a_{\overline{6}|.02} + 1,100(1.02)^{-6} = \$1,144.81.$$

Thus the book value on the date of purchase is \$1,144.81, while the redemption value is only \$1,100, i.e., the book value on the redemption date should be only \$1,100. This \$44.81 must be amortized over these six periods.

BOND INVESTMENT SCHEDULE

<i>Period</i>	<i>Book Value at Beginning of Period</i>	<i>Interest Due on B.V.</i>	<i>Bond Interest Payment</i>	<i>Change in Book Value</i>
1	\$1,144.81	\$ 22.90	\$ 30.00	\$ 7.10
2	1,137.71	22.75	30.00	7.25
3	1,130.46	22.61	30.00	7.39
4	1,123.07	22.46	30.00	7.54
5	1,115.53	22.31	30.00	7.69
6	1,107.84	22.16	30.00	7.84
7	1,100.00			
Totals		\$135.19	\$180.00	\$44.81

Now the yield rate is 2 per cent per 6 months. Thus, for the first period, the buyer expects an interest income of $(.02)(1,144.81) = \$22.90$. The actual bond interest received is \$30.00. Only \$22.90 can be listed as interest earned. The rest, \$7.10, is actually a repayment of capital. Thus at the end

of the first period (or the beginning of the second period) the book value is $1,144.81 - 7.10 = \$1,137.71$. For the second period an interest income of $(.02)(1,137.71) = \$22.75$ is expected. The bond interest payment of \$30 pays this interest and also returns 7.25 of the capital invested. This leaves a book value of \$1,130.46. If this method is continued over the six periods, the original book value of \$1,144.81 will be reduced to the redemption value of \$1,100. The completed schedule is shown on page 114.

The change in book value is subtracted here as the original book value must be decreased to the redemption value. The totals are for check purposes only.

Illustration 2. Construct the bond investment schedule for the bond in part (c) of Illustration 2 of Section 6.2.

Solution. In this problem, the purchase price, or book value on the date of purchase, was calculated to be \$968.85. The yield rate was 3 per cent per 6 months. Thus at the end of the first period the interest due on the book value is $(.03)(968.85) = \$29.07$. But the bond interest payment is only \$25.00. This means that \$4.07 of the interest due is not collected. This is equivalent to saying that the buyer now has \$4.07 more invested than at the beginning of this period. Thus the new book value is $\$968.85 + \$4.07 = \$972.92$. (Note that in this problem the original book value must increase to the maturity value of \$1,000.) Now at the end of the second period the interest due on the book value is $(.03)(972.92) = \$29.19$. Thus \$4.19 will be added to \$972.92 in order to get the new book value of \$977.11. The complete schedule is as follows:

BOND INVESTMENT SCHEDULE

<i>Period</i>	<i>Book Value at Beginning of Period</i>	<i>Interest Due on B.V.</i>	<i>Bond Interest Payment</i>	<i>Change in Book Value</i>
1	\$ 968.85	\$ 29.07	\$ 25.00	\$ 4.07
2	972.92	29.19	25.00	4.19
3	977.11	29.31	25.00	4.31
4	981.42	29.44	25.00	4.44
5	985.86	29.58	25.00	4.58
6	990.44	29.71	25.00	4.71
7	995.15	29.85	25.00	4.85
8	1,000.00			
Totals		\$206.15	\$175.00	\$31.15

If a bond is bought at a premium, it is customary to speak of amortizing the premium. If it is bought at a discount, the usual expression is accumulating the discount.

The book value at the beginning of each period could be calculated independently of the schedule, since this book value is the price that would be paid for this bond at the given yield rate. For example, the book value at the beginning of the fifth period could be calculated as

$$P = 25a_{\overline{31}|.03} + 1,000(1.03)^{-3},$$

$$P = \$985.86.$$

Since any book value can be calculated independently of the schedule, the rest of this line of the investment schedule may be constructed without having the earlier lines in the schedule.

Illustration 3. Without constructing the investment schedule, find what part of the fifth bond interest payment of Illustration 1 goes toward the repayment of capital.

Solution. At the beginning of the fifth period, only two bond interest payments remain. Thus, the book value at the beginning of the fifth period would be

$$\begin{aligned} \text{B.V.} &= 30a_{\overline{2}|.02} + 1,100(1.02)^{-2}, \\ &= \$1,115.53. \end{aligned}$$

Now, the interest expected on this book value for the fifth period is $(.02)(1,115.53) = \$22.31$. Since the bond interest payment is \$30, then $\$30.00 - \$22.31 = \$7.69$ is the part of the fifth bond interest payment that goes toward the repayment of the capital invested. These figures check with the schedule constructed in Illustration 1.

Exercise 25

1. A \$1,000, 4 per cent, AO bond is bought 2 years before its redemption date to yield an investor 2 per cent, $m = 2$. Construct the bond investment schedule.

2. If the yield rate on Problem 1 had been 6 per cent, $m = 2$, construct the bond investment schedule.

3. A \$1,000, 3 per cent, JJ bond is redeemable at 105 on July 1, 1970. If it is purchased on July 1, 1967, to yield 4 per cent, $m = 2$, construct the bond investment schedule.

4. If the bond in Problem 3 had been sold to yield 2 per cent, $m = 2$, construct the bond investment schedule.

5. A \$500, 4 per cent bond, with interest payable on July 15 and January 15, is redeemable at 105 on January 15, 1963. This bond is bought on July 15, 1960, to yield 3 per cent, $m = 2$. Construct the bond investment schedule.

6. Compute directly, without using the schedule (except as a check), the book value of the bond in Problem 5 just after the third bond interest payment has been received. Using this, find the amount of the fourth bond interest payment that is used to replace the capital invested.

7. Find the book value of the bond in Problem 1 of Exercise 23 on October 1, 1969, and construct the rest of the bond investment schedule for this bond.

8. Find the book value of the bond in Problem 4 of Exercise 23 two

years before the redemption date. Construct the rest of the bond investment schedule for this bond.

9. Without using the schedule, determine what part of the interest due on the book value at the time the third bond interest payment is received (i.e., the end of the third period) remains unpaid in Problem 2.

10. Construct the first eight lines of the bond investment schedule for the bond of Problem 8, Exercise 23.

11. Construct the first eight lines of the bond investment schedule for the bond of Problem 11, Exercise 23.

12. Find the book value of the bond in Problem 12 of Exercise 23 on June 1, 1975, and construct the rest of the bond investment schedule for this bond.

13. Find the book value of the bond in Problem 13 of Exercise 23 on June 1, 1976, and construct the rest of the bond investment schedule for this bond.

14. Construct the last six lines of the bond investment schedule for the bond in Problem 10, Exercise 23.

6.5 Purchase Price and Book Value between Bond Interest Dates. If a bond is bought between interest dates, the buyer will receive exactly the same number of bond interest payments, as well as the same redemption value, as if he had purchased the bond on the previous interest date. The price he pays for this bond to yield a given interest rate will be the present value of all the future bond interest payments and the redemption value, calculated at the yield rate. A fraction of an interest period is involved here. By the procedure discussed in Section 3.8, such a present value is found by first calculating the present value on the preceding interest date and then bringing this value up to the desired time by accumulating it at simple interest for the fraction of a period from the preceding interest date to the time the value is desired.

Thus, in order to find the purchase price of a bond bought between bond interest dates, first calculate the price that would have been paid on the preceding bond interest date, using the given yield rate. Now accumulate this price to the actual purchase date, using simple interest at the yield rate.

Illustration 1. A \$1,000, 6 per cent, JJ bond redeemable at 105 on July 1, 1975, is bought to yield 4 per cent, $m = 2$, on March 1, 1960. Find the purchase price.

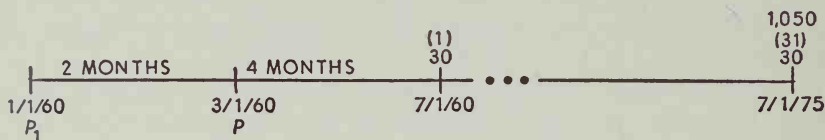
Solution. March 1, 1960, is not a bond interest date. The preceding interest date is January 1, 1960. Let P_1 be the purchase price on January 1, 1960. Since the redemption value is \$1,050 and the buyer will receive

31 bond interest payments of \$30 each, the purchase price P_1 on January 1, 1960, will be:

$$P_1 = 30a_{\overline{31}|.02} + 1,050(1.02)^{-31},$$

$$P_1 = \$1,256.44.$$

From January 1, 1960, to March 1, 1960, is 2 months, or one third of a period. The yield rate is 2 per cent per period. Thus \$1,256.44 must be accumulated at a simple interest rate of 2 per cent per period for one third of a period.



Using the simple interest formula,

$$P = 1,256.44[1 + (.02)(\frac{1}{3})],$$

$$P = 1,256.44\left(\frac{3.02}{3}\right),$$

$$P = \$1,264.82.$$

Thus the buyer would pay \$1,264.82 on March 1, 1960.

This actual purchase price is called the *flat price* of the bond.

Now the seller had this bond of the preceding illustration for 2 months out of the 6 months' bond interest period. Therefore, $\frac{2}{6}$ or $\frac{1}{3}$ of the next bond interest payment had accrued to him at the time of the sale. Thus the buyer figures that out of his purchase price of \$1,264.82, an amount equal to one third of the bond interest payment, that is, $\frac{1}{3}(30) = \$10$, was paid to the seller for his accrued interest. Thus the actual book value of the bond at the time of purchase was the flat price, \$1,264.82, less the accrued interest of \$10. Thus the book value of this bond on March 1, 1960, is \$1,264.82 - \$10.00 = \$1,254.82.

The *accrued interest* is that amount of the next bond interest payment due (accrued to) the seller.

The *book value* of a bond is the *flat price* (actual purchase price) minus the *accrued interest*.

Note that if a bond is purchased on an interest date, the accrued interest is 0. Thus, on a bond interest date, the *flat price* and *book value* are the same. This justifies the use of book value in Section 6.4.

In actual practice, when the price of a bond is quoted on the market, the actual purchase price or flat price is not the price quoted.

But the price quoted is the book value. Thus another term used for the *book value* of a bond is *quoted price*; the buyer pays the *quoted price* and the accrued interest when he purchases a bond. This leads to another expression, the "*and interest*" price. Thus, the three terms, *book value*, "*and interest*" price, and *quoted price* mean precisely the same thing.

Often, the quoted price is not given in dollars and cents, but it is given as a percentage of the face value of the bond. This percentage is given without using the per cent sign and is called the *quotation*. For example, a \$1,000 bond whose *quoted price* is \$1,070 would have a *quotation* of 107. This means the quoted price is 107 per cent of the face value. If the quotation on a \$500 bond is 90, then the quoted price will be $(.90)(500) = \$450$.

Illustration 2. For the bond of Illustration 1, find, on March 1, 1960, the quoted price and the quotation.

Solution. Most of this calculation has already been done above, but the results will be listed here for easy reference.

Flat price.....	\$1,264.82
Accrued interest.....	10.00
Quoted price.....	\$1,254.82

Now the quotation is the ratio of the quoted price to the face of the bond and given as a percentage without the per cent sign.

$$\text{Since } \frac{1,254.82}{1,000} = 1.25482 = 125.48\%,$$

$$\text{quotation} = 125.48.$$

In actual practice the fractional part of the quotation is rounded off to the nearest eighth. In the above example, since .48 is nearest $\frac{4}{8} = \frac{1}{2}$, the market quotation would be $125\frac{1}{2}$.

There is another method for finding the purchase price of a bond between interest dates. The arithmetic computation is a little longer, but it will be given here because the reader may encounter it in other places. This method gives the book value directly, and then the flat price may be found by adding the accrued interest. The method is as follows:

Calculate the purchase price on both the preceding interest date and the next interest date. Now interpolation between these two prices will give the *book value* on the date desired. Adding the accrued interest gives the flat price. It can be shown that this method is entirely equivalent to the method previously described.

The investment schedule for bonds bought between interest dates is constructed precisely the same as for bonds bought on an interest date, except for the first line. The book value calculated here is entered first. The first interest due is for a fraction of a period only. The first bond interest payment will be recorded as the bond interest minus the accrued interest. All the other calculations will be the same.

Exercise 26

1. A \$1,000, 6 per cent, AO bond redeemable on April 1, 1969, is bought on June 1, 1957, to yield the investor 5 per cent, $m = 2$. Find (a) the flat price on June 1, 1957; and (b) the book value of this bond on June 1, 1957.

2. A \$500, 5 per cent, MN bond is redeemable on November 1, 1972. On September 1, 1960, Mr. Applegate purchases this bond to yield him 6 per cent, $m = 2$. Find the "and interest" price on this date. What is the quotation on this bond on September 1, 1960?

3. A \$1,000, 4 per cent, JJ bond has a quoted price of \$1,043.52 on September 1, 1957. Find the flat price on that date.

4. A \$100, 5 per cent effective bond, with interest payments on January 15, is redeemable at 107 on January 15, 1983. Find the flat price and the quoted price on April 15, 1960, to yield the purchaser 4 per cent effective.

5. A \$2,000, 4 per cent, FMAN is redeemable at 103 on August 1, 1980. On June 1, 1965, this bond is offered for sale at a price to yield the purchaser 3 per cent, $m = 4$. Find the purchase price and the book value of this bond on June 1, 1965.

6. A \$5,000, 2 per cent, JJ bond has a book value of \$4,987 on May 1. Find the flat price on that date.

7. A \$1,000, 6 per cent bond, with interest payable quarterly, is bought 2 years and 1 month before the redemption date at a price to yield the investor 4 per cent, $m = 4$. Find (a) the flat price, and (b) the book value on this date.

8. Construct the bond investment schedule for the bond of Problem 7.

9. A \$1,000, 4 per cent, JJ bond is redeemable on July 1, 1967. It is purchased on April 1, 1960, at a price to yield the buyer 6 per cent, $m = 4$. Find (a) the flat price, and (b) the quoted price.

10. A \$1,000, 4 per cent, JJ bond is redeemable on January 1, 1973. On April 1, 1958, this bond is purchased to yield 5 per cent effective. Find (a) the actual purchase price, and (b) the "and interest" price.

11. A \$1,000, 4 per cent bond, with bond interest payable annually on February 1, is quoted at 97 on August 1. Find the flat price on that date.

12. A \$500, 4 per cent bond, with interest payable semiannually, is bought $2\frac{3}{4}$ years before the redemption date. If the yield rate is 6 per cent, $m = 2$, find the (a) flat price, and (b) book value on this date.

13. Construct the bond investment schedule for the bond in Problem 12.

14. A \$100, 6 per cent bond, with interest payable on February 1 and August 1, is quoted at 120 on December 1. Find the flat price on that date.

15. A \$1,000, 6 per cent bond, paying interest on February 1 and August 1, is redeemable at par on August 1, 1974. This bond is purchased to yield the investor 4 per cent compounded semiannually. Find (a) the flat price on May 1, 1969, and (b) the quoted price on May 1, 1969.

16. A \$100, 6 per cent, JJ bond is redeemable at par on July 1, 1978. If the yield rate is 5 per cent, $m = 2$, find the (a) flat price on April 1, 1969, and (b) the "and interest" price on April 1, 1969.

17. A \$100, 4 per cent bond, with interest dates January 1 and July 1, is quoted at 97.5 on February 15. Find the flat price on that date.

6.6 Finding the Yield Rate. One of the important questions in dealing with bonds is this: If a bond is purchased at a stated price, just what rate of interest is the buyer earning on the money he has invested? In other words, given the price, what is the yield rate?

There exist bond tables that give the book value of many bonds calculated for a wide range of yield rates and with the time from the purchase date to the redemption date varying from one period to many periods. If the bond price is given, the particular table covering that type of bond and with the correct number of periods from the purchase date to the redemption time can be found. From this table the yield rate can either be read directly or else computed by interpolation. Investment houses use extensive tables of this type, but such tables are far too large to be included in such a book as this. Even with the largest tables available, situations may arise that will not be in any of the existing tables.

Since the bond tables are not available, the next best thing is to calculate that portion of a bond table needed for each particular problem and then find the yield rate by interpolation. This may sound rather ambiguous and tedious, but in actual practice, it works out very nicely without too much computation. The procedure is this: Select some yield rate and calculate the purchase price of the particular bond. If the purchase price calculated is the same as the given purchase price, then the yield rate used is the desired one. But this is not the usual case. If the purchase price calculated is not the desired price, then another calculation must be made. This time there is a guide as to what yield rate to select next. From the discussion at the end of Section 6.2, it is recalled that as the purchase

price of a bond goes up, the yield rate goes down. Likewise, as the purchase price goes down, the yield rate increases. Therefore, if the calculated price is smaller than the actual purchase price, then the second yield rate tried must be smaller (since the calculated price must increase). On the other hand, if the calculated price was too large, then the next yield rate tried must be larger. The difference in the calculated price and the actual price will help to decide how great a change should be made in the yield rate.

This process is continued until one price is found larger than the actual purchase price and another is found smaller. Now, if the yield rates corresponding to these two calculated purchase prices are such that no other interest rate lies between them in the annuity and compound interest tables used, the actual purchase price has been enclosed as closely as possible between two calculated prices. Interpolation will give the yield rate. But accuracy in interpolation depends on the smallness of the interval over which the interpolation is done. Thus, if there is one or more interest rates in the tables used between the yield rates giving the larger and smaller purchase price, then calculate the purchase prices using these rates until the actual price is enclosed as closely as possible by the two calculated prices. Then interpolate.

If the bond is bought on an interest date and is redeemable at par, the bond rate itself is an excellent choice for the first yield rate to try. This is true, since no actual calculations are necessary here, if it is remembered that under these circumstances the purchase price will be the face of the bond. (See Illustration 2 (b) of Section 6.2 and also the statements following this problem.)

Even if the bond is not redeemable at par, unless other circumstances suggest another rate, it is often wise to take the first trial yield rate as the bond rate. In this case the calculation will actually have to be done.

Illustration. A \$1,000, 6 per cent, JJ bond, redeemable at par on July 1, 1973, is bought for \$1,107.50 on January 1, 1959. Find the yield rate, compounded semiannually.

Solution. This bond has a redemption value of \$1,000 and semiannual bond interest payments of \$30 each. It is bought $14\frac{1}{2}$ years, or 29 half-years, before the redemption date. Thus, 29 bond interest payments of \$30 each are still due. Since the purchase price is greater than the face of the bond (which is redeemable at par), then the yield rate will be less than the bond rate of 3 per cent per 6 months. So the first tried yield rate might

be 2 or $2\frac{1}{2}$ per cent. But since no computation is necessary, it might be noted that if the yield rate is .03 per 6 months, the price will be \$1,000.

\$1,000 is too small, and a higher price is desired, therefore a lower interest rate. Since there is a jump of \$107.50 from this price to the actual price, suppose 2 per cent per 6 months is tried as the next yield rate. Then if the yield rate is 2 per cent per 6 months,

$$P = 30a_{\overline{29}|.02} + 1,000(1.02)^{-29},$$

$$P = \$1,218.44.$$

This price is too large. Note that the two calculated prices, \$1,000 and \$1,218.44, enclose the actual price \$1,107.50. But since the tables contain three interest rates between 2 per cent and 3 per cent, the actual price is not enclosed as closely as possible. If interpolation were tried now, the answer would not be very accurate, not nearly as accurate as the tables permit.

It is convenient to record each calculated price and yield, as well as the actual price, in a table for reference. The table would now appear:

<i>Yield Rate</i>	<i>Price</i>
.03.....	\$1,000.00
?	1,107.50
.02.....	1,218.44

This table clearly shows that \$1,218.44 is too large and a smaller price is desired. Thus a yield rate larger than 2 per cent should be tried. Since \$1,107.50 is approximately halfway between the two calculated prices, try next the rate halfway between 2 and 3 per cent. Thus if the yield rate is .025 per 6 months,

$$P = 30a_{\overline{29}|.025} + 1,000(1.025)^{-29},$$

$$P = \$1,102.27.$$

The table of calculated prices now appears:

<i>Yield Rate</i>	<i>Price</i>
.03.....	\$1,000.00
.025.....	1,102.27
?	1,107.50
.02.....	1,218.44

This last price is close to \$1,107.50, but it is still too small. Thus a larger price is desired, that is, a smaller rate is needed. The only smaller rate in the table which is larger than .02 is .0225.

If the yield rate is .0225 per 6 months,

$$P = 30a_{\overline{29}|.0225} + 1,000(1.0225)^{-29},$$

$$P = \$1,158.49.$$

The complete table of calculated prices now appears:

<i>Yield Rate</i>	<i>Price</i>
.03.....	\$1,000.00
.025.....	1,102.27
?	1,107.50
.0225.....	1,158.49
.02.....	1,218.44

The actual purchase price is now enclosed as closely as possible, and interpolation between .0225 and .025 can now be done as follows:

<i>Yield Rate</i>	<i>Price</i>
.025 $\left[\begin{array}{c} .025 \\ c \\ ? \\ .0225 \end{array} \right]$	$\left[\begin{array}{c} 1,102.27 \\ 50.99 \\ 1,107.50 \\ 1,158.49 \end{array} \right] 56.22$

$$\begin{aligned}\frac{c}{.0025} &= \frac{50.99}{56.22} \\ c &= \frac{(50.99)(.0025)}{56.22} \\ c &= .00227\end{aligned}$$

Thus the yield rate is $.0225 + .00227 = .02477$ per 6 months, or .0495, $m = 2$.

Note that not all of the above calculations were necessary. After the price at .03 had been recorded, the next try should have been .025. If this had been done, the calculation for the price at .02 would never have been made. The rate .02 was tried before .025 in this solution in order to illustrate the various situations that might arise.

An arithmetic method of averages exists that may be used to get an approximate yield rate. But if a very accurate result is desired, the above method (or some other) must be used. The approximate rate only suggests which yield rate to try first when using the interpolation method. Therefore, this approximation method is not given here.

The above illustration was worked for a bond bought on a bond interest date. Precisely the same procedure would be used for a bond bought between interest dates. But the arithmetic computation would be much more complicated, since the calculation of a purchase price between interest dates involves more tedious arithmetic computation.

Exercise 27

1. A \$1,000, 5 per cent bond, with interest payable annually, is redeemable at par. The purchase price 12 years before the redemption date is \$1,097. Find the effective yield rate.

2. If the purchase price of the bond in Problem 1 is \$955, find the annual yield rate.

3. A \$500, 4 per cent, AO bond is redeemable at par on April 1, 1970. Its purchase price on October 1, 1963, is \$522.11. Find the yield rate, compounded semiannually.

4. A \$10,000, 4 per cent, JAJO bond is redeemable on October 1, 1962. If it is bought on April 1, 1958, for \$9,880, find the yield rate, compounded quarterly.

5. A \$1,000, 5 per cent bond, with bond interest payable annually, is redeemable at 105. It is bought 15 years before the redemption date for \$1,035. Find the yield rate.

6. If the purchase price of the bond in Problem 5 is \$1,100, find the yield rate.

7. If the purchase price of the bond in Problem 5 is \$950, find the yield rate.

8. A \$1,000, 4 per cent bond, with bond interest payable annually, is redeemable at 110. Twenty-five years before the redemption date, its price is \$950. Find the yield rate, compounded annually.

9. A \$1,000, 6 per cent, JJ bond is redeemable at par on July 1, 1965. It is purchased on March 1, 1960, for a purchase price (flat price) of \$1,030. Find the yield rate, compounded semiannually.

10. A \$1,000, 3 per cent, AO bond is redeemable at 110. If it is bought 20 years before the redemption date for \$1,000, find the yield rate, compounded semiannually.

11. A \$1,000, 5 per cent bond, with interest payable on January 1 and July 1, is redeemable at par on January 1, 1968. It is purchased on October 1, 1960, for \$985 (flat price). Find the yield rate, compounded semiannually.

12. Find the yield rate, compounded semiannually, of a \$100, 5 per cent bond with interest payable semiannually and redeemable at par in 10 years, if the quoted price is \$95.27.

13. A \$2,000, 6 per cent, JJ bond is redeemable at par on January 1, 1990. If this bond is bought on January 1, 1965, for \$2,106.73, find the yield rate, compounded semiannually.

14. If the purchase price of the bond in Problem 13 is \$1,978.65 on January 1, 1965, find the yield rate, $m = 2$.

15. A \$1,000, 5 per cent bond, paying interest on February 1 and August 1, is redeemable at par on August 1, 1970. If the purchase price of this bond is \$1,094.72 on February 1, 1965, find the yield rate, compounded semiannually.

16. A \$10,000, 6 per cent bond, paying interest on June 1 and December 1, is redeemable at 105 on June 1, 1978. On December 1, 1966, the purchase price is \$9,873.89. Find the yield rate, compounded semiannually.

17. If, on December 1, 1966, the purchase price of the bond of Problem 16 is \$11,001.63, find the yield rate, compounded semiannually.

18. If, on December 1, 1966, the purchase price of the bond of Problem 16 is \$10,000, find the yield rate, compounded semiannually.

19. If the purchase price of the bond in Problem 15 is \$987.49 on February 1, 1965, find the yield rate, compounded semiannually.
20. A \$10,000, 4 per cent, JAJO bond is redeemable at par on October 1, 1980. On April 1, 1968, its purchase price is \$10,987.36. Find the yield rate, compounded quarterly.
21. Find the yield rate, compounded monthly, for the bond in Problem 20.
22. If the bond of Problem 20 has a purchase price of \$9,903.76 on April 1, 1968, find the yield rate, compounded semiannually.

DEPRECIATION, PERPETUITIES, AND CAPITALIZED COST

7.1 Introduction. Through usage, any physical asset, such as a machine or building, decreases in value. Such a decline in value of a capital asset which cannot be corrected by ordinary upkeep is called *depreciation*. Over a period of time, such an asset will depreciate to the point that it must be replaced in order to continue efficient operation. In any business, this depreciation should be determined in advance and a *reserve for depreciation* should be built up so as to reflect the decreasing value of the asset in the accounting records. The periodic entries in the reserve for depreciation account are offset by corresponding entries in a depreciation account which represents (and makes a matter of record) the expense resulting from wear and tear of the asset.

It is not possible to know in advance precisely when a certain machine will have to be replaced, but through engineering experience it is possible to estimate rather accurately the *life* of the asset. At the end of the life of the asset, it may be worthless, or it may have a salvage value. In either case, the estimated value of the asset at the end of its life (before it is replaced) is called the *salvage* or *scrap value*. The difference between the *original cost* and the *scrap value* is called the *wearing value* of the asset. This wearing value is equivalent in amount to what should be in the reserve for depreciation account at the end of the estimated life. The *book value* of the asset at any time is the original cost minus the total of the entries in the reserve for depreciation account up to that particular time. Thus, the book

value of an asset at the end of its life, just before the company disposes of it, is the scrap value of the asset.

The depreciation reserve may be built up by a system of regular entries, or it could be accumulated by irregular entries made at infrequent intervals. Accountants do not approve an irregular entry plan. Although each concern may select any depreciation plan for its own business, the federal government limits the amount of depreciation which can be claimed as a business expense for tax purposes. Thus a system should ordinarily be selected that will stay within the governmental limitations.

Four methods of arriving at measures of depreciation expense will be considered here. All of these will assume that the entries will be made periodically and that they will follow a definite, predetermined plan.

7.2 The Straight Line Method. This is the simplest method of depreciation. The wearing value (the difference between cost and salvage value) is divided by the total number of periods in the life of the asset; the resulting number is the amount to be entered in the reserve for depreciation at the end of each period.

Although a period of any length may be used in a depreciation plan, a year is the most commonly used period.

Illustration. A \$5,000 machine has an estimated life of 8 years and a scrap value of \$200. Find (a) the annual depreciation, and (b) the book value of this machine at the end of 5 years.

Solution. The wearing value is $\$5,000 - \$200 = \$4,800$. Thus

$$\text{annual depreciation} = \frac{4,800}{8} = 600.$$

Since the amount in the depreciation reserve increases by \$600 each year, this reserve will total $(5)(600) = \$3,000$ at the end of 5 years. At this time,

$$\text{book value} = \$5,000 - \$3,000 = \$2,000.$$

The chief advantage of this method is its simplicity. For this reason, it is widely used. An important objection to it, however, lies in the fact that most equipment depreciates more rapidly during the early years of its life and less rapidly in the later years of its life. In the straight line method the depreciation for each year is the same. Thus, the book value of the asset by this method is greater in the early years than its actual value.

7.3 The Sinking Fund Method. In this method, the depreciation reserve is accumulated by making periodic (usually annual) entries in the depreciation accounts in accordance with mathematics of sinking funds, as illustrated below. At the end of the estimated life of the asset, the total of the entries in the reserve for depreciation will be an amount equal to the total wearing value of asset. Because depreciation by this method follows the pattern of sinking fund accumulation, the depreciation charge increases with each year.

It should be noted that entries may be made in the depreciation expense and in the reserve for depreciation accounts in accordance with the mathematics of sinking funds, whether or not a sinking fund as such is actually built up. Moreover, even if a fund is built up in this manner to provide for the replacement of an asset, depreciation expense could be determined by an entirely different method. The accounting entries associated with setting up an actual sinking fund and recording the income due to interest additions to it are separate and distinct from those required to account for depreciation expense and decreased value of the asset. In this section, our only interest is in the mathematics connected with the handling of the reserve for depreciation account. All items are treated as accounting entries only, and no actual fund is set up.

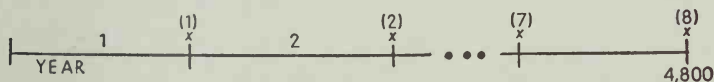
A depreciation schedule can be constructed in the same way as a regular sinking fund schedule, with the addition of a column for book value of the asset.

Illustration 1. A \$5,000 machine has an estimated life of 8 years and a scrap value of \$200. The sinking fund method of depreciation is to be used with "interest accumulation" at 3 per cent effective.

a) Find the annual "payment" into the "sinking fund" reserve for depreciation account.

b) Set up the depreciation schedule.

Solution of (a). Since this machine has a wearing value of \$4,800, the amount in the "sinking fund" at the end of 8 years should be \$4,800. Let x be the size of the annual "payment" into the "sinking fund." The line diagram will be:



$$xs_{\overline{8}|.03} = 4,800$$

$$x = 4,800 \cdot \frac{1}{s_{\overline{8}|.03}}$$

$$x = \$539.79$$

Solution of (b). The depreciation schedule is constructed in essentially the same manner as the sinking fund growth schedule, with a column added for the book value of the machine at the end of each period. This book value is found either by subtracting the amount in the reserve for depreciation account at the end of the period from the original cost (original book value) or by subtracting the total increase in the reserve for any period from the book value at the end of the preceding period.

DEPRECIATION SCHEDULE

Year	<i>In "Sinking Fund" at Beginning of Year</i>	<i>Interest on "Sinking Fund"</i>	<i>Periodic "Payment" into "Sinking Fund"</i>	<i>Depreciation Charge</i>	<i>Book Value at End of Year</i>
1	0	0	\$ 539.79	\$ 539.79	\$4,460.21
2	\$ 539.79	\$ 16.19	539.79	555.98	3,904.23
3	1,095.77	32.87	539.79	572.66	3,331.57
4	1,668.43	50.05	539.79	589.84	2,741.73
5	2,258.27	67.75	539.79	607.54	2,134.19
6	2,865.81	85.97	539.79	625.76	1,508.43
7	3,491.57	104.75	539.79	644.54	863.89
8	4,136.11	124.08	539.81	663.89	200.00
9	4,800.00				
Totals		\$481.66	\$4,318.34	\$4,800.00	

The last "payment" into the "sinking fund" was increased by 2 cents in order to take care of the error introduced by rounding off all computation to two decimal places.

Illustration 2. For the machine in Illustration 1, find, without using the schedule, (a) the book value at the beginning of the sixth year (end of the fifth year); (b) the depreciation charge for the sixth year; and (c) the remaining wearing value at the beginning of the sixth year.

Solution of (a). From Illustration 1, the annual "payment" is \$539.79. Thus, at the end of the fifth year, the reserve for depreciation account will contain entries totaling

$$\$539.79s_{\overline{5}|.03} = \$2,865.82.$$

It will be noted that this is 1 cent more than the schedule gives in the "sinking fund" at this time. This 1 cent was "lost" in the schedule in the rounding off of the interest computation up to this point. (Note that 2 cents had to be added in the last "payment" to compensate for the "loss" in rounding off for the entire 8 years.)

Thus the book value at the end of the fifth (beginning of the sixth) year will be:

$$\text{book value} = \$5,000 - \$2,865.82 = \$2,134.18.$$

Solution of (b). The depreciation charge for the sixth year may be found by two methods. The amount in the depreciation reserve ("sinking fund") at the end of the fifth year may be calculated, and likewise at the end of the sixth year. The difference between these two amounts will be the depreciation charge for the sixth year.

Another method is to calculate the amount in the depreciation reserve at the end of the fifth year to compute the interest this money will "earn" during the period and to add this interest to the yearly "payment." This sum will be the depreciation charge for the sixth year. This method will be used here.

In part (a), it was found that \$2,865.82 is the amount in the depreciation reserve at the end of the fifth year. Interest on this for 1 year will be:

$$(.03)(2,865.82) = \$85.97 .$$

Adding this to the "payment" of \$539.79 gives:

$$\text{depreciation charge} = 539.79 + 85.97 = \$625.76 .$$

This agrees with the entry in the schedule.

Solution of (c). The book value at the beginning of the sixth year is \$2,134.18, and the scrap value is \$200. Thus, at the beginning of the sixth year the

$$\text{remaining wearing value} = 2,134.18 - 200 = \$1,934.18 .$$

The same machine was used in the illustrations of this section and the preceding section. Under the straight line method, this machine had a book value at the end of 5 years of \$2,000. Under the sinking fund method, the book value at the same time was \$2,134.18.

The sinking fund method has the advantage of being a comparatively simple application of the theory of annuities. But, as is seen by the above comparison, the book value in accordance with it will still be higher, in the earlier years, than the actual value of the asset. In fact, under the sinking fund plan, the depreciation charge for each period is greater than for the preceding period. In actual practice, the decrease in resale value normally gets smaller each year.

7.4 The Constant Percentage Method. In this method, a fixed percentage of the book value at the beginning of each period is added to the depreciation reserve at the end of that period.

For example, if the constant percentage rate of depreciation is 10 per cent on a \$5,000 machine, the entry in the reserve for depreciation (and the depreciation charge) for the first year will be

$$(.10)(5,000) = \$500 .$$

This will leave a book value of \$4,500 at the beginning of the second year. Consequently, the amount to be entered in the accounts for the second year will be $(.10)(4,500) = \$450$. The book value at the beginning of the third year is then \$4,050. The depreciation for the third year will be \$405. This process can be continued as long as desired.

Such a depreciation system has the advantage of the heavier depreciation charges coming early in the life of the asset, with the depreciation charges getting smaller each period. It should be clear, though, that no matter how long this process is continued, the book value could never be brought down to a zero scrap value, unless the rate of depreciation is 100 per cent and the life of the asset is 1 year. But this difficulty can be circumvented by reducing the book value to any desired amount (different from zero) in a period 1 year shorter than the life of the asset and writing off the entire remaining book value in the last year.

It should also be clear that the rate of depreciation must be very carefully chosen if the original book value is to be reduced to a specified scrap value in a given time. A formula from which the depreciation rate may be calculated will now be developed.

Let C be *original cost* of the asset, n the *number of periods in life* of this asset, S the *scrap value*, and d the *constant percentage rate* of depreciation. Let $B.V._1$ be the book value of the asset at the end of the first period, $B.V._2$ = book value at end of second period, and, in general, $B.V._k$ = book value at the end of the k th period.

With this notation, the depreciation charge for the first year will be Cd and $B.V._1 = C - Cd = C(1 - d)$. The depreciation charge for the second year will then be $C(1 - d)d$. Thus $B.V._2 = C(1 - d) - C(1 - d)d = C(1 - d)(1 - d) = C(1 - d)^2$. The book values may be tabulated thus:

$$\begin{aligned} B.V._1 &= C - Cd = C(1 - d), \\ B.V._2 &= C(1 - d) - C(1 - d)d = C(1 - d)(1 - d) = C(1 - d)^2, \\ B.V._3 &= C(1 - d)^2 - C(1 - d)^2d = C(1 - d)^2(1 - d) = C(1 - d)^3, \\ B.V._4 &= C(1 - d)^3 - C(1 - d)^3d = C(1 - d)^3(1 - d) = C(1 - d)^4. \end{aligned}$$

From this development it appears that

$$(7) \quad B.V._k = C(1 - d)^k,$$

where k is any positive (or zero) integer less than or equal to n , the number of periods in the life of the asset. By means of mathematical induction, it can be proved that the above formula does follow. The proof will not be given here.

Illustration 1. A \$5,000 machine has an estimated life of 8 years and a scrap value of \$200. If the constant percentage method of depreciation is used, find the annual rate of depreciation.

Solution. Here $C = 5,000$, $n = 8$, and $S = 200$. Now the scrap value is the book value at the end of the life of the machine. Thus $B.V._8 = 200$. Using formula (7):

$$5,000(1 - d)^8 = 200$$

$$(1 - d)^8 = \frac{200}{5,000}$$

Since tables are not available for $(1 - d)^n$, logarithms will be necessary. Now, the right-hand side of this equation can be simplified but, as will be seen in Illustration 2, it is advisable not to make this simplification.

$$\log(1 - d)^8 = \log \frac{200}{5000}$$

$$8 \log(1 - d) = \log 200 - \log 5000$$

$$\log(1 - d) = \frac{\log 200 - \log 5000}{8}$$

$$\log 200 = 12.301030 - 10$$

$$\log 5000 = \frac{3.698970}{8.602060 - 10}$$

$$\log(1 - d) = \frac{8.602060 - 10}{8}$$

$$\log(1 - d) = \frac{14.602060 - 16}{8}$$

$$\log(1 - d) = 1.825258 - 2$$

Taking antilogarithms of both sides,

$$\begin{aligned} 1 - d &= .66874, \\ 1 - .66874 &= d, \\ d &= .33126. \end{aligned}$$

This means that 33.13 per cent of the book value must be written off each year as depreciation in order to get the book value down to the scrap value of \$200 in 8 years.

Illustration 2. For the machine of Illustration 1, find the book value at the end of the fifth year.

Solution. Since $\log(1 - d)$, not d , will be used in the calculation, the approximate value, .3313, for d will not be substituted in the formula. The more accurate value for $\log(1 - d)$ will be substituted at the proper time. Using formula (7):

$$\begin{aligned}
 B.V._5 &= 5000(1 - d)^5, \\
 \log B.V._5 &= \log 5000(1 - d)^5, \\
 \log B.V._5 &= \log 5000 + 5 \log(1 - d).
 \end{aligned}$$

Since the numerical simplification of the equation in Illustration 1 was not done before logarithms were taken, each logarithm desired here can be read directly from the work there without having to use tables. Thus

$$\begin{array}{rcl}
 \log 5000 & = & 3.698970 \\
 5 \log(1 - d) = 5(1.825258 - 2) & = & 9.126290 - 10 \\
 \log B.V._5 & = & 12.825260 - 10
 \end{array}$$

Taking antilogarithms of both sides:

$$B.V._5 = \$668.74.$$

Compare this answer with the answer to Illustration 2 (a) of Section 7.3 and the (b) part of the Illustration of Section 7.2.

Illustration 3. Construct the depreciation schedule for the machine in Illustration 1.

Solution. In order to make the schedule as accurate as possible, the constant rate of depreciation will be used to five decimal places, i.e., $d = .33126$. Each entry in the accounts will be an amount equal to 33.126 per cent of the book value at the beginning of the year for which it is made.

DEPRECIATION SCHEDULE

Year	Book Value at Beginning of Year	Depreciation Charge	Total in Depreciation Reserve at End of Year	Book Value at End of Year
1	\$5,000.00	\$1,656.30	\$1,656.30	\$3,343.70
2	3,343.70	1,107.63	2,763.93	2,236.07
3	2,236.07	740.72	3,504.65	1,495.35
4	1,495.35	495.35	4,000.00	1,000.00
5	1,000.00	331.26	4,331.26	668.74
6	668.74	221.53	4,552.79	447.21
7	447.21	148.14	4,700.93	299.07
8	299.07	99.07	4,800.00	200.00

This schedule gives the book value at the end of 5 years as \$668.74. This is the result calculated in Illustration 2 without using the schedule.

The *declining-balance method* of depreciation, as sanctioned by the 1954 Tax Code, is essentially a special application of the constant percentage method. By it, if the life of the asset is n years, the percentage d allowed each year on the book value at the beginning of the year is $d = \left(\frac{2}{n}\right)(100)$ per cent. For example, if the asset has a 10-year life, the value of d is 20 per cent.

7.5 The Sum of Digits Method. This is a method that gives an arithmetic approximation to the constant percentage method of depreciation. For each period, a certain fractional part of the total wearing value is placed in the depreciation reserve. This fraction is found as follows:

The denominator of the fraction for each period is the sum of the number of periods in the life of the asset and each smaller integer down to and including one. For example, if the life of the asset is 4 periods, then denominator equals $4 + 3 + 2 + 1 = 10$. This will be the denominator of each fraction.

The numerator of the fraction for any particular period is the number of periods remaining in the life of the asset, including the period for which this particular depreciation is being computed. For an asset with a life of 4 periods, the numerator for the first period will be 4, for the second period 3, for the third period 2, and for the fourth period 1. Thus, in this example the fractions will be $\frac{4}{10}$, $\frac{3}{10}$, $\frac{2}{10}$, and $\frac{1}{10}$.

It is easy to show that the fractions formed in this manner will always have a sum of 1, regardless of the number of periods in the life of the asset. Thus, an amount exactly equal to the total wearing value of the asset will be placed in the depreciation reserve.

The depreciation charge is the largest the first period; it decreases each period thereafter.

Illustration. A \$5,000 machine has an estimated life of 8 years and a scrap value of \$200. If the sum of digits method is used, construct the depreciation schedule, using periods of 1 year.

Solution. The wearing value of this machine is \$4,800. Since it has a life of 8 years, the denominator of each fraction will be:

$$8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36.$$

The fractions will be:

$$\frac{8}{36}, \frac{7}{36}, \frac{6}{36}, \frac{5}{36}, \frac{4}{36}, \frac{3}{36}, \frac{2}{36}, \text{ and } \frac{1}{36}.$$

The depreciation for the first year will be:

$$\frac{8}{36}(4,800) = \$1,066.67.$$

For the second year,

$$\frac{7}{36}(4,800) = \$933.33.$$

The completed schedule follows:

DEPRECIATION SCHEDULE

<i>Year</i>	<i>Book Value at Beginning of Year</i>	<i>Depreciation Charge</i>	<i>Total in Depreciation Reserve at End of Year</i>	<i>Book Value at End of Year</i>
1	\$5,000.00	\$1,066.67	\$1,066.67	\$3,933.33
2	3,933.33	933.33	2,000.00	3,000.00
3	3,000.00	800.00	2,800.00	2,200.00
4	2,200.00	666.67	3,466.67	1,533.33
5	1,533.33	533.33	4,000.00	1,000.00
6	1,000.00	400.00	4,400.00	600.00
7	600.00	266.67	4,666.67	333.33
8	333.33	133.33	4,800.00	200.00

Since the same problem was used as an illustration of all four methods, the comparison of the book values at the end of 5 years will be listed here.

Straight line:	$B.V._5 = \$2,000.00$
Sinking fund:	$B.V._5 = \$2,134.18$
Constant percentage:	$B.V._5 = \$668.74$
Sum of digits:	$B.V._5 = \$1,000.00$

Exercise 28

1. A machine costing \$12,000 has an estimated life of 5 years and a scrap value of \$2,000. By the straight line method, find (a) the annual depreciation charge, and (b) the book value at the end of 3 years.

2. If the machine of Problem 1 is depreciated by the sinking fund method and the sinking fund interest rate is 2 per cent annually, find (a) the annual "payment" into the "sinking fund," (b) the book value at the end of 3 years, and (c) the total depreciation charge for the fourth year.

3. Construct the depreciation schedule for Problem 2.

4. If the constant percentage method of depreciation is used on the machine in Problem 1, find (a) the rate of depreciation, and (b) the book value at the end of 3 years.

5. Construct the depreciation schedule for Problem 4.

6. If the sum of digits method of depreciation is used on the machine of Problem 1, construct the depreciation schedule.

7. A \$50,000 building will have to be replaced at the end of 25 years. If the scrap value of the building is \$5,000, find the annual "payment" if the depreciation is computed by the sinking fund method. The sinking fund interest rate is 3 per cent effective.

8. a) Find the book value of the building in Problem 7 at the end of 18 years.

b) Find the depreciation charge for the nineteenth year.

9. Construct the last seven lines of the depreciation schedule for the building of Problem 8.

10. A \$7,500 machine has an estimated scrap value of \$300 and a life of 12 years. If the constant percentage method of depreciation is used, find the rate of depreciation.

11. Find the book value of the machine in Problem 10 at the end of 10 years.

12. Construct the last two lines of the depreciation schedule for the machine in Problem 10.

13. Using the sum of digits method, construct the first four lines of the depreciation schedule of the machine in Problem 10.

14. A \$50,000 building will be depreciated by the constant percentage method at a rate of 20 per cent. Find the book value at the end of 30 years.

15. For the building of Problem 14, find the depreciation charge for the seventeenth year.

16. Construct the depreciation schedule for an asset costing \$10,000, with a life of 5 years and no scrap value. The sinking fund method is to be used, with the sinking fund interest rate at 2 per cent effective.

17. A \$5,000 textile machine has an estimated life of 20 years with a scrap value of \$500. By the sum of digits method find (a) the depreciation for the first year, (b) the depreciation for the tenth year, and (c) the book value at the end of the tenth year.

18. If the constant percentage method of depreciation is used for the machine in Problem 17, find (a) the rate of depreciation, (b) the depreciation for the first year, and (c) the book value at the end of the tenth year.

Compare these answers with those found in Problem 17.

19. Solve Problem 17 by the straight line method.

20. Use the declining-balance method described in Section 7.4 to find the rate of depreciation for the machine in Problem 17. Use this rate to find (a) the depreciation for the first year, and (b) the book value at the end of the tenth year.

21. Richland County purchases \$50,000 of street-cleaning equipment which has an estimated life of 6 years and an expected scrap value of \$10,000. Depreciation is to be provided for by the sinking fund method with the sinking fund interest rate as 3 per cent annually.

a) Find the annual "payment" into the "sinking fund."

b) Find the amount in the "sinking fund" reserve for depreciation at the end of 4 years.

c) Find the book value of the equipment at the end of 4 years.

7.6 Perpetuities. A *perpetuity* is a set of equal periodic payments that continue forever. Thus a perpetuity is a perpetual annuity. Quite clearly, the term *amount* has no meaning here, for such a fund would continue to increase without any bound.

The *present value* of a perpetuity is defined to be the value of all

the future payments one payment period before the first payment. That is, the present value is that sum that could be deposited and from which, beginning one payment period after the deposit, equal periodic withdrawals could be made forever. That the present value of a perpetuity has meaning will be clear from the following example.

Illustration. How much must be deposited now in a fund earning 4 per cent, $m = 2$, in order that \$500 can be withdrawn 6 months after the deposit and at the end of each 6 months thereafter forever.

Solution. Let A be the present value of this perpetuity of \$500, semi-annual payments; $i = .02$ per 6 months. Now A will earn interest for one period before the first \$500 is withdrawn. If this \$500 withdrawal is greater than the interest earned, even by 1 cent, the principal will be reduced, and the interest for the next period will be less, and eventually the principal will be depleted. Thus the \$500 payments could not continue forever. On the other hand, if the \$500 is less than the interest earned for the first period, then the principal for the second period will be larger and will continue to grow larger and larger indefinitely. Thus the original deposit A was too large.

It must be concluded then that the \$500 should be exactly the interest earned during the first period. When the \$500 withdrawal is made, the same principal is left in the account and will earn another \$500 interest to be withdrawn at the end of the next period. This can continue forever. Thus,

$$.02A = 500,$$

$$A = \frac{500}{.02},$$

$$A = \$25,000.$$

7.7 Perpetuity Formula. The above example assumed that the interest period and the payment period were equal. Very often this is not the case in perpetuities. Thus a formula will be developed to cover all cases, whether the payment period is equal to the interest period or not. The symbols A and R are defined exactly as in the case of annuities certain, but their definitions will be repeated here for reference.

Let

R = size of each payment,

i = interest rate per interest period,

A = value of all payments one payment period before the first payment,

k = number of interest periods in a payment period.

In perpetuities, k is usually an integer, but it could be a fraction.

The perpetuity formula could be derived by using the formula for the sum of an infinite geometric progression. But the type of reasoning used in the preceding illustration will be used here.

If the payment period and interest period were the same in length, the reasoning of the above illustration would fit precisely. So let c be the rate per payment period. Then, by the reasoning of the illustration of Section 7.6,

$$cA = R,$$

$$A = \frac{R}{c}.$$

In order to find the relationship between c and i , the method of equivalent interest rates may be used. If \$1 is deposited for one payment period at the rate c per payment period, the $S = (1 + c)^1$. If \$1 is deposited for one payment period at the rate i per interest period, then $S = (1 + i)^k$, since there are k interest periods in a payment period. Since these rates are equivalent,

$$\begin{aligned} 1 + c &= (1 + i)^k, \\ c &= (1 + i)^k - 1. \end{aligned}$$

Now replace c , in the equation $A = \frac{R}{c}$ by this value:

$$A = \frac{R}{(1 + i)^k - 1}.$$

Multiplying the numerator and the denominator by i , and rearranging the fraction,

$$A = \frac{R}{i} \frac{i}{(1 + i)^k - 1},$$

$$(8) \quad A = \frac{R}{i} \cdot \frac{1}{s_{\overline{k}|i}}.$$

This formula (8) gives the present value of a perpetuity one payment period before the first payment. If the value is desired on any other date, compound interest may be used to move A to the desired date.

Illustration 1. Find the present value of a perpetuity of \$10,000 payable at the end of each 10 years if interest is at 4 per cent, $m = 4$.

Solution. Here, $R = \$10,000$, $i = .01$ per 3 months, and $k = 40$, since there are 40 interest periods of 3 months each in 10 years (the pay-

ment period). In the line diagram, the arrow on the right end indicates the payments continue forever.



$$A = \frac{10,000}{.01} \frac{1}{s_{40|.01}}$$

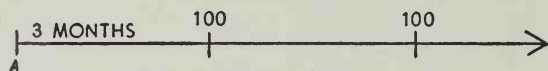
$$A = 1,000,000(.02045560)$$

$$A = \$20,455.60$$

This means that if \$20,455.60 is deposited in an account paying 4 per cent, $m = 4$, then 10 years later, and each 10 years thereafter forever, \$10,000 can be withdrawn from this account.

Illustration 2. Find the present value of a perpetuity of \$100 payable at the end of each 3 months if money is worth 4 per cent, $m = 4$.

Solution. Here $R = \$100$, $i = .01$ per 3 months, and $k = 1$, since both the interest period and the payment period are 3 months in length.



$$A = \frac{100}{.01} \frac{1}{s_{\infty|.01}}$$

Now it is easy to show that $\frac{1}{s_{\infty|i}} = 1$ for all values of i . (The tables also give this value.) Thus

$$A = \frac{100}{.01},$$

$$A = \$10,000.$$

7.8 Capitalized Cost. The *capitalized cost* of an asset is the initial cost plus the present value of the perpetuity of all future replacement costs. That is to say, the capitalized cost of an asset is the total amount of money needed to provide that asset forever.

Illustration. Find the capitalized cost of a building that can be built now for \$125,000 and has to be replaced at the end of each 40 years at a cost of \$100,000. Money is worth 3 per cent effective.

Solution. The \$100,000 replacement costs form a perpetuity with $R = 100,000$, $i = .03$ per year, and $k = 40$. To the present value of this perpetuity must be added the original cost, \$125,000. Let C be the capitalized cost of this building.



$$C = 125,000 + \frac{100,000}{.03} \frac{1}{s_{\overline{40}|.03}}$$

$$C = 125,000 + \frac{(100,000)(.01326238)}{.03}$$

$$C = 125,000 + 44,207.93$$

$$C = \$169,207.93$$

7.9 Comparison of the Cost of Two Assets with Different Life Spans. Capitalized cost is an extremely important concept in modern economic theory. Along with its many applications there, it can be very useful in comparing the values of two assets with different life spans. The following problem will illustrate the method.

Illustration. The Acme Manufacturing Company can purchase a machine for \$11,000. This machine will have to be replaced at the end of each 10 years at a cost of \$10,000. Another machine, doing the same work but having a life span of 12 years, can be bought. If the replacement cost of this machine is the same as the original cost, how much could they afford to pay for the second type machine? Money is worth 4 per cent effective.

Solution. This answer will be found by comparing the capitalized cost of these two machines. For the first machine the capitalized cost will be:

$$C = 11,000 + \frac{10,000}{.04} \frac{1}{s_{\overline{10}|.04}}$$

Let x be the cost of the second machine. Then its capitalized cost will be:

$$C = x + \frac{x}{.04} \frac{1}{s_{\overline{12}|.04}}$$

Equating these two capitalized costs:

$$x + \frac{x}{.04} \frac{1}{s_{\overline{12}|.04}} = 11,000 + \frac{10,000}{.04} \frac{1}{s_{\overline{10}|.04}},$$

$$x + x \frac{.06655217}{.04} = 11,000 + \frac{10,000(.08329094)}{.04},$$

$$x + x (1.66380425) = 11,000 + 20,822.735,$$

$$2.66380425x = 31,822.735,$$

$$x = \frac{31,822.735}{2.66380425},$$

$$x = \$11,946.35.$$

Exercise 29

1. Find the present value of a perpetuity of \$5,000 payable annually if money is worth 4 per cent, $m = 4$.
2. Find the present value of a perpetuity of \$5,000 payable annually if money is worth 4 per cent effective.
3. Mr. I. M. Rich wishes to set up an endowment for scholarships to Weteachem University. The scholarships total \$8,000 annually and are to be paid at the end of each year after the endowment is established. If the endowment earns interest at 3 per cent, $m = 2$, find the amount of money Mr. Rich must place in the endowment fund.
4. Mr. Jones wishes to set up a fund that will pay \$3,000 annually to an orphanage. If the fund earns interest at 3 per cent, $m = 4$, and the first payment of \$3,000 comes 5 years after the fund is established, find the amount Mr. Jones must deposit in the fund.
5. To maintain a certain paved road, \$8,000 will be needed 4 years from now, and annually thereafter. If money is worth 3 per cent effective, find the value now of the perpetual maintenance.
6. Find the capitalized cost of a building which can be constructed for \$200,000 and will have an estimated life of 35 years. At the end of each 35 years, this building will have to be replaced at a total cost of \$210,000. Money is worth 6 per cent, $m = 2$.
7. Find the capitalized cost of a machine which can be bought for \$5,000 and which has a life of 10 years. The machine must be replaced at the same cost at the end of each 10 years. Money is worth 5 per cent effective.
8. A certain section of paved walkways can be constructed for \$5,000. At the end of each 15 years it will cost \$500 to remove the broken pavement and make preparation for the repaving. If the repaving costs the same as the original paving, find the capitalized cost of this project at 3 per cent effective.
9. A taxi company must maintain a fleet of 30 cabs. If each cab costs \$2,000 and has a scrap value of \$500 at the end of 3 years, find the capitalized cost of this fleet of cabs at 4 per cent.
10. A certain machine costing \$3,000 and having a life of 8 years has a scrap value of \$300. If money is worth 3 per cent, $m = 2$, how much could be paid for another machine which does the same work but has a life of 10 years and no scrap value.
11. Solve Problem 10 if the second machine could do 50 per cent more work than the first machine.
12. Find the amount of money necessary to endow a library so as to provide for the following:
 - a) Construction of a building at an initial cost of \$100,000 and its replacement every 30 years at a cost of \$90,000.
 - b) The purchase of books worth \$100,000 at the time of construction and the purchase of \$10,000 worth of books at the end of each year.

c) Maintenance cost of \$15,000 annually, the first payment being made 1 year after the construction.

Money is worth 3 per cent compounded annually.

13. Find the capitalized cost of a machine which may be bought now for \$10,000, must be repaired at the end of each year at a cost of \$500, lasts 7 years, and has a scrap value of \$2,000. Money is worth 4 per cent effective.

14. Dane County purchases four school busses at a total cost of \$20,000. These busses will have a total scrap value of \$4,000 at the end of 6 years, at which time it will be necessary to replace them. If money is worth 5 per cent, $m = 1$, what is the capitalized cost of these busses?

15. A certain machine costs \$1,500, lasts for 8 years, and has a final scrap value of \$400. If money is worth 4 per cent compounded annually, how much could a purchaser afford to pay for another machine for the same purpose, whose life would be 15 years and with a scrap value zero?

16. The interior of a room can be painted at a cost of \$40.00, and the painting must be renewed every 3 years. If money is worth 4 per cent annually, how much could the owner afford to pay for papering the room if the paper would need renewing at the end of every 5 years?

LIFE ANNUITIES

8.1 Introduction. A *contingent annuity*, the term was defined in Chapter 4, is a set of payments in which the number of payments is not definitely fixed in advance. This number of payments depends upon conditions that develop during the time of the annuity payments. Both life annuities and life insurance furnish examples of such contingent annuities. In this and the following chapter, a very brief discussion of the mathematics involved is presented. A more complete study would involve the mathematical theory of probability and would constitute a full course in itself.¹

A *whole life annuity* is a set of periodic payments which are to be made to a given individual as long as he is alive and to cease at the time of his death. Thus, the number of payments is contingent on the length of life of the individual.

The history of life annuities goes back to at least the Babylonian period, and it is conjectured that the annuity practices in Babylon may have been adopted from the Hindus and Chinese. It is known that the Egyptians were using annuities in the period 1700–1100 B.C. Records show that Hepd'efal, a Prince ruling in Sint in the Middle Empire, purchased an annuity. Under the Romans, the use of annuities developed to such an extent that crude annuity tables were constructed and used extensively.²

8.2 Mortality Tables. It is not possible to foretell exactly when any given person will die. But, by studying the death statistics of large groups, it is possible to predict with a very high degree of accu-

¹ For example, see Robert E. Larson and Erwin A. Gauminitz, *Life Insurance Mathematics* (New York: John Wiley & Sons, Inc., 1951).

² For a more complete discussion, see Edwin W. Kopf, "The Early History of the Annuity," *Proceedings of the Casualty Actuarial Society*, 1926–1927, pp. 225–66.

racy the number of persons out of any given large group that will die in any year or period of time. A *mortality table* is a record of a fixed group showing the number of persons living at each age level from the beginning of the study period until the age when there are no survivors.

The Roman Pretorian Prefect Ulpianus constructed a mortality table about A.D. 225. This was the most accepted table for more than 1500 years. In 1693, Edmund Halley constructed the first mortality table based on recorded mortality in modern form. His table was constructed from material collected by the Breslau theologian, Kasper Neumann. Although Halley's tables were based on actual experience and made use of compound interest tables, they did not include commutation columns (to be described later). These commutation columns were invented by the Danish Mathematician John Nicholas Tetens and published in 1786. Since this time, many others, including Abraham DeMoivre, Richard Price, James Dodson, and Thomas Simpson of England, have helped to place the theory of life annuities on a sound scientific basis.³

There are a great many mortality tables in existence now. Most insurance companies use a mortality table in computing life annuities which is different from the one used in the calculations for life insurance. If people die less rapidly than predicted in the mortality table, an insurance company will have to pay out more in life annuities than it has provided for. On the other hand, if people die more rapidly than predicted, the insurance company must pay its life insurance policies earlier than anticipated. Thus, the mortality table chosen for life annuities is usually different from the one used for life insurance. At present, the table most widely used for life annuities is the 1937 Standard Annuity Table. Most life insurance currently issued is based on the 1941 Commissioner's Standard Ordinary Mortality Table. This table is usually referred to as the CSO Table. Prior to the construction of the CSO Table, the most widely used table was the American Experience Table, published in 1868.

Since the mathematical theory of life annuities and life insurance is not affected by the particular mortality table, all work in this text will be based on the CSO Table (Table 10 in the Appendix). The reader should keep in mind that in actual practice the 1937 Standard Annuity Table should be used for computing life annuities. This 1937 table assumes that the last person (male) out of a group of 100,000

³ *Ibid.*

living at age 5 will die during the one hundred ninth year of life. It is assumed that females will live 5 years longer.

8.3 Net Premiums and Loading Factor. In order to determine the price or *premium* a person must pay for a life annuity or a life insurance policy, the insurance company must allow for the interest earned on the premiums which it invests until the money is paid out to the policyholders. Usually, a rather conservative rate of interest is used in these calculations. In this book, a rate of $2\frac{1}{2}$ per cent per year will be used for all problems. Furthermore, it will be assumed that all premiums or annuities are paid annually.

Based on the rate of interest that is to be earned by the invested premiums and on the particular mortality table in use, each company calculates a *net premium*. The *gross premium* actually paid by the purchaser is found by adding to this net premium a *loading factor*. This loading factor is added to take care of the expenses of the company, such as agent's commissions and other costs of operation. Since the methods of computing the loading factor vary with the different companies, only *net premiums* will be discussed in this book. Just remember that the gross premium paid will be larger by the amount of the loading factor.

8.4 Mortality Table Symbols. Let l_0 represent the number of persons born in any given year. In the CSO Table, $l_0 = 1,023,102$. Now a certain number of these persons will die before reaching their first birthday. Let d_0 represent this number. (For the CSO Table, $d_0 = 23,102$.) Now let l_1 represent the number who are still alive on their first birthday, while d_1 will represent the number who die between the first and second birthday.

In general, let l_x represent the number of persons living at age x and d_x represent the number who die between ages x and $x + 1$. In the CSO Table, $l_{45} = 852,554$, that is to say, 852,554 of the original 1,023,102 persons live to reach the age 45. Since $d_{45} = 7,340$, this means 7,340 of these persons living at age 45 will die before they reach age 46.

It is clear that the number who die during any year is equal to the number living at the beginning of that year minus the number living at the end of this year (or the beginning of the next year). Thus

$$(9) \quad d_x = l_x - l_{x+1}.$$

Let q_x represent the rate of dying at the age x . Since this rate is the

quotient of the number dying during the year divided by the number living at the beginning of this year,

$$(10) \quad q_x = \frac{d_x}{l_x}.$$

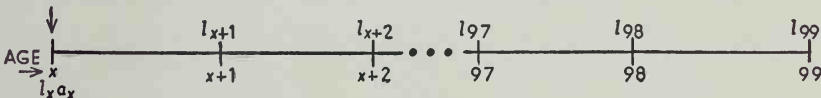
q_x can be read directly from the mortality table. Since $q_{28} = .00325$, it is assumed that .325 per cent of these living at age 28 will die before reaching the age 29. Thus q_x is the mathematical probability that a person aged x will die before reaching the age $x + 1$.

The largest x for which $l_x > 0$ is denoted by ω . $\omega + 1$ is called the limiting age of the mortality table. For the CSO Table, $\omega = 99$. This gives a limiting age of 100.

8.5 Whole Life Annuities. The price paid for a whole life annuity will depend upon the age of the purchaser, called the *annuitant*, and the time at which the first annuity payment will be made. For a *whole life annuity immediate*, the first annuity payment will be made 1 year after the date of purchase. For a *whole life annuity due*, the first payment will be made at the time of purchase.

In order to find the purchase price of a whole life annuity immediate paying \$1 each year and purchased by a person whose age is x , we can reason as follows:

Let a_x be the price paid for this annuity of 1 per year and assume that each person of age x as listed in the mortality table purchases such an annuity. That is to say, l_x persons will deposit a_x dollars each, giving a fund which contains $l_x a_x$ dollars at the time of purchase. One period later, l_{x+1} persons will be alive and each will receive \$1. Thus l_{x+1} dollars will then be taken out of the fund. At the end of the next period, the l_{x+2} persons living will each receive \$1, or a total of l_{x+2} dollars will be withdrawn. This process will continue until there are no survivors. In the case of the CSO Table, this last withdrawal will be l_{99} dollars at age 99. This information may be placed on the time line as follows:



Using the age x as a comparison date the equation of value becomes:

$$l_x a_x = l_{x+1}(1+i)^{-1} + l_{x+2}(1+i)^{-2} + \dots + l_{98}(1+i)^{x-98} + l_{99}(1+i)^{x-99}$$

$$a_x = \frac{l_{x+1}(1+i)^{-1} + l_{x+2}(1+i)^{-2} + \dots + l_{99}(1+i)^{x-99}}{l_x}.$$

If x is relatively small, there will be a larger number of terms in the numerator and the arithmetic computation may be very tedious. In order to avoid this tedious computation and in order to introduce some similarity of subscripts and exponents, multiply both numerator and denominator by $(1+i)^{-x}$. This last equation now becomes:

$$a_x = \frac{l_{x+1}(1+i)^{-(x+1)} + l_{x+2}(1+i)^{-(x+2)} + \dots + l_{99}(1+i)^{-99}}{l_x(1+i)^{-x}}$$

Now define the *commutation symbols*:⁴

$$(11) \quad D_x = l_x (1+i)^{-x},$$

$$(12) \quad N_x = D_x + D_{x+1} + \dots + D_{98} + D_{99}.$$

NOTE: In general, the last term in N_x is D_ω . For the CSO Table, $\omega = 99$.

This new notation gives:

$$a_x = \frac{D_{x+1} + D_{x+2} + \dots + D_{98} + D_{99}}{D_x},$$

or

$$a_x = \frac{N_{x+1}}{D_x}.$$

The values of D_x and N_x are given in the commutation columns of a mortality table. The commutation columns for the CSO Table with $i = .025$ are given in Table 11 in the Appendix.

The a_x given in the above formula is the price to be paid by a person age x for a whole life annuity immediate of \$1 per year. If an annuity of R dollars per year is desired, then the premium is Ra_x . Let A represent this net premium,

$$(13) \quad A = Ra_x = R \frac{N_{x+1}}{D_x}.$$

Illustration 1. John Smith, at age 39, wishes to purchase a life annuity paying him \$1,000 at the age of 40 and \$1,000 each year thereafter as long as he lives. Find the price Mr. Smith must pay for this annuity.

Solution. As stated earlier, the interest rate is assumed to be .025 per year in all problems.

⁴ The notation and symbols used in this text follow the new actuarial notation and symbols adopted in 1946-47 by the Executive Council of the Permanent Committee, International Congress of Actuaries. This notation differs in some respects from that used in books written prior to that time.

By formula (13):

$$\begin{aligned} A &= 1,000a_{39} = 1,000 \frac{N_{40}}{D_{39}}, \\ &= 1,000 \frac{6,708,572.66}{339,178.75}, \\ &= \$19,778.87. \end{aligned}$$

Illustration 2. Mrs. A. P. Jones, at age 51, uses the proceeds of a \$10,000 insurance policy to purchase a life annuity, with the first payment to be made 1 year later. What annual payment will she receive?

Solution. The \$10,000 is the present value or price of a whole life annuity immediate at age 51. Let R be the size of the annual payment.

$$\begin{aligned} A &= 10,000 \\ Ra_{51} &= 10,000 \\ R \frac{N_{52}}{D_{51}} &= 10,000 \\ R &= 10,000 \frac{D_{51}}{N_{52}} \\ R &= 10,000 \frac{227,335.15}{3,386,227.40} = \$671.35 \end{aligned}$$

For a *whole life annuity due* of \$1 per year, each of the l_x persons will receive \$1 immediately in addition to precisely the same amounts received under a whole life annuity immediate. Since \$1 now is worth exactly one dollar at the time of purchase, each purchaser will have to pay exactly \$1 more for a whole life annuity due than for a whole life annuity immediate. Let $\ddot{a}_x$ be the price paid for a whole life annuity due of \$1 per year, then

$$\begin{aligned} \ddot{a}_x &= 1 + a_x = 1 + \frac{N_{x+1}}{D_x} = \frac{D_x + N_{x+1}}{D_x}, \\ \ddot{a}_x &= \frac{D_x + D_{x+1} + D_{x+2} + \dots + D_{99}}{D_x}, \\ (14) \quad \ddot{a}_x &= \frac{N_x}{D_x}. \end{aligned}$$

This formula for the whole life annuity due is given here because of the frequency of its use in life annuities and life insurance. It will be used specifically in the next chapter in the development of one of the fundamental formulas there.

Illustration 3. If Mr. Smith of Illustration 1 wishes to receive his first \$1,000 at time of purchase (age 39), find his purchase price.

Solution. This is now a whole life annuity due.

$$\begin{aligned}\text{Net premium} &= 1,000\ddot{a}_x = 1,000 \frac{N_{39}}{D_{39}} \\ &= \frac{(7,047,751.41)(1,000)}{339,178.75} = \$20,778.87\end{aligned}$$

NOTE: This result is precisely the answer of Illustration 1 plus the initial payment of 1,000: $19,778.87 + 1,000 = 20,778.87$.

Illustration 4. If Mrs. Jones of Illustration 2 wishes to receive her first annuity payment at age 51, find the size of the annual payments she will receive.

Solution. This is now a whole life annuity due and

$$\begin{aligned}R\ddot{a}_{51} &= 10,000, \\ R \frac{N_{51}}{D_{51}} &= 10,000, \\ R &= 10,000 \frac{D_{51}}{N_{51}}, \\ R &= 10,000 \frac{227,335.15}{3,613,562.55} = \$629.12.\end{aligned}$$

Exercise 30

- Using the CSO Table, find the values of (a) l_{18} , (b) l_{52} , (c) l_{80} , and (d) l_{92} .
- Using the CSO Table, find the value of (a) l_{29} , (b) l_{43} , (c) l_{75} , and (d) l_{95} .
- Using the CSO Table, find the values of (a) d_{23} , (b) d_{37} , (c) d_{75} , and (d) d_{91} .
- Using the CSO Table, find the values of (a) d_{17} , (b) d_{41} , (c) d_{69} , and (d) d_{84} .
- Find the percentage of persons expected to die during the (a) thirteenth year, (b) twenty-seventh year, (c) fifty-eighth year, and (d) ninety-second year of life.
- Find the percentage of persons expected to die during the (a) eleventh year, (b) thirty-fifth year, (c) forty-ninth year, and (d) ninety-seventh year of life.
- Mr. Obie Nelson, age 53, purchases a whole life annuity immediate of \$5,000 a year. Find the net single premium he pays.
- Mr. Willie Jones, age 42, wins a whole life annuity of \$2,000 a year with the first payment to be received at age 43. Mr. Jones elects to receive immediately the cash value (present value) of this whole life annuity. How much money does he receive?
- Mr. Walter Reed, at age 63, receives \$10,000 from an insurance policy and immediately reinvests it in a life annuity with the first payment due at age 64. Find the yearly income from this life annuity.

10. Mr. W. E. Beam retires at age 65. From the sale of his business he invests \$25,000 in a life income with the first annual payment to be made at age 66. Find the size of the annual payment.

11. If Mr. Reed, in Problem 9, wishes to receive his first payment of the life annuity immediately (age 63), find the size of the yearly income he will receive.

12. If Mr. Beam, in Problem 10, wishes to receive his first income at the time of the purchase of his life annuity (age 65), find the size of the annual payment to him.

13. Find the net single premium that will purchase a \$3,000 whole life annuity due at age 49.

14. What is the net single premium for a whole life annuity due of \$5,000 a year if it is purchased at age 67?

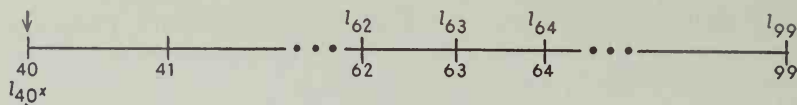
15. The Williamsburg Manufacturing Company provides a plan whereby its employees will each receive \$2,000 upon retirement, at age 65, and an additional \$2,000 at the end of each year as long as the employee lives. Each employee will have the option of taking this life annuity or the cash equivalent of it at the time of retirement. Mr. Smith chooses the single cash payment. Find the amount of money he receives at age 65.

16. Mr. Oscar Rennebohm is the recipient of an income of \$3,600 a year for life, with the option that at any time he can take the present value of all future payments due him, thus ending the contract. What would Mr. Rennebohm receive if he chooses, at age 72, to take the single payment? This payment is to include the \$3,600 due him at age 72.

8.6 Other Types of Life Annuities. In addition to the two types of life annuities already discussed, there are several others which occur frequently enough to warrant their discussion here. Although a definite formula can be given for each type, the equation of value method, as used in the derivation of the formula for a_x , can be used in each individual situation. The following illustrations will show how this method may be applied to the cases defined. The reader should then be able to work any ordinary problem that should arise.

Illustration 1. Mr. Carlton Truax, age 40, wishes to purchase a life annuity that will pay him \$1,500 a year for life, with the first payment being made at the time Mr. Truax reaches the age 62, if he is still alive at that time. Find the price Mr. Truax must pay at age 40.

Solution. Such a life annuity is called a *deferred whole life annuity* because the payments are to begin on a date in the future and are to continue for the entire life of the annuitant. Let x be the price paid at age 40. Assuming that all l_{40} persons will purchase such an annuity paying \$1 at the end of each year and setting up the equation of value, we get:



$$l_{40}x = l_{62}(1.025)^{-22} + l_{63}(1.025)^{-23} + \dots + l_{99}(1.025)^{-59}$$

Multiply both sides of this equation by $(1.025)^{-40}$.

NOTE: Multiplication of both sides of this equation by $(1.025)^{-40}$ is equivalent to moving the comparison date to the time of birth. In general, in the equations of value to follow, multiplication by $(1+i)^{-x}$ gives the same result as taking the comparison date at age 0 rather than at age x .

$$l_{40}(1.025)^{-40}x = l_{62}(1.025)^{-62} + l_{63}(1.025)^{-63} + \dots + l_{99}(1.025)^{-99}$$

Using the commutation symbols:

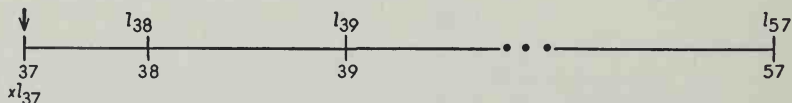
$$\begin{aligned} D_{40}x &= D_{62} + D_{63} + \dots + D_{99}, \\ D_{40}x &= N_{62}, \\ x &= \frac{N_{62}}{D_{40}}. \end{aligned}$$

This is the price paid, assuming he would receive \$1 per year. In order to

$$\begin{aligned} \text{receive \$1,500 a year his net premium} &= 1,500x = 1,500 \frac{N_{62}}{D_{40}}, \\ &= 1,500 \frac{1,565,274.55}{329,983.61}, \\ &= \$7,115.23. \end{aligned}$$

Illustration 2. Mr. Avril Jones, at age 37, purchases a life annuity that, starting at age 38, will pay him \$1,000 a year. These payments will stop after the twentieth payment or at the death of Mr. Jones, whichever occurs first. Find the price Mr. Jones pays.

Solution. This type of annuity is called a *temporary life annuity*. With x representing the price paid for a \$1 temporary life annuity, the equation of value becomes:



$$xl_{37} = l_{38}(1.025)^{-1} + l_{39}(1.025)^{-2} + \dots + l_{57}(1.025)^{-20}$$

Multiply both sides by $(1.025)^{-37}$ and replace $l_x(1.025)^{-x}$ by D_x :

$$xl_{37}(1.025)^{-37} = l_{38}(1.025)^{-38} + \dots + l_{57}(1.025)^{-57},$$

$$xD_{37} = D_{38} + D_{39} + \dots + D_{57}.$$

Now add this equation to $N_{58} = D_{58} + D_{59} + \dots + D_{99}$:

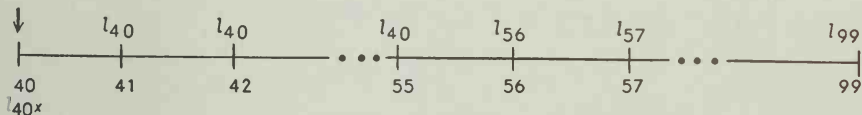
$$\begin{aligned}
 xD_{37} + N_{58} &= D_{38} + D_{39} + \dots + D_{57} + (D_{58} + D_{59} + \dots + D_{99}), \\
 xD_{37} + N_{58} &= N_{38}, \\
 xD_{37} &= N_{38} - N_{58}, \\
 x &= \frac{N_{38} - N_{58}}{D_{37}}.
 \end{aligned}$$

Thus, Mr. Jones's net premium P will be:

$$\begin{aligned}
 P &= 1,000x = 1,000 \frac{N_{38} - N_{58}}{D_{37}}, \\
 P &= 1,000 \frac{7,397,318.31 - 2,197,265.32}{360,161.02}, \\
 P &= \$14,438.13.
 \end{aligned}$$

Illustration 3. At the age of 40, Bob Welsh purchases a \$1,000 life annuity with the first payment at age 41 and with the first 15 payments guaranteed. That is to say, if Mr. Welsh dies before he reaches the age of 55, the company will continue to pay that annuity to his beneficiary until a total of 15 payments in all have been made by the company. If Mr. Welsh dies after he has reached the age of 55, the payments will immediately stop.

Solution. The guaranteed feature of this policy means that each of the l_{40} persons purchasing such an annuity will receive \$1 each year for 15 years. That is to say l_{40} dollars will be paid out each year for the first 15 years. But at the end of 16 years, only those living, l_{56} , will receive a payment. If x is the price paid for an annuity of \$1 per year, the equation of value will be:



The first 15 payments of l_{40} form an ordinary annuity certain, and their value at age 40 will be $l_{40}a_{\overline{15}|.025}$. Thus the equation of value becomes:

$$xl_{40} = l_{40}a_{\overline{15}|.025} + l_{56}(1.025)^{-16} + l_{57}(1.025)^{-17} + \dots + l_{99}(1.025)^{-59}.$$

Multiplying by $(1.025)^{-40}$ and using the commutation symbols, this becomes:

$$x l_{40}(1.025)^{-40} = l_{40}(1.025)^{-40} a_{\overline{15}|.025} + l_{56}(1.025)^{-56} + l_{57}(1.025)^{-57} + \dots + l_{99}(1.025)^{-99}.$$

$$x D_{40} = D_{40} a_{\overline{15}|.025} + N_{56}$$

$$x = a_{\overline{15}|.025} + \frac{N_{56}}{D_{40}}$$

$$x = 12.38137773 + \frac{2,560,828.18}{328,983.61}$$

$$x = 12.38137773 + 7.78406006$$

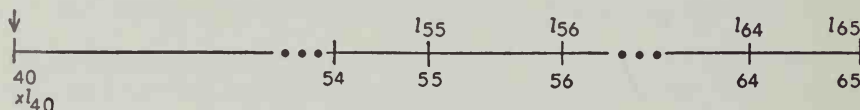
$$x = 20.16543779$$

Thus, for an annuity paying \$1,000 a year, Mr. Welsh will pay:

$$P = 1,000x = \$20,165.44.$$

Illustration 4. Mr. Wallace Q. Smithy, at the age of 40, wishes to purchase an annuity of \$500 a year, with the first payment to be made at the time he is 55, provided he is alive then. The payments are to stop with the one made at age 65 or at his death, whichever occurs first. Find the price Mr. Smithy must pay for this annuity.

Solution. This is called a *deferred temporary life annuity*. Letting x be the cost of such an annuity of \$1 per year, the equation of value will be:



$$xl_{40} = l_{55}(1.025)^{-15} + l_{56}(1.025)^{-16} + \dots + l_{64}(1.025)^{-24} + l_{65}(1.025)^{-25}$$

Multiplying both sides by $(1.025)^{-40}$ and using the commutation symbols, this becomes:

$$\begin{aligned} xl_{40}(1.025)^{-40} &= l_{55}(1.025)^{-55} + l_{56}(1.025)^{-56} + \dots + l_{65}(1.025)^{-65}, \\ xD_{40} &= D_{55} + D_{56} + \dots + D_{65}. \end{aligned}$$

Add this equation to the identity:

$$\begin{aligned} N_{66} &= D_{66} + D_{67} + \dots + D_{99}, \\ xD_{40} + N_{66} &= D_{55} + D_{56} + \dots + D_{65} + (D_{66} + D_{67} + \dots + D_{99}), \\ xD_{40} + N_{66} &= N_{55}, \\ xD_{40} &= N_{55} - N_{66}, \\ x &= \frac{N_{55} - N_{66}}{D_{40}}. \end{aligned}$$

The price P Mr. Smithy will pay for a \$500 annuity of this type will be:

$$P = 500x = 500 \frac{N_{55} - N_{66}}{D_{40}}.$$

The arithmetic computation will be left as an exercise for the reader.

A very special case of the deferred temporary life annuity illustrated in Illustration 4 is of sufficient importance to warrant giving it a special name. If the annuitant is to receive only one payment to be made at some specific time in the future, provided he is alive at that time, this single payment is called a *pure endowment*. If the annuitant purchases this pure endowment of \$1 at age x , with the endowment to be paid n years later, the value of this pure endowment

is denoted by ${}_nE_x$. Setting up the time line and equation of value, we get:



$${}_nE_x l_x = l_{x+n}(1+i)^{-n}$$

Multiplying both sides by $(1+i)^{-x}$, and using the commutation symbols, this becomes:

$$\begin{aligned} {}_nE_x l_x (1+i)^{-x} &= l_{x+n} (1+i)^{-(x+n)}, \\ {}_nE_x D_x &= D_{x+n}, \\ (15) \quad {}_nE_x &= \frac{D_{x+n}}{D_x}. \end{aligned}$$

Although it is not necessary to use this formula (any such problem can be worked by the method used in Illustration 4), the student may find it convenient at times.

Illustration 5. A man, age 50, wishes to purchase a pure endowment of \$5,000 to be paid to him, if he is alive, at the age of 65.

Solution. This is a pure endowment with $x = 50$ and $n = 15$. If P represents the price he will pay, then

$$P = 5,000 {}_{15}E_{50} = 5,000 \frac{D_{65}}{D_{50}},$$

$$P = 5,000 \frac{116,088.15}{235,925.04},$$

$$P = \$2,460.28.$$

Exercise 31

1. At age 27, Mr. E. S. Vance purchases a life annuity of \$2,000, with the first payment to be made at age 65, provided he is alive at that time. Find the net single premium he pays.

2. Willie Q. Trimble, at age 31, receives an inheritance. Out of this money he wishes to purchase a \$5,000 life annuity with first payment due at age 60. Find the net single premium he must pay.

3. Oscar Peabody, at age 35, inherits \$10,000, which he invests in a life annuity with first payment due at age 65. Find the size of the annual income he will receive provided he lives to the age of 65.

4. What will be the annual income from a life annuity whose cost at age 42 is \$15,000 and with first payment due at age 70?

5. In order to provide for his son's education, Mr. Quattlebaum purchases a \$2,000, five-year temporary life annuity on his son. If his

son is 8 years old at the time of purchase and the first \$2,000 is due at age 17, find the net single premium Mr. Quattlebaum pays.

6. What is the present value of a 20-year temporary life annuity of \$5,000 purchased at age 32 and with first payment due at age 45?

7. Mr. W. E. Smith, at age 40, is the beneficiary of a \$10,000 life insurance policy. He invests this money in a 15-year temporary life annuity with the first payment due at age 50. Find the annual income he will receive provided he is still alive at age 50.

8. If a man, age 35, invests \$25,000 in a 20-year life annuity with the first payment due at age 45, find the size of the annual payment.

9. Mr. Colfax purchases a 10-year life annuity of \$1,000 a year. If Mr. Colfax is 27 at the time of purchase and he is to receive the first \$1,000 at age 28, find the purchase price.

10. What is the present value of a 20-year, \$2,400, temporary life annuity purchased at age 41 and with first payment due at age 42?

11. Mr. William Baumgartner, at age 35, purchases a \$2,500 life annuity, with first payment at age 36 and with the first 14 payments guaranteed. Payments are to continue as long as he lives after that. Find the net single premium for this life annuity.

12. Mr. Grotto, at age 35, wishes to purchase a life annuity of \$2,000 per year, with the first payment due at age 55. The first 10 payments are guaranteed, and after that, the payments will continue as long as he lives. Find the present value of this annuity.

13. Mr. Walter Roark deposits \$100 each 3 months into a savings account paying interest at 2 per cent, $m = 4$. He makes his first deposit at age 22. Just after making his deposit at age 47, he uses the money in this account to purchase a life annuity, with first payment due at age 65. Find the size of the annual income he expects to receive.

14. Mr. Bill Rogers, beginning at age 25, deposits \$50 a month into a fund earning 3 per cent, $m = 12$. After making his \$50 deposit at age 37, financial conditions prevent any more deposits. At age 42, he withdraws the money from this fund and purchases a life annuity, with the first payment due him at age 70. Find the size of the annual payment he expects to receive.

15. Find the present value, at age 24, of a pure endowment of \$8,000 at age 42.

16. A man purchases, at age 42, an \$8,000 pure endowment which is to be paid to him at age 65, provided he is alive at that time. Find the net single premium he pays.

17. Mrs. Hallsworth, at age 37, uses the proceeds of an \$8,000 insurance policy to purchase a 15-year pure endowment. Find the size of this endowment.

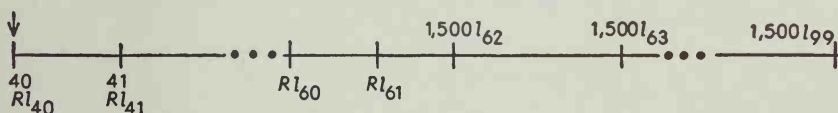
18. Find the size of a 20-year pure endowment that can be purchased at age 39 for \$7,000.

8.7 Life Annuities Purchased by Making a Set of Equal Payments. In all the discussion and illustrations given in the preceding

sections of this chapter, it is assumed that the annuitant pays for his annuity with a single, lump-sum payment. In the case of life annuities immediate and life annuities due this is generally the case. On the other hand, in deferred life annuities, the purchaser may desire to spread his payments over a period of years. Thus, his payments become a life annuity also. In order to calculate the size of such payments, it will be necessary to equate the value of two equivalent life annuities: the one he pays to the insurance company, and the one the insurance company is to pay him, if he lives to receive it. The present date, the time of purchase, will be used as the comparison date. The following illustrations will show the techniques used in this type of problem.

Illustration 1. Mr. Carlton Truax, age 40, wishes to purchase a life annuity that will pay him \$1,500 a year for life, with the first payment being made at the time he reaches the age 62, if he is still alive at that time. Rather than making a lump-sum payment at age 40, Mr. Truax wishes to make a set of equal annual payments of R dollars each, with the first payment made at age 40 and the last at age 61, if he lives to that age.

Solution. This is the same problem as Illustration 1 of Section 8.6 with the payment being made as a temporary life annuity due rather than a lump sum. The time line and equation of value will now be:



$$Rl_{40} + Rl_{41}(1.025)^{-1} + \dots + Rl_{61}(1.025)^{-21} = 1,500l_{62}(1.025)^{-22} \\ + \dots + 1,500l_{99}(1.025)^{-59}$$

Multiplying both sides by $(1.025)^{-40}$ and using commutation symbols, we get

$$R[l_{40}(1.025)^{-40} + \dots + l_{61}(1.025)^{-61}] \\ = 1,500[l_{62}(1.025)^{-62} + \dots + l_{99}(1.025)^{-99}] ,$$

$$\text{Now } R[D_{40} + \dots + D_{61}] = 1,500 [D_{62} + \dots + D_{99}] . \\ D_{40} + \dots + D_{61} = [D_{40} + \dots + D_{61} + D_{62} + \dots + D_{99}] \\ - [D_{62} + \dots + D_{99}] ,$$

$$= N_{40} - N_{62} ,$$

$$R[N_{40} - N_{62}] = 1,500 N_{62} ,$$

$$R = 1,500 \frac{N_{62}}{N_{40} - N_{62}} .$$

The reader can finish the arithmetic computation.

Illustration 2. Mr. Wallace Q. Smithy, at the age 40, purchases an annuity of \$500 a year, with the first payment to be made to him at the time

he is 55, provided he is alive then. The payments are to stop with the one made at age 65 or at his death, whichever occurs first. In order to pay for this annuity Mr. Smithy wishes to make 10 annual payments, provided he lives that long, of R dollars each, the first due at age 40. Find the size R of each payment.

Solution. This is Illustration 4 of Section 8.6 with the lump-sum payment replaced by a set of payments. The time line and equation of value will be as follows:



$$Rl_{40} + Rl_{41}(1.025)^{-1} + \dots + Rl_{49}(1.025)^{-9} \\ = 500l_{55}(1.025)^{-15} + \dots + 500l_{65}(1.025)^{-25}$$

Multiplying both sides of this equation by $(1.025)^{-40}$ and using the commutation symbols, this becomes:

$$Rl_{40}(1.025)^{-40} + \dots + Rl_{49}(1.025)^{-49} \\ = 500l_{55}(1.025)^{-55} + \dots + 500l_{65}(1.025)^{-65}, \\ R[D_{40} + D_{41} + \dots + D_{49}] = 500[D_{55} + \dots + D_{65}].$$

Since

$$D_{40} + \dots + D_{49} = N_{40} - N_{50} \text{ and } D_{55} + \dots + D_{65} = N_{55} - N_{66},$$

this equation becomes:

$$R[N_{40} - N_{50}] = 500[N_{55} - N_{66}], \\ R = 500 \frac{N_{55} - N_{66}}{N_{40} - N_{50}}.$$

The arithmetic computation is left to the reader.

Exercise 32

1. Complete the solution of Illustration 1.
2. Complete the solution of Illustration 2.
3. At age 27, Mr. E. S. Wilson purchases a life annuity of \$2,000, with the first payment due him at age 65. Mr. Wilson is to pay for this life annuity by making annual payments, the first due at age 27 and the last due at age 64. Find the size of these net annual premiums.
4. Solve Problem 3 if there are only 20 of these annual premiums, the first due at age 27.
5. Bob Smith, at age 31, receives an inheritance. Out of this money he wishes to purchase a \$5,000 life annuity with first payment due him at age 60. If Mr. Smith pays annual premiums, the first due at age 31 and the last due at age 59, find the size of these net premiums.
6. Solve Problem 5 if Mr. Smith's last premium is due at age 35.

7. Mr. Will Reuters wishes to provide money for his son's education. In order to do this, he purchases a \$2,000, 5-year temporary life annuity on his son, with the first annuity payment due at age 17. This temporary life annuity is purchased when the son's age is 8. Mr. Reuter contracts to pay for this annuity by making 8 equal annual payments, the first due at the time of purchase. The contract further provides that if the son should die before reaching the age 15, the premium payments would cease immediately. Find the size of the net annual premium Mr. Reuter pays.

8. A 20-year temporary life annuity of \$5,000 is purchased at age 32, with the first annuity payment due at 45. This temporary life annuity is to be paid for by annual premiums, the first due at age 32 and the last due at age 44. Find the size of this net annual premium.

9. Mr. Broadman, at age 27, purchases a 10-year life annuity of \$1,000 a year, with the first payment due him at age 48. If he pays for this annuity by annual premiums, the first due at time of purchase and the last due 1 year before he receives his first annuity payment, find the size of the net annual premium.

10. What is the annual net premium on a 20-year, \$2,400 temporary life annuity purchased at age 41 and with first payment due at age 55? The annual premiums begin at the time of purchase and the last is due at age 50.

11. Mr. Campbell, at age 42, purchases an \$8,000 pure endowment to be paid him at age 65, if he is alive then. He pays for this pure endowment with annual premiums, the first due at the time of purchase and the last due at age 56. Find the size of the annual net premium.

12. Solve Problem 11 if the pure endowment is to be paid at age 72.

LIFE INSURANCE

9.1 Life Insurance. In a life insurance system, relatively small but regular contributions are paid in by a large number of persons in order to build up a fund sufficiently large to help provide for the survivors of those who participate in the system. A *life insurance company* is the agency which collects the *premiums* or regular payments and issues each participant a contract called a *policy*. This policy binds the company to make a definite payment, called the *face* of the policy, to the *beneficiary* designated by the policyholder. This payment is to be made at the time of the death of the policyholder.

The number of premiums paid by the insured is contingent upon the length of his life. Thus, these premiums form a type of life annuity. As in the case of life annuities, only *net premiums* will be considered here. To this must be added the particular company *loading factor* to get the *gross premium* paid by the policyholder. Furthermore, we shall assume that each insurance payment is made at the end of the year in which the policyholder dies. In actual practice, the insurance companies pay each claim as soon as proof of death is furnished. An adjustment, included in the loading factor, is made to the net premium to take care of this fact.

9.2 Single Premium, Whole Life Insurance. We shall first derive the formula for getting the single premium A_x for a policy purchased at age x which will provide for a payment of \$1 at the death of the insured. We will assume that the \$1 payment will be made at the end of the year in which the insured dies. The age x is taken as the nearest age, in whole years, of the purchaser at the time he buys the policy. For example, a person age 16 years and 3 months would

list his age as 16, while one age 16 years and 7 months would consider his age as 17.

Assume that the insurance company sells a \$1 policy to each of l_x persons living at the age x . During the first year, d_x of these will die. Thus, d_x dollars must be paid out at the end of the first year. Again, at the end of the second year, d_{x+1} dollars must be paid out, and so on. If the CSO mortality table is used, the final payment will be d_{99} at the age 100. The time line will appear thus:



Taking the age x as a comparison date, the equation of value becomes:

$$A_x l_x = d_x(1+i)^{-1} + d_{x+1}(1+i)^{-2} + \dots + d_{99}(1+i)^{-(100-x)}.$$

Multiplying both sides by $(1+i)^{-x}$, this becomes:

$$A_x l_x (1+i)^{-x} = d_x(1+i)^{-(x+1)} + d_{x+1}(1+i)^{-(x+2)} + \dots + d_{99}(1+i)^{-100}.$$

Now define the two commutation symbols:

$$(16) \quad C_x = d_x(1+i)^{-(x+1)},$$

$$(17) \quad M_x = C_x + C_{x+1} + \dots + C_{99}.$$

NOTE: The last term in M_x is C_ω . Since we are using the CSO table which has $\omega = 99$, the definition has been given to fit this table. In using any other mortality table, the subscript on the last C should be changed to fit that table.

Numerical values for both M_x and C_x are given in the commutation columns for the mortality table used. See Table 11 in the Appendix.

By using the commutation symbols defined here and in Chapter 8, this equation of value becomes

$$A_x D_x = C_x + C_{x+1} + \dots + C_{99},$$

$$A_x D_x = M_x,$$

$$A_x = \frac{M_x}{D_x}.$$

If the face of this policy is to be F dollars instead of \$1, the single premium A is given by

$$(18) \quad A = F A_x = F \frac{M_x}{D_x}.$$

Illustration. Mr. Jones purchases a \$10,000 whole life insurance policy at age 25. Find the size of the single net premium he pays.

Solution. Let A = size of this premium. By formula (18),

$$A = 10,000 A_{25} = 10,000 \frac{M_{25}}{D_{25}},$$

$$A = 10,000 \frac{189,700.8750}{506,594.02},$$

$$A = \$3,744.63.$$

9.3 Annual Premium, Whole Life Insurance. Very few people are able to pay for a whole life, also called straight life, insurance policy with a single premium. Most people desire to spread their payments out over the entire period of the life of the policy, or else over some fixed interval of time, contingent upon their living that length of time. If the premiums are paid annually for the entire life of the policy, it is called an *ordinary life policy*. If the premiums are to be paid for some fixed time (or until the death of the insured, whichever occurs first), say 15 years, then it is called a *15-pay* or *15-payment life policy*. The n -payment policies will be considered in Section 9.4.

The purchaser, or policyholder, pays his first premium of R dollars at the time of the purchase. If he pays R dollars annually as long as he lives, his insurance premiums form a whole life annuity due of R dollars each. By formula (14) of Section 8.4 the present value of these premiums is

$$R \ddot{a}_x = R \frac{N_x}{D_x}.$$

But by formula (18), Section 9.2, the present value of the whole life insurance policy of F dollars is

$$F A_x = F \frac{M_x}{D_x}.$$

Since the two present values should be equal,

$$R \frac{N_x}{D_x} = F \frac{M_x}{D_x},$$

$$(19) \quad R = F \frac{M_x}{N_x}.$$

Illustration. Mr. Jones purchases a \$10,000 ordinary life policy at age 25. Find his annual net premium.

Solution. Let R be the size of the net premium. By formula (19), with $F = 10,000$ and $x = 25$,

$$R = 10,000 \frac{M_{25}}{N_{25}},$$

$$R = 10,000 \frac{189,700.8750}{12,992,619.10},$$

$$R = \$146.01.$$

Exercise 33

1. Mr. Willie Jones, age 23, purchases a \$10,000 whole life insurance policy. Find the size of the single net premium he pays.

2. Find the single net premium paid at age 43 for a \$5,000 whole life insurance policy.

3. What net single premium must be paid, at age 59, for a \$15,000 whole life policy?

4. What net single premium must be paid, at age 27, for a \$10,000 whole life policy?

5. Mrs. A. P. Wilson, age 39, wishes to invest \$8,000. Find the size of a whole life insurance policy she can purchase with this sum as a single net premium.

6. How much whole life insurance can be purchased for a net single premium of \$12,000 at age 47?

7. Suppose, instead of purchasing life insurance, Mrs. Wilson of Problem 5 invests her \$8,000 in a fund which earns interest at 3 per cent compounded semiannually. She will leave this money in this fund until her death. Find the amount her heirs will receive if she dies at age (a) 42, (b) 63, and (c) 82.

Compare the answer for each part with the amount that would have been received by her heirs if she had purchased the insurance.

8. A man, age 47, deposits \$12,000 in a fund earning interest at 4 per cent, $m = 4$. If this money is left in this account until his death, find the amount his heirs will receive from it provided he dies at age (a) 50, (b) 61, and (c) 87.

9. If Mr. Jones, of Problem 1, wishes to pay his premiums annually, find the size of these net premiums.

10. Find the annual net premium paid on a \$5,000 ordinary life insurance policy purchased at age 43.

11. A \$15,000 ordinary life insurance policy is purchased at age 59. Find the net annual premium.

12. A \$10,000 ordinary life policy is purchased at age 27. Find the net annual premium.

13. Mr. Olson, at age 32, can invest \$300 at the beginning of each year. He wishes to use this money as the net annual premium on an ordinary life insurance policy. Find the face of this policy.

14. How much ordinary life insurance can be purchased, at age 49, for a net annual premium of \$200?

15. Mr. Smith, age 37, wishes to purchase a life insurance policy that will pay his heirs \$15,000 if he dies any time during the next 10 years, and which will pay \$10,000 if he dies at any time after the next 10 years. By using an equation of value, find the net single premium for such a policy.

16. Mr. Hyers, age 29, wishes to purchase a life insurance policy that will pay \$20,000 if he dies any time during the next 15 years, and which will pay \$10,000 if he dies at any time after the next 15 years. By using an equation of value, find the net single premium for this policy.

17. Find the net annual premium for the policy described in Problem 15.

18. Find the net annual premium for the policy described in Problem 16.

19. Find the net annual premium for a \$5,000 ordinary life policy purchased at age 75.

9.4 Limited Payment Life Insurance. A limited payment plan of life insurance furnishes the insured protection for his entire life, but his number of premiums are limited by the particular policy. For example, in a 20-payment life policy, no premiums will be paid after the first 20 years, no matter how much longer the insured person lives. Of course, if the insured dies before the 20 payments are completed, all payments stop at his death and his beneficiary receives the face of the policy. The present value of his benefits will be the same as in an ordinary whole life policy, but his premiums will constitute a temporary life annuity due and its present value can be calculated by the methods discussed in Section 8.6. The following illustrations will make this method clear.

Illustration 1. Mr. Jones, age 25, purchases a 20-year-pay life insurance policy of \$10,000. Find the size of his annual premium.

Solution. For this policy Mr. Jones will pay at most 20 annual payments (fewer if he dies before the twentieth payment is due). Let R be the size of each payment, and assume that l_x persons, age x , purchase such a policy.

If each policy pays \$1, the time line and equation of value becomes:

$$\begin{array}{c}
 \downarrow \\
 \begin{array}{ccccccc}
 | & & | & & | & & | & & | & & | \\
 25 & & 26 & & 27 & \cdots & 44 & & 45 & \cdots & 100 \\
 \hline
 25 & & 26 & & 27 & & 44 & & 45 & & 100 \\
 Rl_{25} & & Rl_{26} & & Rl_{27} & & Rl_{44} & & & &
 \end{array}
 \end{array}$$

$$\begin{aligned}
 Rl_{25} + Rl_{26}(1.025)^{-1} + \cdots + Rl_{44}(1.025)^{-19} \\
 = d_{25}(1.025)^{-1} + \cdots + d_{99}(1.025)^{-75}
 \end{aligned}$$

Multiplying both sides of this equation by $(1.025)^{-25}$ and using the commutation symbols, this becomes:

$$\begin{aligned}
 R[l_{25}(1.025)^{-25} + l_{26}(1.025)^{-26} + \dots + l_{44}(1.025)^{-44}] \\
 = d_{25}(1.025)^{-26} + \dots + d_{99}(1.025)^{-100}, \\
 R[D_{25} + D_{26} + \dots + D_{44}] = C_{25} + C_{26} + \dots + C_{99}.
 \end{aligned}$$

Now

$$\begin{aligned}
 D_{25} + \dots + D_{44} &= D_{25} + \dots + D_{99} - [D_{45} + \dots + D_{99}], \\
 &= N_{25} - N_{45}.
 \end{aligned}$$

$$\therefore R[N_{25} - N_{45}] = M_{25},$$

$$R = \frac{M_{25}}{N_{25} - N_{45}}.$$

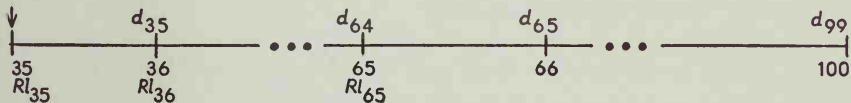
Since the policy is for \$10,000 instead of \$1, the annual premium will be:

$$P = 10,000 R = 10,000 \frac{M_{25}}{N_{25} - N_{45}}.$$

The arithmetic computation will be left to the reader.

Illustration 2. Mr. I. M. Hewlet, age 35, purchases a \$10,000 paid-up life policy at age 65. Find his annual premium.

Solution. The statement "paid-up life policy at age 65" means that no premium will be paid after the one due at age 65, that is to say, the last premium is due at age 65, provided the insured is living at that time. Let l_{35} persons purchase such a policy with face value \$1, and let R be the annual premium due on each policy. The time line and equation of value becomes:



$$\begin{aligned}
 Rl_{35} + Rl_{35}(1.025)^{-1} + \dots + Rl_{65}(1.025)^{-30} \\
 = d_{35}(1.025)^{-1} + \dots + d_{99}(1.025)^{-65}
 \end{aligned}$$

Multiply both sides by $(1.025)^{-35}$ and substitute the commutation symbols.

$$\begin{aligned}
 Rl_{35}(1.025)^{-35} + \dots + Rl_{66}(1.025)^{-65} \\
 = d_{35}(1.025)^{-36} + \dots + d_{99}(1.025)^{-100}
 \end{aligned}$$

$$R[D_{35} + \dots + D_{65}] = C_{35} + \dots + C_{99}$$

$$R[N_{35} - N_{66}] = M_{35}$$

$$R = \frac{M_{35}}{N_{35} - N_{66}}.$$

$\therefore$ Premium P on a \$10,000 policy will be

$$P = 10,000 R = 10,000 \frac{M_{35}}{N_{35} - N_{66}}.$$

By using Table 11 in the Appendix, the reader may compute the numerical value of P .

Exercise 34

1. Complete the solution of Illustration 1.
2. Find the net annual premium on a 25-year-pay life insurance policy for \$8,000 if it is purchased at age 40.
3. Mr. Overton, age 23, purchases a 20-year-pay life insurance policy of \$10,000. Find the net annual premium.
4. Find the net annual premium paid for a 15-year-pay life insurance policy of \$5,000 purchased at age 43.
5. What net annual premium must be paid, at age 59, for a 10-year-pay life insurance policy of \$15,000?
6. What net annual premium must be paid, at age 27, for a \$10,000, 25-year-pay life insurance policy?
7. Complete the solution of Illustration 2.
8. Find the net annual premium on a \$20,000 paid-up life policy at age 60 if it is purchased at age 29.
9. Mr. W. T. Onley, age 29, purchases an \$8,000 paid-up life policy at age 70. Find the net annual premium.
10. Find the net annual premium on a \$15,000 paid-up life policy at age 55 that is purchased by a person aged 22.
11. How much 20-year-pay life insurance can be purchased, at age 39, for a net annual premium of \$400?
12. Find the face of a 15-year-pay life insurance policy purchased at age 21 for a net annual premium of \$250.
13. Al Comstock, age 23, purchases a paid-up life policy at age 60. If his net annual premiums are \$200, find the face of the policy.
14. A paid-up life policy at age 65 is purchased by a man, age 41, for a net annual premium of \$200. Find the amount of the policy.
15. Find the annual net premium for the policy described in Problem 15, Exercise 33, if Mr. Smith pays for this policy by the 20-year-pay plan.
16. Find the annual net premium for the policy described in Problem 16, Exercise 33, if Mr. Hyers pays for this policy by the 15-year-pay plan.
17. Find the net annual premium on a 25-year-pay life insurance policy for \$5,000 that is purchased at age 75. Compare this answer with that of Problem 19, Exercise 33.

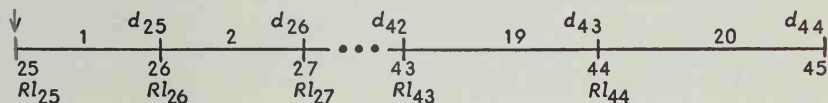
9.5 Term Insurance. Term insurance is a temporary type of insurance. It offers protection for a certain fixed number of years, called the term of the insurance policy. If the insured dies during the term of the policy, his beneficiary is paid the face of the policy. After the term has elapsed, the insured has no insurance protection. Since no money is ever paid out on many of the term policies, the annual

premiums for term insurance at a given age will be smaller than for those of a corresponding ordinary life policy. This makes term insurance less costly when one desires a heavy protection for a relatively short period.

The following illustration shows how an annual premium may be calculated on a term insurance policy.

Illustration. At age 25, Mr. Jones purchases a \$10,000, twenty-year term life insurance policy. Calculate the annual premium Mr. Jones must pay at the beginning of each year during the term of this policy. Compare this answer with the annual premium on a straight life policy as given in the Illustration of Section 9.3.

Solution. Let R be the size of the annual premium for a \$1 policy and assume l_{25} persons purchase such a policy at age 25. The time diagram and equation of value then become:



$$Rl_{25} + Rl_{26}(1.025)^{-1} + \dots + Rl_{44}(1.025)^{-19} \\ = d_{25}(1.025)^{-1} + \dots + d_{44}(1.025)^{-20}$$

Multiplying both sides of this equation by $(1.025)^{-25}$ and using the commutation symbols, this becomes:

$$Rl_{25}(1.025)^{-25} + Rl_{26}(1.025)^{-26} + \dots + Rl_{44}(1.025)^{-44} \\ = d_{25}(1.025)^{-26} + \dots + d_{44}(1.025)^{-45},$$

$$R[D_{25} + \dots + D_{44}] = C_{25} + \dots + C_{44},$$

$$R[N_{25} - N_{45}] = M_{25} - M_{45},$$

$$R = \frac{M_{25} - M_{45}}{N_{25} - N_{45}}.$$

Let P represent the premium for the \$10,000 policy. Then

$$P = 10,000 R = 10,000 \frac{M_{25} - M_{45}}{N_{25} - N_{45}}.$$

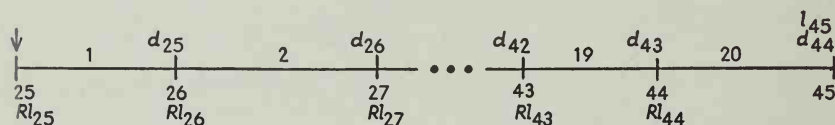
The reader should complete this computation and compare the result with the answers to the Illustration of Section 9.3 and Illustration 1 of Section 9.4. This will give some idea of the relative size of premiums for these three types of problems.

9.6 Endowment Insurance. An endowment insurance policy is one which will pay the face of the policy to the beneficiary if the insured person dies within a given period. But if the policyholder lives to the end of this period, then he will receive the face of the policy at

that time. Thus, an endowment policy really is a term insurance policy plus a pure endowment. To find the single premium for such a \$1 policy, the premium ${}_nE_x$ [Formula (15), Section 8.6] for a \$1 pure endowment can be added to the premium for a \$1 term policy. The entire premium may be calculated by setting up the particular equation of value as in the following illustration. If an annual premium is desired, the equation of value procedure is probably better. Unless otherwise stated, the annual premium is considered as paid at the first of each year of the term.

Illustration. At age 25, Mr. Jones purchases a \$10,000 twenty-year endowment policy. Find the annual premium Mr. Jones pays for this policy.

Solution. Let R be the size of the annual premium of a \$1 policy, and assume l_{25} persons purchase this type policy at age 25. The time diagram and the equation of value are as follows. Notice the parallel with the work in the Illustration of Section 9.5. In addition to payments made in that problem, the insurance company must pay l_{45} dollars to the survivors at age 45.



$$\begin{aligned}
 Rl_{25} + Rl_{26}(1.025)^{-1} + \dots + Rl_{44}(1.025)^{-19} \\
 = d_{25}(1.025)^{-1} + \dots + d_{44}(1.025)^{-20} + l_{45}(1.025)^{-30}
 \end{aligned}$$

Multiplying both sides by $(1.025)^{-25}$ and substituting the commutation symbols, this becomes:

$$R[D_{25} + \dots + D_{44}] = C_{25} + \dots + C_{44} + D_{45},$$

$$R[N_{25} - N_{45}] = M_{25} - M_{45} + D_{45},$$

$$R = \frac{M_{25} - M_{45} + D_{45}}{N_{25} - N_{45}}.$$

9.7 Brief Summary. There are many other types of insurance policies issued, but most of them are combinations of the illustrations given here. By using the ideas developed in these illustrations and the equation of value, a student can work many problems involving a wide variety of insurance policies. As has been said in the beginning of Chapter 8, this material should be considered only as an introduction to the study of the mathematics of life insurance. Only one other topic, reserves, will be discussed briefly in Section 9.8. It is

hoped that these brief chapters will stimulate the student to continue his study of life insurance mathematics.

In order that the reader can see some comparisons in the premium cost for the various types of insurance and pure endowment discussed in these two chapters, both the single premium and annual premium for a \$10,000 policy, purchased at age 25, are listed here for several types of insurance. Each of these premiums has been computed in an illustration or the problem has been given as an exercise to the student.

\$10,000 POLICY PURCHASED AT AGE 25, NET PREMIUMS

<i>Type</i>	<i>Single Premium</i>	<i>Annual Premium</i>
Whole life	\$3,744.63	\$146.01
20-year payment		242.26
20-year term	690.18	44.65
20-year pure endowment	5,539.72	358.39
20-year endowment insurance	6,229.90	403.04

Exercise 35

1. Complete the solution of the Illustration of Section 9.5.
2. At age 43, Mr. Upjohn purchases a \$20,000, 15-year term life insurance policy. Find the net annual premium Mr. Upjohn pays.
3. Find the net single premium for a 20-year term policy for \$10,000 and bought at age 25.
4. Find the net single premium for a \$20,000 15-year term life insurance policy purchased at age 43.
5. Find the annual net premium on a 25-year term policy for \$12,000 and purchased at age 29.
6. A \$7,000 10-year term policy is purchased at age 42. Find the net annual premium.
7. Complete the solution of the Illustration of Section 9.6.
8. At age 33, a 15-year endowment insurance policy of \$22,000 is purchased. Find the net annual premium.
9. Find the net single premium for a 20-year endowment insurance policy for \$10,000 if it is purchased at age 25.
10. Find the net single premium for a 15-year term endowment insurance policy of \$22,000 purchased at age 33.
11. Mr. Ogleby, at age 39, purchases a 20-year term policy with an annual net premium of \$200. Find the face of this policy.
12. Find the face of a 15-year term policy purchased at age 37 if the annual net premium is \$300.
13. Find the face of a 20-year endowment insurance policy that can be purchased at age 39 for an annual net premium of \$200.

14. If the net annual premium of a 15-year endowment insurance policy, purchased at age 37, is \$300, find the face of the policy.

15. Find the net annual premium on a 25-year term insurance policy for \$5,000 that is purchased at age 75. Compare this answer with that of Problem 19, Exercise 33 and Problem 17, Exercise 34. From your comparisons, which policy would be the best? Hint: Assume that no payments are made to the insured person before his death, and consider a person who lives to age, for example, 102.

16. Find the net annual premium on a 25-year endowment insurance policy purchased at age 75. Compare your answer with that of Problem 15.

9.8 Reserves. For a 1-year term insurance policy, the sum of the net premiums collected from each of the l_x persons living at age x is just sufficient to pay off the death claims of the d_x persons who die during that year. This premium on a 1-year term policy is called the *natural* premium. If Mr. Jones, age 25, purchases a \$10,000 one-year term policy, his premium will be \$28.10. In the Illustration of Section 9.3, the annual premium on a \$10,000 whole life policy, age 25, is \$146.01. Thus, Mr. Jones pays the insurance company \$146.01 — \$28.10 = \$117.91 more than is necessary for the cost of his insurance for this year. This “overpayment” is deposited by the insurance company in a reserve fund, earning interest at the rate used by this company to calculate the size of the premium. We have assumed a rate of $2\frac{1}{2}$ per cent. At age 52, the natural premium is \$139.51. If Mr. Jones is still alive, he will pay the same premium as before, \$146.01. Thus only \$6.50 of his payment will go into the reserve fund. At age 53, the natural premium is \$150.54. Mr. Jones’ payment of \$146.01 is not sufficient to pay the cost of insurance for that particular year, so the “underpayment” of \$4.43 will have to be taken out of this reserve fund to help bear the cost of insurance for that year. At age 60, the natural premium is \$259.42. Thus the “underpayment” of $259.42 - 146.01 = \$113.41$ must be taken from the reserve fund. That is to say, this reserve fund is built up in the earlier years in order to take care of the later years when the premium does not pay the cost of the insurance. For a whole life policy, this reserve fund builds up gradually, year by year, until at the limit of the mortality table used, the reserve is equal to the face of the policy, provided the insured is still alive at that time. In the case of the CSO table, the reserve should equal the face of the policy at age 100. For an endowment policy, the reserve increases until it reaches the face value of the policy at the end of the term. In fact, a whole life plan of in-

insurance is really an endowment plan at age 100 (for CSO table). Actually, it is customary for many insurance companies to pay the face amount of the policy to the insured if he lives to the limiting age of the mortality table used. Thus any one who lives to be 100 with a whole life policy based on the CSO table would receive the face amount at that time.

The reserve fund does not belong to the insurance company, but it is money belonging to the policyholder and held in trust by the company. In this sense it is much like a savings account in a bank. If the insured person discontinues his insurance for any reason, the company must refund the policyholder the amount to his credit in his reserve fund. A charge, called a surrender charge, is usually made so that the policyholder does not receive quite the entire amount.

As indicated by the above discussion, a schedule of the growth of the reserve fund could be constructed, making use of the same principles as those employed in constructing a sinking fund schedule as used in Section 5.8. In the reserve fund, the payments into the fund are not equal, and in fact, after a certain point, they become withdrawals. Furthermore, in order to find out the amount in the reserve fund for each individual policyholder, it is best to set up the reserve fund for all the l_x persons living at age x . At the end of each year, the total amount in the reserve fund can be divided by the number of survivors in order to get each individual policyholder's share of the reserve fund. But it is possible to find out the amount in the reserve fund at any time without constructing the complete schedule.

The amount in the reserve fund at the end of any year, after the interest has been added, but before the next premium is due, is called the *terminal reserve*. Now the terminal reserves at any time plus the present value of the future premiums should equal the present value of the future benefits, that is, the single premium payment at that time which would provide for the policy from then on. The following illustration shows how this terminal reserve may be calculated.

Illustration. Mr. Jones, age 25, purchases a \$10,000 whole life insurance policy with annual premiums. If Mr. Jones is still alive at age 52, find his terminal reserve at the end of his fifty-first year of life.

Solution. In the Illustration of Section 9.3 the annual premium on this policy was calculated to be \$146.01. The premiums of this amount form a life annuity due at age 52, whose present value is given by formula (14). This present value would be:

$$\begin{aligned}
 146.01 \bar{a}_{52} &= 146.01 \frac{N_{52}}{D_{52}}, \\
 &= 146.01 \frac{3,386,227.40}{218,847.25}, \\
 &= (146.01) 15.473018, \\
 &= \$2,259.22.
 \end{aligned}$$

The single premium on a \$10,000 whole life insurance policy, age 52, is given by formula

$$(18) \quad A = FA_x = F \frac{M_x}{D_x}.$$

Thus the value, at age 52, of his future benefits is:

$$\begin{aligned}
 A &= 10,000 A_{52} = 10,000 \frac{M_{52}}{D_{52}}, \\
 &= 10,000 \frac{136,256.3361}{218,847.25}, \\
 &= 10,000 (.62260932), \\
 &= \$6,226.09.
 \end{aligned}$$

Thus the terminal reserve V is:

$$\begin{aligned}
 V &= \$6,226.09 - 2,259.22, \\
 V &= \$3,966.87.
 \end{aligned}$$

This illustration was for a whole life policy. The same methods will give the terminal reserve for any type of policy. The technique used is simply this: The terminal reserve at age x is equal to the single premium necessary to purchase this policy at age x minus the present value of all premiums due (including the one due at age x).

Exercise 36

1. Mr. Jones, age 23, purchases a \$10,000 whole life insurance policy with annual premiums. If Mr. Jones is still alive at age 60, find his terminal reserve at the end of his fifty-ninth year of life. (See Problem 9, Exercise 33.)

2. A \$5,000 ordinary life insurance policy, with annual premiums, is purchased at age 43. If the purchaser is still alive at age 65, find his terminal reserve at the end of his sixty-fourth year of life. (See Problem 10, Exercise 33.)

3. A \$15,000 ordinary life insurance policy, with annual premiums, is purchased at age 59. Find the terminal reserve of this policy provided the purchaser is still alive at the end of his sixty-fifth year.

4. A \$10,000 ordinary life policy, with annual premiums, is purchased at age 27. Find the terminal reserve of this policy provided the purchaser is still alive at the end of his fiftieth year.

5. Mr. Overton, age 23, purchases a 20-year-pay life insurance policy of \$10,000, with annual premiums. Find Mr. Overton's terminal reserve at the end of his fortieth year of life, provided he is still alive. (See Problem 3, Exercise 34.)

6. A 15-year-pay life insurance policy of \$5,000, with annual premiums, is purchased at age 43. Find the terminal reserve for this policy if the purchaser is still alive at the end of his fifty-second year. (See Problem 4, Exercise 34.)

7. Find the terminal reserve for the policy described in Problem 5 at the end of Mr. Overton's sixty-fifth year of life, provided he is still alive.

8. Find the terminal reserve for the policy described in Problem 6 provided the purchaser is still alive at the end of his sixtieth year of life.

9. A \$12,000 twenty-five-year term policy, with annual premiums, is purchased at age 29. If the purchaser is still alive at the end of his forty-ninth year, find his terminal reserve at that time. What is the terminal reserve at the end of his fifty-fourth year? (See Problem 5, Exercise 35.)

10. A \$7,000 ten-year term policy, with annual premiums, is purchased at age 42. If the policyholder is still alive at the end of his forty-seventh year, find his terminal reserve.

11. For the policy of Problem 1, calculate the deposits that will be made into the reserve fund for each of the first 5 years of this policy.

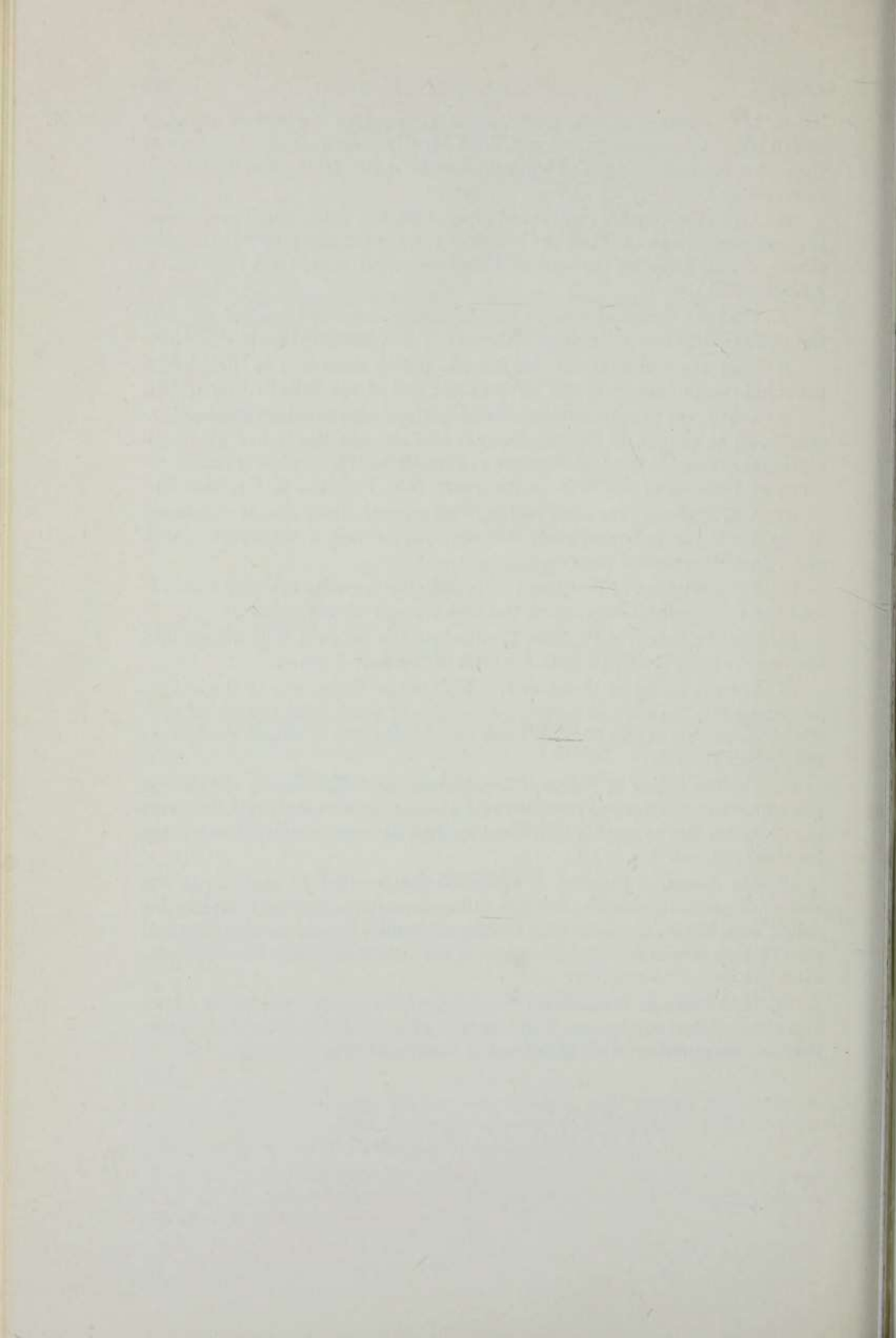
12. For the policy of Problem 2, calculate the deposits that will go into the reserve fund for this policy for each of the first 5 years.

13. For the policy of Problem 1, calculate the differences in the annual premiums and the natural premiums for each of the 5 years beginning with the premium due at age 60. Will these be deposits into or withdrawals from the reserve fund?

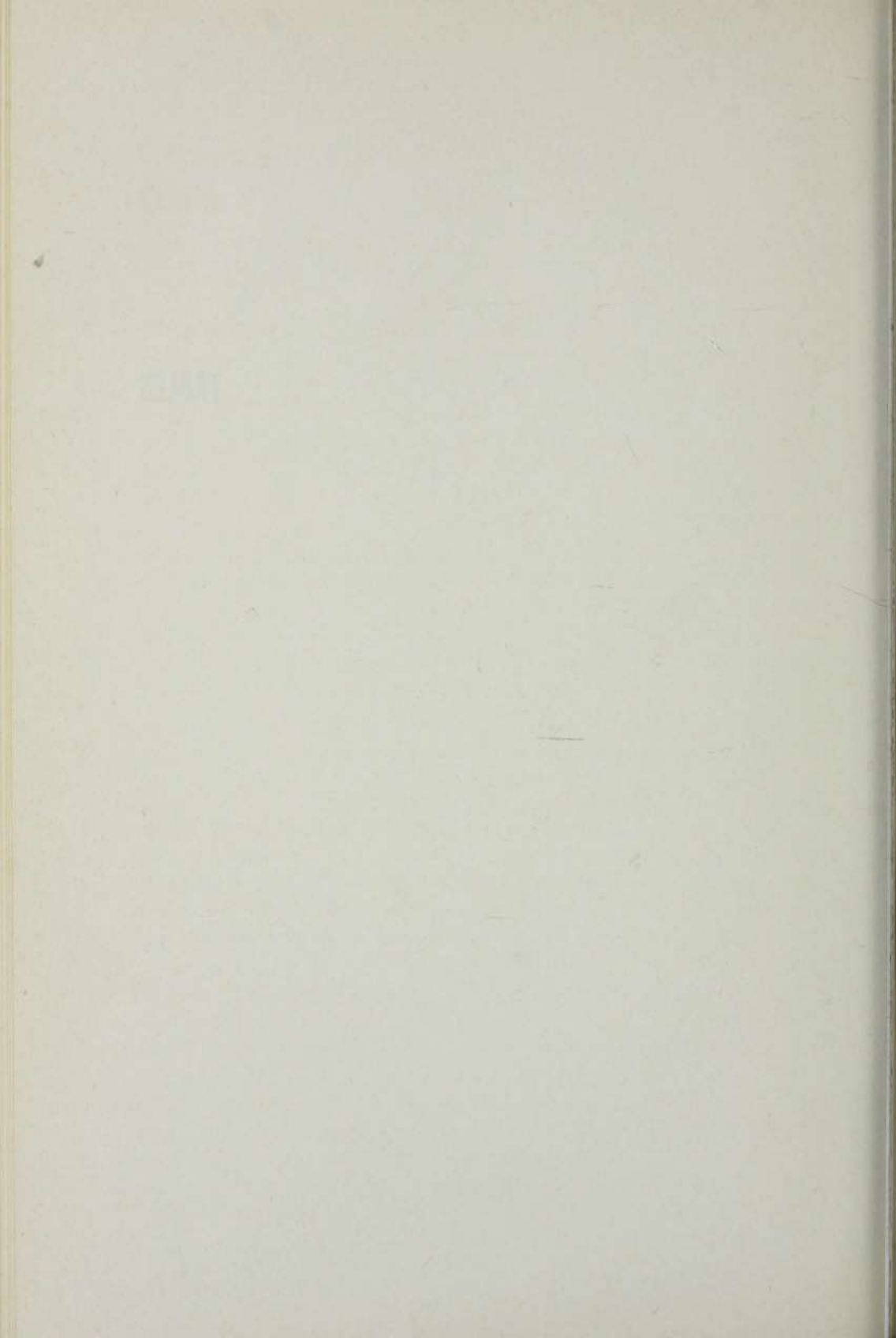
14. For the policy of Problem 2, calculate the differences in the annual premiums and the natural premiums for each of the 5 years, beginning with the premium due at age 65. Will these be deposits into or withdrawals from the reserve fund?

15. Mr. Jones, of Problem 1, at age 60 decides that he cannot pay the insurance premium due then or any future premiums. Rather than lose his policy completely, the insurance company permits him to use his terminal reserve as a single payment on a whole life policy purchased at that age. Find the face of this policy.

16. If the man in Problem 2 discontinues his premium payments at the end of his sixty-fourth year, find the face of a whole life insurance policy that he can purchase with his terminal reserve at that time.



TABLES



TABLES

The following tables have been carefully selected so as to be sufficient for the problems in this book as well as for a large number of practical applications that arise in ordinary business transactions.

Table 1, a six-place table of logarithms, is reproduced from *Mathematics of Business, Accounting, and Finance*, Revised Edition, by Kenneth L. Trefftz and E. J. Hills, by special arrangement with Professor Kenneth L. Trefftz and the publishers, Harper and Brothers. Copyright by Harper and Brothers, 1947, 1956.

Tables 2, 3, 4, 5, 6, 7, and 8 are taken from James W. Glover's *Tables of Applied Mathematics*, published by George Wahr, Ann Arbor, Michigan. The tables of Glover include a complete seven-place table of logarithms, the compound interest and annuity tables computed to eight decimal places, and the logarithms of each of these tables to seven decimal places. Interest rates from $\frac{1}{4}$ per cent to $8\frac{1}{2}$ per cent are given.

Table 9 was computed by the author and the values for $\frac{1}{s \sqrt{\frac{1}{p} i}}$ were checked with existing tables wherever possible.

Tables 10 and 11 are taken from Volume III of *Actuarial Tables* compiled and published by the Actuarial Society of America and the American Institute of Actuaries, predecessors of the Society of Actuaries. These tables were copyrighted in 1945. The Actuarial Society publishes seven volumes of these tables with interest rates ranging from 2 per cent to $3\frac{1}{2}$ per cent.

For the person who finds a need for more extensive tables than those given here, the following tables, in addition to those listed above, will be found helpful. *Compound Interest and Annuity Tables* by Frederick C. Kent and Maude E. Kent, and published by McGraw-Hill Book Company, gives values to ten decimal places for interest rates from $\frac{1}{4}$ per cent to $10\frac{1}{2}$ per cent. It does not give the logarithms of these compound interest and annuity values. *Financial Compound Interest and Annuity Tables*, copyright 1942, 1947, is published by the Financial Publishing Company, Boston, Massachusetts.

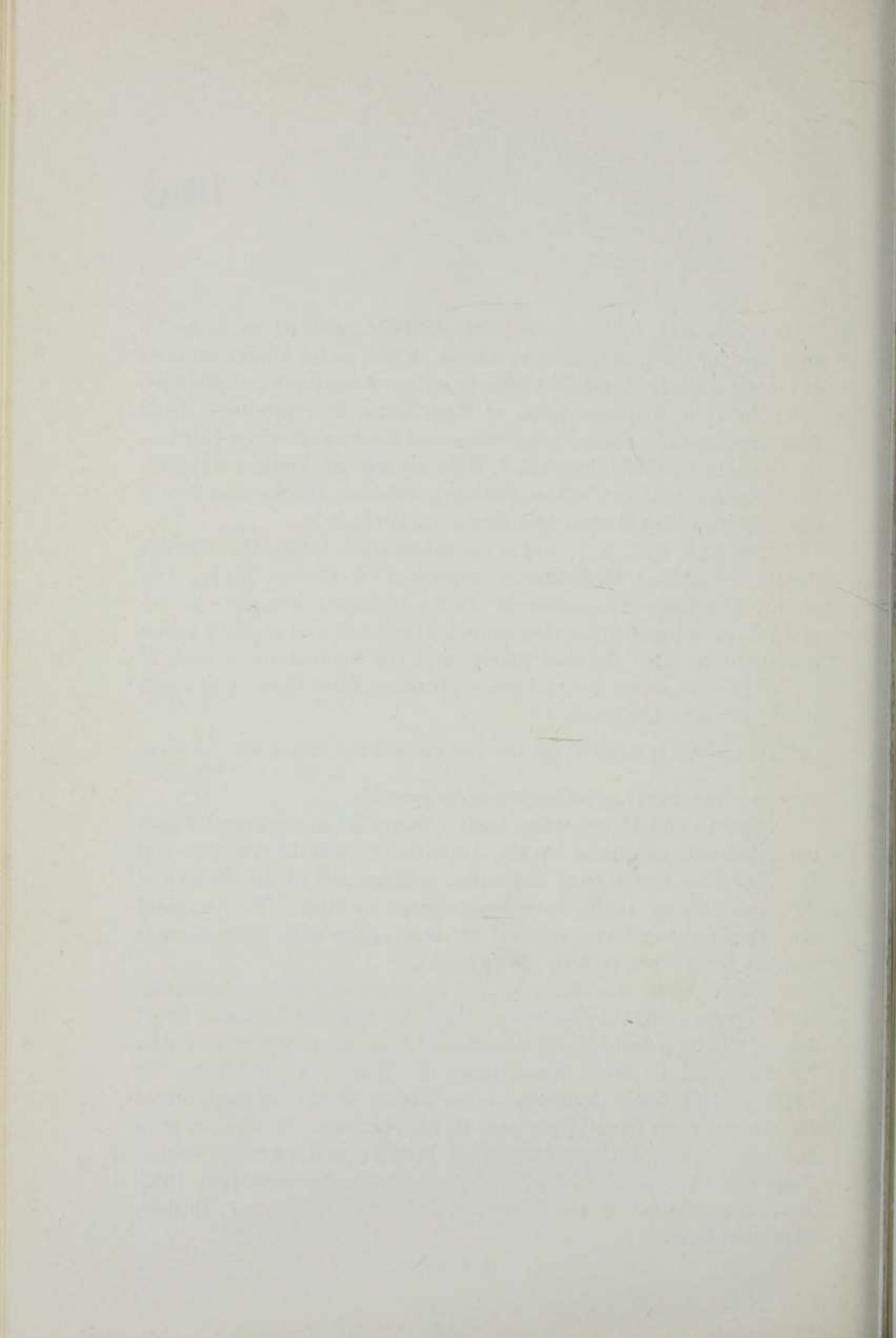


TABLE 1

LOGARITHMS

SIX-PLACE MANTISSAS

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
100	00 0000	00 0434	00 0868	00 1301	00 1734	00 2166	00 2598	00 3029	00 3461	00 3891	432
101	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
102	8600	9026	9451	9876	01 0300	01 0724	01 1147	01 1570	01 1993	01 2415	424
103	01 2837	01 3259	01 3680	01 4100	4521	4940	5360	5779	6197	6616	420
104	7033	7451	7868	8284	8700	9116	9532	9947	02 0361	02 0775	416
105	02 1189	02 1603	02 2016	02 2428	02 2841	02 3252	02 3664	02 4075	4486	4896	412
106	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
107	9384	9789	03 0195	03 0600	03 1004	03 1408	03 1812	03 2216	03 2619	03 3021	404
108	03 3424	03 3826	4227	4628	5029	5430	5830	6230	6629	7028	400
109	7426	7825	8223	8620	9017	9414	9811	04 0207	04 0602	04 0996	397
110	04 1393	04 1787	04 2182	04 2576	04 2969	04 3362	04 3755	4148	4540	4932	393
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
112	9218	9606	9993	05 0380	05 0766	05 1153	05 1538	05 1924	05 2309	05 2694	386
113	05 3078	05 3463	05 3846	4230	4613	4996	5378	5760	6142	6524	383
114	6905	7286	7666	8046	8426	8805	9185	9563	9942	06 0320	379
115	06 0698	06 1075	06 1452	06 1829	06 2206	06 2582	06 2958	06 3333	06 3709	4083	376
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
117	8186	8557	8928	9298	9668	07 0038	07 0407	07 0776	07 1145	07 1514	370
118	07 1882	07 2250	7617	7985	8352	8718	9085	9451	9816	5182	366
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	9182	9543	9904	08 0266	08 0626	08 0987	08 1347	08 1707	08 2067	08 2426	360
121	08 2785	08 3144	08 3503	3861	4219	4576	4934	5291	5647	6004	357
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
123	9905	09 0258	09 0611	09 0963	09 1315	09 1667	09 2018	09 2370	09 2721	09 3071	352
124	09 3422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
125	6910	7257	7604	7951	8298	8644	8990	9335	9681	10 0026	346
126	10 0371	10 0715	10 1059	10 1403	10 1747	10 2091	10 2434	10 2777	10 3119	3462	343
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
128	7210	7549	7888	8227	8565	8903	9241	9579	9916	11 0253	338
129	11 0590	11 0926	11 1263	11 1599	11 1934	11 2270	11 2605	11 2940	11 3275	3609	335
130	3943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
131	7271	7603	7934	8265	8595	8926	9256	9586	9915	12 0245	330
132	12 0574	12 0903	12 1231	12 1560	12 1888	12 2216	12 2544	12 2871	12 3198	3525	328
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
134	7105	7429	7753	8076	8399	8722	9045	9368	9690	13 0012	323
135	13 0334	13 0655	13 0977	13 1298	13 1619	13 1939	13 2260	13 2580	13 2900	3219	321
136	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
137	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
138	9879	10 0194	14 0508	14 0822	14 1136	14 1450	14 1763	14 2076	14 2389	14 2702	314
139	14 3015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	6128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
141	9219	9527	9835	15 0142	15 0449	15 0756	15 1063	15 1370	15 1676	15 1982	307
142	15 2288	15 2594	15 2900	3205	3510	3815	4120	4424	4728	5032	305
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
144	8362	8664	8965	9266	9567	9868	16 0168	16 0469	16 0769	16 1068	301
145	16 1368	16 1667	16 1967	16 2266	16 2564	16 2863	3161	3460	3758	4055	299
146	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
147	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
148	17 0262	17 0555	17 0848	17 1141	17 1434	17 1726	17 2019	17 2311	17 2603	17 2895	293
149	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	6091	6381	6670	6959	7248	7536	7825	8113	8401	8689	289

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
150	17 6091	17 6381	17 6670	17 6959	17 7248	17 7536	17 7825	17 8113	17 8401	17 8689	289
151	8977	9264	9552	9839	18 0126	18 0413	18 0699	18 0986	18 1272	18 1558	287
152	18 1844	18 2129	18 2415	18 2700	2985	3270	3555	3839	4123	4407	285
153	4691	4975	52 59	5542	5825	6108	6391	6674	6956	7239	283
154	7521	7803	8084	8366	8647	8928	9209	9490	9771	19 0051	281
155	19 0332	19 0612	19 0892	19 1171	19 1451	19 1730	19 2010	19 2289	19 2567	2846	279
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
157	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
158	8657	8932	9206	9481	9755	20 0029	20 0303	20 0577	20 0850	20 1124	274
159	20 1397	20 1670	20 1943	20 2216	20 2488	2761	3033	3305	3577	3848	272
160	4120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
161	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
162	9515	9783	21 0051	21 0319	21 0586	21 0853	21 1121	21 1388	21 1654	21 1921	267
163	21 2188	21 2454	2720	2986	3252	3518	3783	4049	4314	4579	266
164	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
165	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
166	22 0108	22 0370	22 0631	22 0892	22 1153	22 1414	22 1675	22 1936	22 2196	22 2456	261
167	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
168	5209	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
169	7887	8144	8400	8657	8913	9170	9426	9682	9938	23 0193	256
170	23 0449	23 0704	23 0960	23 1215	23 1470	23 1724	23 1979	23 2234	23 2488	2742	255
171	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
172	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
173	8046	8297	8548	8799	9049	9299	9550	9800	24 0050	24 0300	250
174	24 0549	24 0799	24 1048	24 1297	24 1546	24 1795	24 2044	24 2293	2541	2790	249
175	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
176	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
177	7973	8219	8464	8709	8954	9198	9443	9687	9932	25 0176	245
178	25 0420	25 0664	25 0908	25 1151	25 1395	25 1638	25 1881	25 2125	25 2368	2610	243
179	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	5273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
182	26 0071	26 0310	26 0548	26 0787	26 1025	26 1263	26 1501	26 1739	26 1976	26 2214	238
183	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
184	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
185	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
186	9513	9746	9980	27 0213	27 0446	27 0679	27 0912	27 1144	27 1377	27 1609	233
187	27 1842	27 2074	27 2306	2538	2770	3001	3233	3464	3696	3927	232
188	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
189	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
190	8754	8982	9211	9439	9667	9895	28 0123	28 0351	28 0578	28 0806	228
191	28 1033	28 1261	28 1488	28 1715	28 1942	28 2169	2396	2622	2849	3075	227
192	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
194	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
195	29 0035	29 0257	29 0480	29 0702	29 0925	29 1147	29 1369	29 1591	29 1813	29 2034	222
196	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
197	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
198	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
199	8853	9071	9289	9507	9725	9943	30 0161	30 0378	30 0595	30 0813	218
200	30 1030	30 1247	30 1464	30 1681	30 1898	30 2114	2331	2547	2764	2980	217

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
200	30 1030	30 1247	30 1464	30 1681	30 1898	30 2114	30 2331	30 2547	30 2764	30 2980	217
201	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
202	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
203	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
204	9630	9843	31 0056	31 0268	31 0481	31 0693	31 0906	31 1118	31 1330	31 1542	212
205	31 1754	31 1966	2177	2389	2600	2812	3023	3234	3445	3656	211
206	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
207	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
208	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
209	32 0146	32 0354	32 0562	32 0769	32 0977	32 1184	32 1391	32 1598	32 1805	32 2012	207
210	2219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
211	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
212	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
213	8380	8583	8787	8991	9194	9398	9601	9805	33 0008	33 0211	203
214	33 0414	33 0617	33 0819	33 1022	33 1225	33 1427	33 1630	33 1832	2034	2236	202
215	2438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
216	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
217	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
218	8456	8656	8855	9054	9253	9451	9650	9849	34 0047	34 0245	199
219	34 0444	34 0642	34 0841	34 1039	34 1237	34 1435	34 1632	34 1830	2028	2225	198
220	2423	2620	2817	3014	3212	3409	3606	3802	3999	4196	197
221	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
222	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
223	8305	8500	8694	8889	9083	9278	9472	9666	9860	35 0054	194
224	35 0248	35 0442	35 0636	35 0829	35 1023	35 1216	35 1410	35 1603	35 1796	1989	193
225	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
226	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
227	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
228	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
229	9835	36 0025	36 0215	36 0404	36 0593	36 0783	36 0972	36 1161	36 1350	36 1539	189
230	36 1728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
231	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
232	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
233	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
234	9216	9401	9587	9772	9958	37 0143	37 0328	37 0513	37 0698	37 0883	185
235	37 1068	37 1253	37 1437	37 1622	37 1806	1991	2175	2360	2544	2728	184
236	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
239	8398	8580	8761	8943	9124	9306	9487	9668	9849	38 0030	181
240	38 0211	38 0392	38 0573	38 0754	38 0934	38 1115	38 1296	38 1476	38 1656	1837	181
241	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
242	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
243	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
244	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
245	9166	9343	9520	9698	9875	39 0051	39 0228	39 0405	39 0582	39 0759	177
246	39 0935	39 1112	39 1288	39 1464	39 1641	1817	1993	2169	2345	2521	176
247	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
248	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
249	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	7940	8114	8287	8461	8634	8808	8981	9154	9328	9501	173

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
250	39 7940	39 8114	39 8287	39 8461	39 8634	39 8808	39 8981	39 9154	39 9328	39 9501	173
251	9674	9847	40 0020	40 0192	40 0365	40 0538	40 0711	40 0883	40 1056	40 1228	173
252	40 1401	40 1573	1745	1917	2089	2261	2433	2605	2777	2949	172
253	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
254	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
255	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
256	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
257	9933	41 0102	41 0271	41 0440	41 0609	41 0777	41 0946	41 1114	41 1283	41 1451	169
258	41 1620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
259	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	4973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
261	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
262	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
263	9956	42 0121	42 0286	42 0451	42 0616	42 0781	42 0945	42 1110	42 1275	42 1439	165
264	42 1604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
265	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
266	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
267	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
268	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
269	9752	9914	43 0075	43 0236	43 0398	43 0559	43 0720	43 0881	43 1042	43 1203	161
270	43 1364	43 1525	1685	1846	2007	2167	2328	2488	2649	2809	161
271	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
272	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
273	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
274	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
275	9333	9491	9648	9806	9964	44 0122	44 0279	44 0437	44 0594	44 0752	158
276	44 0909	44 1066	44 1224	44 1381	44 1538	1695	1852	2009	2166	2323	157
277	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
278	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
279	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155
280	7158	7313	7468	7623	7778	7933	8088	8242	8397	8552	155
281	8706	8861	9015	9170	9324	9478	9633	9787	9941	45 0095	154
282	45 0249	45 0403	45 0557	45 0711	45 0865	45 1018	45 1172	45 1326	45 1479	1633	154
283	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
284	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
285	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
286	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
287	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
288	9392	9543	9694	9845	9995	46 0146	46 0296	46 0447	46 0597	46 0748	151
289	46 0898	46 1048	46 1198	46 1348	46 1499	1649	1799	1948	2098	2248	150
290	2398	2548	2697	2847	2997	3146	3296	3445	3594	3744	150
291	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
292	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
293	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
294	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
295	9822	9969	47 0116	47 0263	47 0410	47 0557	47 0704	47 0851	47 0998	1145	147
296	47 1292	47 1438	1585	1732	1878	2025	2171	2318	2464	2610	146
297	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
298	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
299	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145
300	7121	7266	7411	7555	7700	7844	7989	8133	8278	8422	145

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
300	47 7121	47 7266	47 7411	47 7555	47 7700	47 7844	47 7989	47 8133	47 8278	47 8422	145
301	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
302	48 0007	48 0151	48 0294	48 0438	48 0582	48 0725	48 0869	48 1012	48 1156	48 1299	144
303	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
304	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
305	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
306	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
307	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
308	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
309	9958	49 0099	49 0239	49 0380	49 0520	49 0661	49 0801	49 0941	49 1081	49 1222	140
310	49 1362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
311	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
312	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
313	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
314	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
315	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
316	9687	9824	9962	50 0099	50 0236	50 0374	50 0511	50 0648	50 0785	50 0922	137
317	50 1059	50 1196	50 1333	1470	1607	1744	1880	2017	2154	2291	137
318	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
319	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	5150	5286	5421	5557	5693	5828	5964	6099	6234	6370	136
321	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
322	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
323	9203	9337	9471	9606	9740	9874	51 0009	51 0143	51 0277	51 0411	134
324	51 0545	51 0679	51 0813	51 0947	51 1081	51 1215	1349	1482	1616	1750	134
325	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
326	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
327	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
328	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
329	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	8514	8646	8777	8909	9040	9171	9303	9434	9566	9697	131
331	9828	9959	52 0090	52 0221	52 0353	52 0484	52 0615	52 0745	52 0876	52 1007	131
332	52 1138	52 1269	1400	1530	1661	1792	1922	2053	2183	2314	131
333	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
334	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
335	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
336	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
337	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
338	8917	9045	9174	9302	9430	9559	9687	9815	9943	53 0072	128
339	53 0200	53 0328	53 0456	53 0584	53 0712	53 0840	53 0968	53 1096	53 1223	1351	128
340	1479	1607	1734	1862	1990	2117	2245	2372	2500	2627	128
341	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
342	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
343	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
344	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
345	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
346	9076	9202	9327	9452	9578	9703	9829	9954	54 0079	54 0204	125
347	54 0329	54 0455	54 0580	54 0705	54 0830	54 0955	54 1080	54 1205	1330	1454	125
348	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
349	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	4068	4192	4316	4440	4564	4688	4812	4936	5060	5183	124

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
350	54 4068	54 4192	54 4316	54 4440	54 4564	54 4688	54 4812	54 4936	54 5060	54 5183	124
351	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
352	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
353	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
354	9003	9126	9249	9371	9494	9616	9739	9861	9984	55 0106	123
355	55 0228	55 0351	55 0473	55 0595	55 0717	55 0840	55 0962	55 1084	55 1206	1328	122
356	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
357	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
358	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
359	5054	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	6303	6423	6544	6664	6785	6905	7026	7146	7267	7387	120
361	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
362	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
363	9907	56 0026	56 0146	56 0265	56 0385	56 0504	56 0624	56 0743	56 0863	56 0982	119
364	56 1101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
365	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
366	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
367	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	8202	8319	8436	8554	8671	8788	8905	9023	9140	9257	117
371	9374	9491	9608	9725	9842	9959	57 0076	57 0193	57 0309	57 0426	117
372	57 0543	57 0660	57 0776	57 0893	57 1010	57 1126	1243	1359	1476	1592	117
373	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
374	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
375	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
376	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
377	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
378	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
379	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114
380	9784	9898	58 0012	58 0126	58 0241	58 0355	58 0469	58 0583	58 0697	58 0811	114
381	58 0925	58 1039	1153	1267	1381	1495	1608	1722	1836	1950	114
382	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	114
383	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
384	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
385	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	113
386	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	112
387	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
388	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	112
389	9950	59 0061	59 0173	59 0284	59 0396	59 0507	59 0619	59 0730	59 0842	59 0953	112
390	59 1065	1176	1287	1399	1510	1621	1732	1843	1955	2066	111
391	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
392	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	111
393	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
394	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	110
395	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
396	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	110
397	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
398	9883	9992	60 0101	60 0210	60 0319	60 0428	60 0537	60 0646	60 0755	60 0864	109
399	60 0973	60 1082	1191	1299	1408	1517	1625	1734	1843	1951	109
400	2060	2169	2277	2386	2494	2603	2711	2819	2928	3036	108

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
400	60 2060	60 2169	60 2277	60 2386	60 2494	60 2603	60 2711	60 2819	60 2928	60 3036	108
401	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
402	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	108
403	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
404	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
405	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
406	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	107
407	9594	9701	9808	9914	61 0021	61 0128	61 0234	61 0341	61 0447	61 0554	107
408	61 0660	61 0767	61 0873	61 0979	1086	1192	1298	1405	1511	1617	106
409	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	2784	2890	2996	3102	3207	3313	3419	3525	3630	3736	106
411	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	106
412	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
413	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
414	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
415	8048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
416	9093	9198	9302	9406	9511	9615	9719	9824	9928	62 0032	104
417	62 0136	62 0240	62 0344	62 0448	62 0552	62 0656	62 0760	62 0864	62 0968	1072	104
418	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
419	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
420	3249	3353	3456	3559	3663	3766	3869	3973	4076	4179	103
421	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
422	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	103
423	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
424	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	102
425	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
426	9410	9512	9613	9715	9817	9919	63 0021	63 0123	63 0224	63 0326	102
427	63 0428	63 0530	63 0631	63 0733	63 0835	63 0936	1038	1139	1241	1342	102
428	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
429	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
430	3458	3569	3670	3771	3872	3973	4074	4175	4276	4376	101
431	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	101
432	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	100
433	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	100
434	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	100
435	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	100
436	9486	9586	9686	9785	9885	9984	64 0084	64 0183	64 0283	64 0382	99
437	64 0481	64 0581	64 0680	64 0779	64 0879	64 0978	1077	1177	1276	1375	99
438	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	99
439	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	3453	3551	3650	3749	3847	3946	4044	4143	4242	4340	98
441	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	98
442	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
443	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
444	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
445	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	97
446	9335	9432	9530	9627	9724	9821	9919	65 0016	65 0113	65 0210	97
447	65 0308	65 0405	65 0502	65 0599	65 0696	65 0793	65 0890	0987	1084	1181	97
448	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
449	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
450	3213	3309	3405	3502	3598	3695	3791	3888	3984	4080	96

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
450	65 3213	65 3309	65 3405	65 3502	65 3598	65 3695	65 3791	65 3888	65 3984	65 4080	96
451	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	96
452	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
453	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	96
454	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
455	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	95
456	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	95
457	9916	66 0011	66 0106	66 0201	66 0296	66 0391	66 0486	66 0581	66 0676	66 0771	95
458	66 0865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
459	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	95
460	2758	2852	2947	3041	3135	3230	3324	3418	3512	3607	94
461	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
462	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
463	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
464	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
465	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
466	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
467	9317	9410	9503	9596	9689	9782	9875	9967	67 0060	67 0153	93
468	67 0246	67 0339	67 0431	67 0524	67 0617	67 0710	67 0802	67 0895	0988	1080	93
469	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	2098	2190	2283	2375	2467	2560	2652	2744	2836	2929	92
471	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
472	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
473	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
474	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
475	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
476	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
477	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
478	9428	9519	9610	9700	9791	9882	9973	68 0063	68 0154	68 0245	91
479	68 0336	68 0426	68 0517	68 0607	68 0698	68 0789	68 0879	0970	1060	1151	91
480	1241	1332	1422	1513	1603	1693	1784	1874	1964	2055	90
481	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
482	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
483	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
484	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
485	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
486	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
487	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
488	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
489	9309	9398	9486	9575	9664	9753	9841	9930	69 0019	69 0107	89
490	69 0196	69 0285	69 0373	69 0462	69 0550	69 0639	69 0728	69 0816	0905	0993	89
491	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
492	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
493	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
494	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
495	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
496	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
497	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
498	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
499	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
500	8970	9057	9144	9231	9317	9404	9491	9578	9664	9751	87

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
500	69 8970	69 9057	69 9144	69 9231	69 9317	69 9404	69 9491	69 9578	69 9664	69 9751	87
501	9838	9924	70 0011	70 0098	70 0184	70 0271	70 0358	70 0444	70 0531	70 0617	87
502	70 0704	70 0790	0877	0963	1050	1136	1222	1309	1395	1482	86
503	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
504	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
505	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
506	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
507	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
508	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
509	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
510	7570	7655	7740	7826	7911	7996	8081	8166	8251	8336	85
511	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
512	9270	9355	9440	9524	9609	9694	9779	9863	9948	71 0033	85
513	71 0117	71 0202	71 0287	71 0371	71 0456	71 0540	71 0625	71 0710	71 0794	0879	85
514	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
515	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
516	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
517	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
518	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
519	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84
520	6003	6087	6170	6254	6337	6421	6504	6588	6671	6754	83
521	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
522	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
523	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
524	9331	9414	9497	9580	9663	9745	9828	9911	9994	72 0077	83
525	72 0159	72 0242	72 0325	72 0407	72 0490	72 0573	72 0655	72 0738	72 0821	0903	83
526	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
527	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
528	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
529	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	4276	4358	4440	4522	4604	4685	4767	4849	4931	5013	82
531	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
532	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
533	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
534	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
535	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
536	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
537	9974	73 0055	73 0136	73 0217	73 0298	73 0378	73 0459	73 0540	73 0621	73 0702	81
538	73 0782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
539	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	2394	2474	2555	2635	2715	2796	2876	2956	3037	3117	80
541	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
542	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
543	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
544	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
545	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
546	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
547	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
548	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
549	9572	9651	9731	9810	9889	9968	74 0047	74 0126	74 0205	74 0284	79
550	74 0363	74 0442	74 0521	74 0600	74 0678	74 0757	0836	0915	0994	1073	79

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
550	74 0363	74 0442	74 0521	74 0600	74 0678	74 0757	74 0836	74 0915	74 0994	74 1073	79
551	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
552	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	79
554	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
555	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
556	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
557	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
559	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	8188	8266	8343	8421	8498	8576	8653	8731	8808	8885	77
561	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
562	9736	9814	9891	9968	75 0045	75 0123	75 0200	75 0277	75 0354	75 0431	77
563	75 0508	75 0586	75 0663	75 0740	0817	0894	0971	1048	1125	1202	77
564	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
565	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
566	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
567	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
568	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
569	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	5875	5951	6027	6103	6180	6256	6332	6408	6484	6560	76
571	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
572	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
573	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
574	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
575	9668	9743	9819	9894	9970	76 0045	76 0121	76 0196	76 0272	76 0347	75
576	76 0422	76 0498	76 0573	76 0649	76 0724	0799	0875	0950	1025	1101	75
577	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
578	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
579	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75
580	3428	3503	3578	3653	3727	3802	3877	3952	4027	4101	75
581	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	75
582	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
583	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
584	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
585	7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
586	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
587	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
588	9377	9451	9525	9599	9673	9746	9820	9894	9968	77 0042	74
589	77 0115	77 0189	77 0263	77 0336	77 0410	77 0484	77 0557	77 0631	77 0705	0778	74
590	0852	0926	0999	1073	1146	1220	1293	1367	1440	1514	74
591	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
594	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
595	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
596	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
597	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
598	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
599	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
600	8151	8224	8296	8368	8441	8513	8585	8658	8730	8802	72

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
600	77 8151	77 8224	77 8296	77 8368	77 8441	77 8513	77 8585	77 8658	77 8730	77 8802	72
601	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
602	9596	9669	9741	9813	9885	9957	78 0029	78 0101	78 0173	78 0245	72
603	78 0317	78 0389	78 0461	78 0533	78 0605	78 0677	0749	0821	0893	0965	72
604	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
605	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
606	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
607	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
608	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
609	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	5330	5401	5472	5543	5615	5686	5757	5828	5899	5970	71
611	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
612	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
613	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
614	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
615	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
616	9581	9651	9722	9792	9863	9933	79 0004	79 0074	79 0144	79 0215	70
617	79 0285	79 0356	79 0426	79 0496	79 0567	79 0637	0707	0778	0848	0918	70
618	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	70
619	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
620	2392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
621	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
622	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
623	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
624	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
625	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
626	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
627	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
628	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
629	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69
630	9341	9409	9478	9547	9616	9685	9754	9823	9892	9961	69
631	80 0029	80 0098	80 0167	80 0236	80 0305	80 0373	80 0442	80 0511	80 0580	80 0648	69
632	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
633	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
634	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	68
635	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
636	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
637	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
638	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
639	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
640	6180	6248	6316	6384	6451	6519	6587	6655	6723	6790	68
641	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
642	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
643	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
644	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
645	9560	9627	9694	9762	9829	9896	9964	81 0031	81 0098	81 0165	67
646	81 0233	81 0300	81 0367	81 0434	81 0501	81 0569	81 0636	0703	0770	0837	67
647	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
648	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
649	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
650	2913	2980	3047	3114	3181	3247	3314	3381	3448	3514	67

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
550	81 2913	81 2980	81 3047	81 3114	81 3181	81 3247	81 3314	81 3381	81 3448	81 3514	67
551	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
552	4288	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
553	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
554	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
555	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
556	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
557	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
558	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
559	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
560	9544	9610	9676	9741	9807	9873	9939	82 0004	82 0070	82 0136	66
561	82 0201	82 0267	82 0333	82 0399	82 0464	82 0530	82 0595	0661	0727	0792	66
562	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
563	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
564	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
565	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
566	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
567	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
568	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
569	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
570	6075	6140	6204	6269	6334	6399	6464	6528	6593	6658	65
571	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
572	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
573	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
574	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
575	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
576	9947	83 0011	83 0075	83 0139	83 0204	83 0268	83 0332	83 0396	83 0460	83 0525	64
577	83 0589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
578	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
579	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
580	2509	2573	2637	2700	2764	2828	2892	2956	3020	3083	64
581	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	64
582	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
583	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
584	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
585	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
586	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
587	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
588	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
589	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
590	8849	8912	8975	9038	9101	9164	9227	9289	9352	9415	63
591	9478	9541	9604	9667	9729	9792	9855	9918	9981	84 0043	63
592	84 0106	84 0169	84 0232	84 0294	84 0357	84 0420	84 0482	84 0545	84 0608	0671	63
593	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
594	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
595	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
596	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
597	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
598	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
599	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62
700	5098	5160	5222	5284	5346	5408	5470	5532	5594	5656	62

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
700	84 5098	84 5160	84 5222	84 5284	84 5346	84 5408	84 5470	84 5532	84 5594	84 5656	62
701	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
702	6337	6399	6461	6523	5685	6646	6708	6770	6832	6894	62
703	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
704	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
705	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
706	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
707	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
708	85 0033	85 0095	85 0156	85 0217	85 0279	85 0340	85 0401	85 0462	85 0524	85 0585	61
709	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	1258	1320	1381	1442	1503	1564	1625	1686	1747	1809	61
711	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
712	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
713	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
714	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
715	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
716	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
717	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
718	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
719	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
720	7332	7393	7453	7513	7574	7634	7694	7755	7815	7875	60
721	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
722	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
723	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
724	9739	9799	9859	9918	9978	86 0038	86 0098	86 0158	86 0218	86 0278	60
725	86 0338	86 0398	86 0458	86 0518	86 0578	0637	0697	0757	0817	0877	60
726	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
727	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
728	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
729	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	3323	3382	3442	3501	3561	3620	3680	3739	3799	3858	59
731	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
732	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
733	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
734	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
735	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
736	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
737	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
738	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
739	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
740	9232	9290	9349	9408	9466	9525	9584	9642	9701	9760	59
741	9818	9877	9935	9994	87 0053	87 0111	87 0170	87 0228	87 0287	87 0345	59
742	87 0404	87 0462	87 0521	87 0579	0638	0696	0755	0813	0872	0930	58
743	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
744	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
745	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
746	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
747	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
748	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
749	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
750	5061	5119	5177	5235	5293	5351	5409	5466	5524	5582	58

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
750	87 5061	87 5119	87 5177	87 5235	87 5293	87 5351	87 5409	87 5466	87 5524	87 5582	58
751	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
752	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
753	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
754	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
755	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
756	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
757	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
758	9669	9726	9784	9841	9898	9956	88 0013	88 0070	88 0127	88 0185	57
759	88 0242	88 0299	88 0356	88 0413	88 0471	88 0528	0585	0642	0699	0756	57
760	0814	0871	0928	0985	1042	1099	1156	1213	1271	1328	57
761	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
762	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
763	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
764	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
765	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
766	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
767	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
768	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
769	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	6491	6547	6604	6660	6716	6773	6829	6885	6942	6998	56
771	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
772	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
773	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
774	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
775	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
776	9862	9918	9974	89 0030	89 0086	89 0141	89 0197	89 0253	89 0309	89 0365	56
777	89 0421	89 0477	89 0533	0589	0645	0700	0756	0812	0868	0924	56
778	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
779	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	2095	2150	2206	2262	2317	2373	2429	2484	2540	2595	56
781	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
782	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
783	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
784	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
785	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
786	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
787	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
788	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
789	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	7627	7682	7737	7792	7847	7902	7957	8012	8067	8122	55
791	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
792	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
793	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
794	9821	9875	9930	9985	90 0039	90 0094	90 0149	90 0203	90 0258	90 0312	55
795	90 0367	90 0422	90 0476	90 0531	0586	0640	0695	0749	0804	0859	55
796	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
797	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
798	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
799	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
800	3090	3144	3199	3253	3307	3361	3416	3470	3524	3578	54

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
800	90 3090	90 3144	90 3199	90 3253	90 3307	90 3361	90 3416	90 3470	90 3524	90 3578	54
801	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
802	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
803	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
804	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
805	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
806	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
807	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
808	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
809	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
810	8485	8539	8592	8646	8699	8753	8807	8860	8914	8967	54
811	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
812	9556	9610	9663	9716	9770	9823	9877	9930	9984	91 0037	53
813	91 0091	91 0144	91 0197	91 0251	91 0304	91 0358	91 0411	91 0464	91 0518	0571	53
814	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
815	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
816	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
817	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
818	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
819	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
820	3814	3867	3920	3973	4026	4079	4132	4184	4237	4290	53
821	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
822	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
823	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
824	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
825	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
826	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
827	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
828	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
829	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	9078	9130	9183	9235	9287	9340	9392	9444	9496	9549	52
831	9601	9653	9706	9758	9810	9862	9914	9967	92 0019	92 0071	52
832	92 0123	92 0176	92 0228	92 0280	92 0332	92 0384	92 0436	92 0489	0541	0593	52
833	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
834	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
835	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
836	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
837	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
838	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
839	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	4279	4331	4383	4434	4486	4538	4589	4641	4693	4744	52
841	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
842	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
843	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
844	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
845	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
846	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
847	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
848	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
849	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
850	9419	9470	9521	9572	9623	9674	9725	9776	9827	9879	51

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
850	92 9419	92 9470	92 9521	92 9572	92 9623	92 9674	92 9725	92 9776	92 9827	92 9879	51
851	9930	9981	93 0032	93 0083	93 0134	93 0185	93 0236	93 0287	93 0338	93 0389	51
852	93 0440	93 0491	0542	0592	0643	0694	0745	0796	0847	0898	51
853	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
854	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
855	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
856	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
857	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
858	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
859	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	4498	4549	4599	4650	4700	4751	4801	4852	4902	4953	50
861	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
862	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
863	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
864	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
865	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
866	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
867	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
868	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
869	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
870	9519	9569	9619	9669	9719	9769	9819	9869	9918	9968	50
871	94 0018	94 0068	94 0118	94 0168	94 0218	94 0267	94 0317	94 0367	94 0417	94 0467	50
872	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
873	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
874	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
875	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
876	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
877	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
878	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
879	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
880	4483	4532	4581	4631	4680	4729	4779	4828	4877	4927	49
881	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
885	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
888	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	9390	9439	9488	9536	9585	9634	9683	9731	9780	9829	49
891	9878	9926	9975	95 0024	95 0073	95 0121	95 0170	95 0219	95 0267	95 0316	49
892	95 0365	95 0414	95 0462	0511	0560	0608	0657	0706	0754	0803	49
893	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
894	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
895	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
896	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
897	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
898	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
899	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	4243	4291	4339	4387	4435	4484	4532	4580	4628	4677	48

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
900	95 4243	95 4291	95 4339	95 4387	95 4435	95 4484	95 4532	95 4580	95 4628	95 4677	48
901	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
902	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
903	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
904	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
905	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
906	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
907	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
908	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
909	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	9041	9089	9137	9185	9232	9280	9328	9375	9423	9471	48
911	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
912	9995	96 0042	96 0090	96 0138	96 0185	96 0233	96 0280	96 0328	96 0376	96 0423	48
913	96 0471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
914	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	48
915	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
916	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
917	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
918	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
919	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	3788	3835	3882	3929	3977	4024	4071	4118	4165	4212	47
921	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
922	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
923	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
924	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
925	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
926	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
927	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
928	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
929	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	8483	8530	8576	8623	8670	8716	8763	8810	8856	8903	47
931	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
932	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
933	9882	9928	9975	97 0021	97 0068	97 0114	97 0161	97 0207	97 0254	97 0300	47
934	97 0347	97 0393	97 0440	0486	0533	0579	0626	0672	0719	0765	46
935	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
936	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
939	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
940	3128	3174	3220	3266	3313	3359	3405	3451	3497	3543	46
941	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
942	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
943	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
944	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
945	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
946	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	7724	7769	7815	7861	7906	7952	7998	8043	8089	8135	46

TABLE 1. Logarithms. Six-Place Mantissas

N	0	1	2	3	4	5	6	7	8	9	D
950	97 7724	97 7769	97 7815	97 7861	97 7906	97 7952	97 7998	97 8043	97 8089	97 8135	46
951	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
952	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
953	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
954	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
955	98 0003	98 0049	98 0094	98 0140	98 0185	98 0231	98 0276	98 0322	98 0367	98 0412	45
956	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
958	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
959	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	2271	2316	2362	2407	2452	2497	2543	2588	2633	2678	45
961	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
962	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
963	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
964	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
966	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
967	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
968	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
969	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	6772	6817	6861	6906	6951	6996	7040	7085	7130	7175	45
971	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
972	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
973	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
974	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
975	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
976	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
977	9895	9939	9983	99 0028	99 0072	99 0117	99 0161	99 0206	99 0250	99 0294	44
978	99 0339	99 0383	99 0428	0472	0516	0561	0605	0650	0694	0738	44
979	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	1226	1270	1315	1359	1403	1448	1492	1536	1580	1625	44
981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
983	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
984	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
985	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
989	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	5635	5679	5723	5767	5811	5854	5898	5942	5986	6030	44
991	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
993	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
994	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
995	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
996	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
998	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
1000	00 0000	00 0043	00 0087	00 0130	00 0174	00 0217	00 0260	00 0304	00 0347	00 0391	43

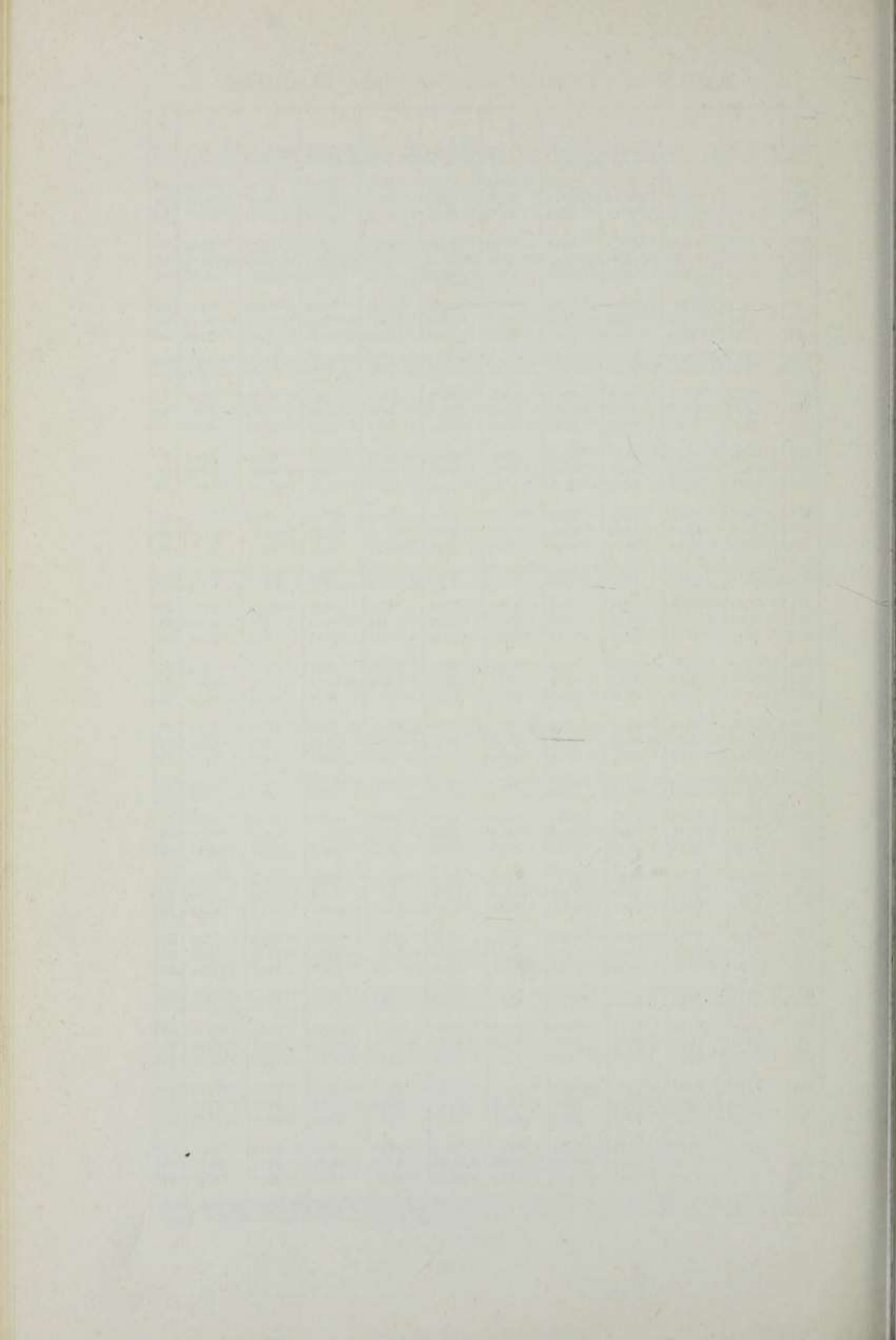


TABLE 2
LOGARITHMS
SEVEN-PLACE MANTISSAS

TABLE 2. Logarithms. Seven-Place Mantissas

N.	0	1	2	3	4	5	6	7	8	9	D.
1000	000 0000	0434	0869	1303	1737	2171	2605	3039	3473	3907	434
1001	4341	4775	5208	5642	6076	6510	6943	7377	7810	8244	434
1002	8677	9111	9544	9977	*0411	*0844	*1277	*1710	*2143	*2576	433
1003	001 3009	3442	3875	4308	4741	5174	5607	6039	6472	6905	433
1004	7337	7770	8202	8635	9067	9499	9932	*0364	*0796	*1228	432
1005	002 1661	2093	2525	2957	3389	3821	4253	4685	5116	5548	432
1006	5980	6411	6843	7275	7706	8138	8569	9001	9432	9863	431
1007	003 0295	0726	1157	1588	2019	2451	2882	3313	3744	4174	431
1008	4605	5036	5467	5898	6328	6759	7190	7620	8051	8481	431
1009	8912	9342	9772	*0203	*0633	*1063	*1493	*1924	*2354	*2784	430
1010	004 3214	3644	4074	4504	4933	5363	5793	6223	6652	7082	430
1011	7512	7941	8371	8800	9229	9659	*0088	*0517	*0947	*1376	429
1012	005 1805	2234	2663	3092	3521	3950	4379	4808	5237	5666	429
1013	6094	6523	6952	7380	7809	8238	8666	9094	9523	9951	429
1014	006 0380	0808	1236	1664	2092	2521	2949	3377	3805	4233	428
1015	4660	5088	5516	5944	6372	6799	7227	7655	8082	8510	428
1016	8937	9365	9792	*0219	*0647	*1074	*1501	*1928	*2355	*2782	427
1017	007 3210	3637	4064	4490	4917	5344	5771	6198	6624	7051	427
1018	7478	7904	8331	8757	9184	9610	*0037	*0463	*0889	*1316	426
1019	008 1742	2168	2594	3020	3446	3872	4298	4724	5150	5576	426
1020	6002	6427	6853	7279	7704	8130	8556	8981	9407	9832	426
1021	009 0257	0683	1108	1533	1959	2384	2809	3234	3659	4084	425
1022	4509	4934	5359	5784	6208	6633	7058	7483	7907	8332	425
1023	8756	9181	9605	*0030	*0454	*0878	*1303	*1727	*2151	*2575	424
1024	010 3000	3424	3848	4272	4696	5120	5544	5967	6391	6815	424
1025	7239	7662	8086	8510	8933	9357	9780	*0204	*0627	*1050	424
1026	011 1474	1897	2320	2743	3166	3590	4013	4436	4859	5282	423
1027	5704	6127	6550	6973	7396	7818	8241	8664	9086	9509	423
1028	9931	*0354	*0776	*1198	*1621	*2043	*2465	*2887	*3310	*3732	422
1029	012 4154	4576	4998	5420	5842	6264	6685	7107	7529	7951	422
1030	8372	8794	9215	9637	*0059	*0480	*0901	*1323	*1744	*2165	422
1031	013 2587	3008	3429	3850	4271	4692	5113	5534	5955	6376	421
1032	6797	7218	7639	8059	8480	8901	9321	9742	*0162	*0583	421
1033	014 1003	1424	1844	2264	2685	3105	3525	3945	4365	4785	420
1034	5205	5625	6045	6465	6885	7305	7725	8144	8564	8984	420
1035	9403	9823	*0243	*0662	*1082	*1501	*1920	*2340	*2759	*3178	420
1036	015 3598	4017	4436	4855	5274	5693	6112	6531	6950	7369	419
1037	7788	8206	8625	9044	9462	9881	*0300	*0718	*1137	*1555	419
1038	016 1974	2392	2810	3229	3647	4065	4483	4901	5319	5737	418
1039	6155	6573	6991	7409	7827	8245	8663	9080	9498	9916	418
1040	017 0333	0751	1168	1586	2003	2421	2838	3256	3673	4090	417
1041	4507	4924	5342	5759	6176	6593	7010	7427	7844	8260	417
1042	8677	9094	9511	9927	*0344	*0761	*1177	*1594	*2010	*2427	417
1043	018 2843	3259	3676	4092	4508	4925	5341	5757	6173	6589	416
1044	7005	7421	7837	8253	8669	9084	9500	9916	*0332	*0747	416
1045	019 1163	1578	1994	2410	2825	3240	3656	4071	4486	4902	415
1046	5317	5732	6147	6562	6977	7392	7807	8222	8637	9052	415
1047	9467	9882	*0296	*0711	*1126	*1540	*1955	*2369	*2784	*3198	415
1048	020 3613	4027	4442	4856	5270	5684	6099	6513	6927	7341	414
1049	7755	8169	8583	8997	9411	9824	*0238	*0652	*1066	*1479	414
1050	021 1893	2307	2720	3134	3547	3961	4374	4787	5201	5614	413
N.	0	1	2	3	4	5	6	7	8	9	D.

TABLE 2. Logarithms. Seven-Place Mantissas

N.	0	1	2	3	4	5	6	7	8	9	D.
I 050	021 1893	2307	2720	3134	3547	3961	4374	4787	5201	5614	413
1051	6027	6440	6854	7267	7680	8093	8506	8919	9332	9745	413
1052	022 0157	0570	0983	1396	1808	2221	2634	3046	3459	3871	413
1053	4284	4696	5109	5521	5933	6345	6758	7170	7582	7994	412
1054	8406	8818	9230	9642	*0054	*0466	*0878	*1289	*1701	*2113	412
1055	023 2525	2936	3348	3759	4171	4582	4994	5405	5817	6228	411
1056	6639	7050	7462	7873	8284	8695	9106	9517	9928	*0339	411
1057	024 0750	1161	1572	1982	2393	2804	3214	3625	4036	4446	411
1058	4857	5267	5678	6088	6498	6909	7319	7729	8139	8549	410
1059	8960	9370	9780	*0190	*0600	*1010	*1419	*1829	*2239	*2649	410
I 060	025 3059	3468	3878	4288	4697	5107	5516	5926	6335	6744	410
1061	7154	7563	7972	8382	8791	9200	9609	*0018	*0427	*0836	409
1062	026 1245	1654	2063	2472	2881	3289	3698	4107	4515	4924	409
1063	5333	5741	6150	6558	6967	7375	7783	8192	8600	9008	408
1064	9416	9824	*0233	*0641	*1049	*1457	*1865	*2273	*2680	*3088	408
1065	027 3496	3904	4312	4719	5127	5535	5942	6350	6757	7165	408
1066	7572	7979	8387	8794	9201	9609	*0016	*0423	*0830	*1237	407
1067	028 1644	2051	2458	2865	3272	3679	4086	4492	4899	5306	407
1068	5713	6119	6526	6932	7339	7745	8152	8558	8964	9371	406
1069	9777	*0183	*0590	*0996	*1402	*1808	*2214	*2620	*3026	*3432	406
I 070	029 3838	4244	4649	5055	5461	5867	6272	6678	7084	7489	406
1071	7895	8300	8706	9111	9516	9922	*0327	*0732	*1138	*1543	405
1072	030 1948	2353	2758	3163	3568	3973	4378	4783	5188	5592	405
1073	5997	6402	6807	7211	7616	8020	8425	8830	9234	9638	405
1074	031 0043	0447	0851	1256	1660	2064	2468	2872	3277	3681	404
1075	4085	4489	4893	5296	5700	6104	6508	6912	7315	7719	404
1076	8123	8526	8930	9333	9737	*0140	*0544	*0947	*1350	*1754	403
1077	032 2157	2560	2963	3367	3770	4173	4576	4979	5382	5785	403
1078	6188	6590	6993	7396	7799	8201	8604	9007	9409	9812	403
1079	033 0214	0617	1019	1422	1824	2226	2629	3031	3433	3835	402
I 080	4238	4640	5042	5444	5846	6248	6650	7052	7453	7855	402
1081	8257	8659	9060	9462	9864	*0265	*0667	*1068	*1470	*1871	402
1082	034 2273	2674	3075	3477	3878	4279	4680	5081	5482	5884	401
1083	6285	6686	7087	7487	7888	8289	8690	9091	9491	9892	401
1084	035 0293	0693	1094	1495	1895	2296	2696	3096	3497	3897	400
1085	4297	4698	5098	5498	5898	6298	6698	7098	7498	7898	400
1086	8298	8698	9098	9498	9898	*0297	*0697	*1097	*1496	*1896	400
1087	036 2295	2695	3094	3494	3893	4293	4692	5091	5491	5890	399
1088	6289	6688	7087	7486	7885	8284	8683	9082	9481	9880	399
1089	037 0279	0678	1076	1475	1874	2272	2671	3070	3468	3867	399
I 090	4265	4663	5062	5460	5858	6257	6655	7053	7451	7849	398
1091	8248	8646	9044	9442	9839	*0237	*0635	*1033	*1431	*1829	398
1092	038 2226	2624	3022	3419	3817	4214	4612	5009	5407	5804	398
1093	6202	6599	6996	7393	7791	8188	8585	8982	9379	9776	397
1094	039 0173	0570	0967	1364	1761	2158	2554	2951	3348	3745	397
1095	4141	4538	4934	5331	5727	6124	6520	6917	7313	7709	397
1096	8106	8502	8898	9294	9690	*0086	*0482	*0878	*1274	*1670	396
1097	040 2066	2462	2858	3254	3650	4045	4441	4837	5232	5628	396
1098	6023	6419	6814	7210	7605	8001	8396	8791	9187	9582	395
1099	9977	*0372	*0767	*1162	*1557	*1952	*2347	*2742	*3137	*3532	395
I 100	041 3927	4322	4716	5111	5506	5900	6295	6690	7084	7479	395
N.	0	1	2	3	4	5	6	7	8	9	D.

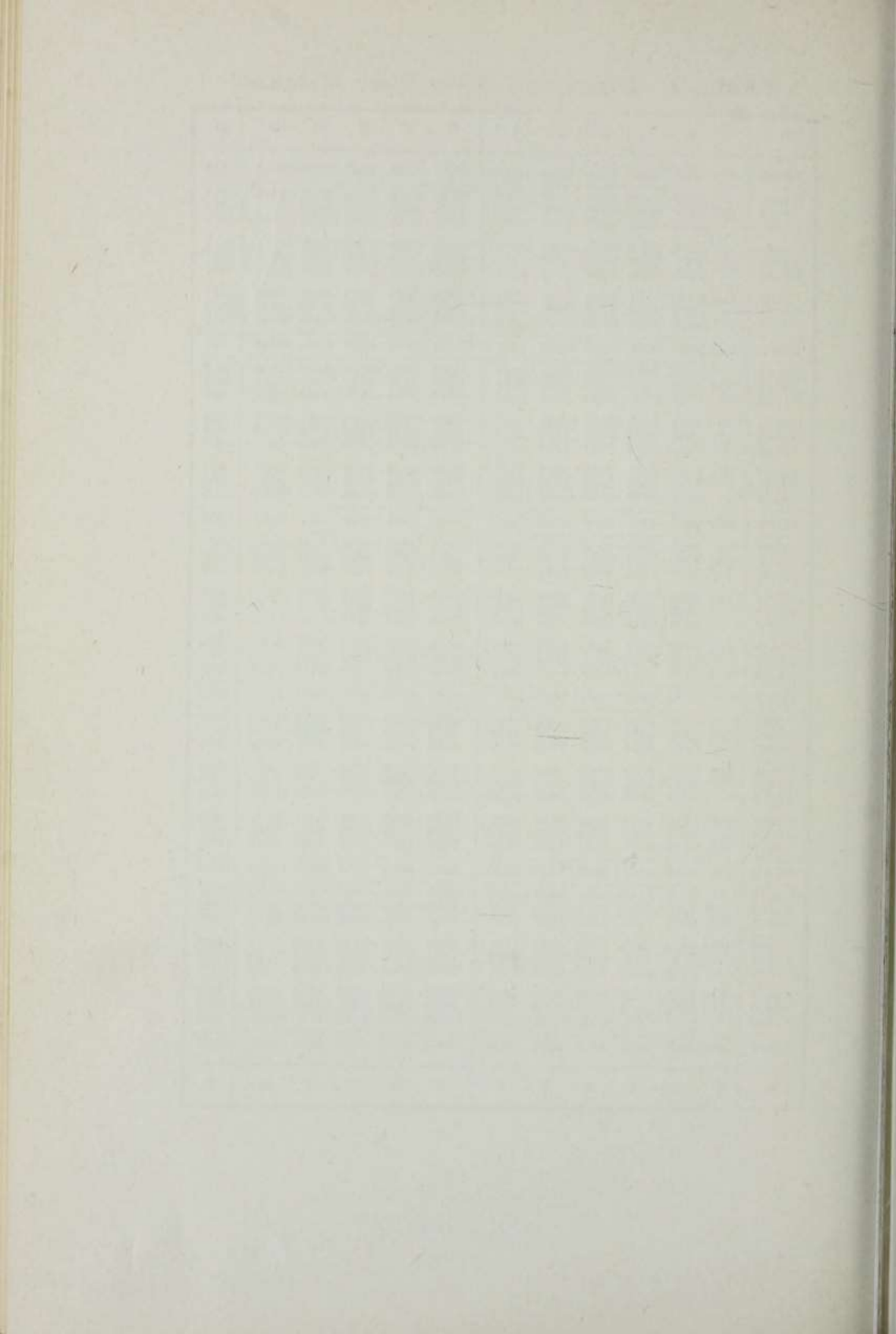


TABLE 3

$(1 + i)^n$ and $\log (1 + i)^n$

TABLE 3. $(1 + i)^n$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
1	1.0025 0000	1.0033 3333	1.0050 0000	1.0066 6667	1
2	1.0050 0625	1.0066 7778	1.0100 2500	1.0133 7778	2
3	1.0075 1877	1.0100 3337	1.0150 7513	1.0201 3363	3
4	1.0100 3756	1.0134 0015	1.0201 5050	1.0269 3452	4
5	1.0125 6266	1.0167 7815	1.0252 5125	1.0337 8075	5
6	1.0150 9406	1.0201 6741	1.0303 7751	1.0406 7262	6
7	1.0176 3180	1.0235 6797	1.0355 2940	1.0476 1044	7
8	1.0201 7588	1.0269 7986	1.0407 0704	1.0545 9451	8
9	1.0227 2632	1.0304 0313	1.0459 1058	1.0616 2514	9
10	1.0252 8313	1.0338 3780	1.0511 4013	1.0687 0264	10
11	1.0278 4634	1.0372 8393	1.0563 9583	1.0758 2732	11
12	1.0304 1596	1.0407 4154	1.0616 7781	1.0829 9951	12
13	1.0329 9200	1.0442 1068	1.0669 8620	1.0902 1950	13
14	1.0355 7448	1.0476 9138	1.0723 2113	1.0974 8763	14
15	1.0381 6341	1.0511 8369	1.0776 8274	1.1048 0422	15
16	1.0407 5882	1.0546 8763	1.0830 7115	1.1121 6958	16
17	1.0433 6072	1.0582 0326	1.0884 8651	1.1195 8404	17
18	1.0459 6912	1.0617 3060	1.0939 2894	1.1270 4794	18
19	1.0485 8404	1.0652 6971	1.0993 9858	1.1345 6159	19
20	1.0512 0550	1.0688 2060	1.1048 9558	1.1421 2533	20
21	1.0538 3352	1.0723 8334	1.1104 2006	1.1497 3950	21
22	1.0564 6810	1.0759 5795	1.1159 7216	1.1574 0443	22
23	1.0591 0927	1.0795 4448	1.1215 5202	1.1651 2046	23
24	1.0617 5704	1.0831 4296	1.1271 5978	1.1728 8793	24
25	1.0644 1144	1.0867 5344	1.1327 9558	1.1807 0718	25
26	1.0670 7247	1.0903 7595	1.1384 5955	1.1885 7857	26
27	1.0697 4015	1.0940 1053	1.1441 5185	1.1965 0242	27
28	1.0724 1450	1.0976 5724	1.1498 7261	1.2044 7911	28
29	1.0750 9553	1.1013 1609	1.1556 2197	1.2125 0897	29
30	1.0777 8327	1.1049 8715	1.1614 0008	1.2205 9236	30
31	1.0804 7773	1.1086 7044	1.1672 0708	1.2287 2964	31
32	1.0831 7892	1.1123 6601	1.1730 4312	1.2369 2117	32
33	1.0858 8687	1.1160 7389	1.1789 0833	1.2451 6731	33
34	1.0886 0159	1.1197 9414	1.1848 0288	1.2534 6843	34
35	1.0913 2309	1.1235 2679	1.1907 2689	1.2618 2489	35
36	1.0940 5140	1.1272 7187	1.1966 8052	1.2702 3705	36
37	1.0967 8653	1.1310 2945	1.2026 6393	1.2787 0530	37
38	1.0995 2850	1.1347 9955	1.2086 7725	1.2872 3000	38
39	1.1022 7732	1.1385 8221	1.2147 2063	1.2958 1153	39
40	1.1050 3301	1.1423 7748	1.2207 9424	1.3044 5028	40
41	1.1077 9559	1.1461 8541	1.2268 9821	1.3131 4661	41
42	1.1105 6508	1.1500 0603	1.2330 3270	1.3219 0092	42
43	1.1133 4149	1.1538 3938	1.2391 9786	1.3307 1360	43
44	1.1161 2485	1.1576 8551	1.2453 9385	1.3395 8502	44
45	1.1189 1516	1.1615 4446	1.2516 2082	1.3485 1559	45
46	1.1217 1245	1.1654 1628	1.2578 7892	1.3575 0569	46
47	1.1245 1673	1.1693 0100	1.2641 6832	1.3665 5573	47
48	1.1273 2802	1.1731 9867	1.2704 8916	1.3756 6610	48
49	1.1301 4634	1.1771 0933	1.2768 4161	1.3848 3721	49
50	1.1329 7171	1.1810 3303	1.2832 2581	1.3940 6946	50

TABLE 3. $\log (1 + i)^n$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
1	0.001 0844	0.001 4452	0.002 1661	0.002 8857	1
2	0.002 1688	0.002 8905	0.004 3321	0.005 7714	2
3	0.003 2531	0.004 3357	0.006 4982	0.008 6571	3
4	0.004 3375	0.005 7810	0.008 6642	0.011 5428	4
5	0.005 4219	0.007 2262	0.010 8303	0.014 4284	5
6	0.006 5063	0.008 6714	0.012 9964	0.017 3141	6
7	0.007 5907	0.010 1167	0.015 1624	0.020 1998	7
8	0.008 6751	0.011 5619	0.017 3285	0.023 0855	8
9	0.009 7594	0.013 0072	0.019 4946	0.025 9712	9
10	0.010 8438	0.014 4524	0.021 6606	0.028 8569	10
11	0.011 9282	0.015 8976	0.023 8267	0.031 7426	11
12	0.013 0126	0.017 3429	0.025 9927	0.034 6283	12
13	0.014 0970	0.018 7881	0.028 1588	0.037 5139	13
14	0.015 1813	0.020 2334	0.030 3249	0.040 3996	14
15	0.016 2657	0.021 6786	0.032 4909	0.043 2853	15
16	0.017 3501	0.023 1239	0.034 6570	0.046 1710	16
17	0.018 4345	0.024 5691	0.036 8230	0.049 0567	17
18	0.019 5189	0.026 0143	0.038 9891	0.051 9424	18
19	0.020 6032	0.027 4596	0.041 1552	0.054 8281	19
20	0.021 6876	0.028 9048	0.043 3212	0.057 7138	20
21	0.022 7720	0.030 3501	0.045 4873	0.060 5995	21
22	0.023 8564	0.031 7953	0.047 6534	0.063 4851	22
23	0.024 9408	0.033 2405	0.049 8194	0.066 3708	23
24	0.026 0252	0.034 6858	0.051 9855	0.069 2565	24
25	0.027 1095	0.036 1310	0.054 1515	0.072 1422	25
26	0.028 1939	0.037 5763	0.056 3176	0.075 0279	26
27	0.029 2783	0.039 0215	0.058 4837	0.077 9136	27
28	0.030 3627	0.040 4667	0.060 6497	0.080 7993	28
29	0.031 4471	0.041 9120	0.062 8158	0.083 6850	29
30	0.032 5314	0.043 3572	0.064 9819	0.086 5706	30
31	0.033 6158	0.044 8025	0.067 1479	0.089 4563	31
32	0.034 7002	0.046 2477	0.069 3140	0.092 3420	32
33	0.035 7846	0.047 6929	0.071 4800	0.095 2277	33
34	0.036 8690	0.049 1382	0.073 6461	0.098 1134	34
35	0.037 9533	0.050 5834	0.075 8122	0.100 9991	35
36	0.039 0377	0.052 0287	0.077 9782	0.103 8848	36
37	0.040 1221	0.053 4739	0.080 1443	0.106 7705	37
38	0.041 2065	0.054 9191	0.082 3103	0.109 6562	38
39	0.042 2909	0.056 3644	0.084 4764	0.112 5418	39
40	0.043 3753	0.057 8096	0.086 6425	0.115 4275	40
41	0.044 4596	0.059 2549	0.088 8085	0.118 3132	41
42	0.045 5440	0.060 7001	0.090 9746	0.121 1989	42
43	0.046 6284	0.062 1454	0.093 1407	0.124 0846	43
44	0.047 7128	0.063 5906	0.095 3067	0.126 9703	44
45	0.048 7972	0.065 0358	0.097 4728	0.129 8560	45
46	0.049 8815	0.066 4811	0.099 6388	0.132 7417	46
47	0.050 9659	0.067 9263	0.101 8049	0.135 6274	47
48	0.052 0503	0.069 3716	0.103 9710	0.138 5130	48
49	0.053 1347	0.070 8168	0.106 1370	0.141 3987	49
50	0.054 2191	0.072 2620	0.108 3031	0.144 2844	50

TABLE 3. $(1 + i)^n$

n	$\frac{1}{2}\%$	$\frac{3}{4}\%$	1%	$1\frac{1}{2}\%$	n
51	1.1358 0414	1.1849 6981	1.2896 4194	1.4033 6325	51
52	1.1386 4365	1.1889 1971	1.2960 9015	1.4127 1901	52
53	1.1414 9026	1.1928 8277	1.3025 7060	1.4221 3713	53
54	1.1443 4398	1.1968 5905	1.3090 8346	1.4316 1805	54
55	1.1472 0484	1.2008 4858	1.3156 2867	1.4411 6217	55
56	1.1500 7285	1.2048 5141	1.3222 0702	1.4507 6992	56
57	1.1529 4804	1.2088 6758	1.3288 1805	1.4604 4172	57
58	1.1558 3041	1.2128 9714	1.3354 6214	1.4701 7799	58
59	1.1587 1998	1.2169 4013	1.3421 3946	1.4799 7918	59
60	1.1616 1678	1.2209 9659	1.3488 5015	1.4898 4571	60
61	1.1645 2082	1.2250 6658	1.3555 9440	1.4997 7801	61
62	1.1674 3213	1.2291 5014	1.3623 7238	1.5097 7653	62
63	1.1703 5071	1.2332 4730	1.3691 8424	1.5198 4171	63
64	1.1732 7658	1.2373 5813	1.3760 3016	1.5299 7399	64
65	1.1762 0977	1.2414 8266	1.3829 1031	1.5401 7381	65
66	1.1791 5030	1.2456 2093	1.3898 2486	1.5504 4164	66
67	1.1820 9817	1.2497 7300	1.3967 7399	1.5607 7792	67
68	1.1850 5342	1.2539 3891	1.4037 5785	1.5711 8310	68
69	1.1880 1605	1.2581 1871	1.4107 7664	1.5816 5766	69
70	1.1909 8609	1.2623 1244	1.4178 3053	1.5922 0204	70
71	1.1939 6356	1.2665 2015	1.4249 1968	1.6028 1672	71
72	1.1969 4847	1.2707 4188	1.4320 4428	1.6135 0217	72
73	1.1999 4084	1.2749 7769	1.4392 0450	1.6242 5885	73
74	1.2029 4069	1.2792 2761	1.4464 0052	1.6350 8724	74
75	1.2059 4804	1.2834 9170	1.4536 3252	1.6459 8782	75
76	1.2089 6291	1.2877 7001	1.4609 0069	1.6569 6107	76
77	1.2119 8532	1.2920 6258	1.4682 0519	1.6680 0748	77
78	1.2150 1528	1.2963 6945	1.4755 4622	1.6791 2753	78
79	1.2180 5282	1.3006 9068	1.4829 2395	1.6903 2172	79
80	1.2210 9795	1.3050 2632	1.4903 3857	1.7015 9053	80
81	1.2241 5070	1.3093 7841	1.4977 9026	1.7129 3446	81
82	1.2272 1108	1.3137 4099	1.5052 7921	1.7243 5403	82
83	1.2302 7910	1.3181 2013	1.5128 0561	1.7358 4972	83
84	1.2333 5480	1.3225 1386	1.5203 6964	1.7474 2205	84
85	1.2364 3819	1.3269 2224	1.5279 7148	1.7590 7153	85
86	1.2395 2928	1.3313 4532	1.5356 1134	1.7707 9868	86
87	1.2426 2811	1.3357 8314	1.5432 8940	1.7826 0400	87
88	1.2457 3468	1.3402 3575	1.5510 0585	1.7944 8803	88
89	1.2488 4901	1.3447 0320	1.5587 6087	1.8064 5128	89
90	1.2519 7114	1.3491 8554	1.5665 5468	1.8184 9429	90
91	1.2551 0106	1.3536 8283	1.5743 8745	1.8306 1758	91
92	1.2582 3882	1.3581 9510	1.5822 5939	1.8428 2170	92
93	1.2613 8441	1.3627 2242	1.5901 7069	1.8551 0718	93
94	1.2645 3787	1.3672 6483	1.5981 2154	1.8674 7456	94
95	1.2676 9922	1.3718 2238	1.6061 1215	1.8799 2439	95
96	1.2708 6847	1.3763 9512	1.6141 4271	1.8924 5722	96
97	1.2740 4564	1.3809 8310	1.6222 1342	1.9050 7360	97
98	1.2772 3075	1.3855 8638	1.6303 2449	1.9177 7409	98
99	1.2804 2383	1.3902 0500	1.6384 7611	1.9305 5925	99
100	1.2836 2489	1.3948 3902	1.6466 6849	1.9434 2965	100

TABLE 3. $\log (1 + i)^n$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
51	0.055 3034	0.073 7073	0.110 4691	0.147 1701	51
52	0.056 3878	0.075 1525	0.112 6352	0.150 0558	52
53	0.057 4722	0.076 5978	0.114 8013	0.152 9415	53
54	0.058 5566	0.078 0430	0.116 9673	0.155 8272	54
55	0.059 6410	0.079 4882	0.119 1334	0.158 7129	55
56	0.060 7254	0.080 9335	0.121 2995	0.161 5985	56
57	0.061 8097	0.082 3787	0.123 4655	0.164 4842	57
58	0.062 8941	0.083 8240	0.125 6316	0.167 3699	58
59	0.063 9785	0.085 2692	0.127 7976	0.170 2556	59
60	0.065 0629	0.086 7145	0.129 9637	0.173 1413	60
61	0.066 1473	0.088 1597	0.132 1298	0.176 0270	61
62	0.067 2316	0.089 6049	0.134 2958	0.178 9127	62
63	0.068 3160	0.091 0502	0.136 4619	0.181 7984	63
64	0.069 4004	0.092 4954	0.138 6279	0.184 6841	64
65	0.070 4848	0.093 9407	0.140 7940	0.187 5697	65
66	0.071 5692	0.095 3859	0.142 9601	0.190 4554	66
67	0.072 6535	0.096 8311	0.145 1261	0.193 3411	67
68	0.073 7379	0.098 2764	0.147 2922	0.196 2268	68
69	0.074 8223	0.099 7216	0.149 4583	0.199 1125	69
70	0.075 9067	0.101 1669	0.151 6243	0.201 9982	70
71	0.076 9911	0.102 6121	0.153 7904	0.204 8839	71
72	0.078 0755	0.104 0573	0.155 9564	0.207 7696	72
73	0.079 1598	0.105 5026	0.158 1225	0.210 6552	73
74	0.080 2442	0.106 9478	0.160 2886	0.213 5409	74
75	0.081 3286	0.108 3931	0.162 4546	0.216 4266	75
76	0.082 4130	0.109 8383	0.164 6207	0.219 3123	76
77	0.083 4974	0.111 2835	0.166 7868	0.222 1980	77
78	0.084 5817	0.112 7288	0.163 9528	0.225 0837	78
79	0.085 6661	0.114 1740	0.171 1189	0.227 9694	79
80	0.086 7505	0.115 6193	0.173 2849	0.230 8551	80
81	0.087 8349	0.117 0645	0.175 4510	0.233 7408	81
82	0.088 9193	0.118 5098	0.177 6171	0.236 6264	82
83	0.090 0036	0.119 9550	0.179 7831	0.239 5121	83
84	0.091 0880	0.121 4002	0.181 9492	0.242 3978	84
85	0.092 1724	0.122 8455	0.184 1152	0.245 2835	85
86	0.093 2568	0.124 2907	0.186 2813	0.248 1692	86
87	0.094 3412	0.125 7360	0.188 4474	0.251 0549	87
88	0.095 4256	0.127 1812	0.190 6134	0.253 9406	88
89	0.096 5099	0.128 6264	0.192 7795	0.256 8263	89
90	0.097 5943	0.130 0717	0.194 9456	0.259 7119	90
91	0.098 6787	0.131 5169	0.197 1116	0.262 5976	91
92	0.099 7631	0.132 9622	0.199 2777	0.265 4833	92
93	0.100 8475	0.134 4074	0.201 4437	0.268 3690	93
94	0.101 9318	0.135 8526	0.203 6098	0.271 2547	94
95	0.103 0162	0.137 2979	0.205 7759	0.274 1404	95
96	0.104 1006	0.138 7431	0.207 9419	0.277 0261	96
97	0.105 1850	0.140 1884	0.210 1080	0.279 9118	97
98	0.106 2694	0.141 6336	0.212 2740	0.282 7975	98
99	0.107 3537	0.143 0788	0.214 4401	0.285 6831	99
100	0.108 4381	0.144 5241	0.216 6062	0.288 5688	100

TABLE 3. $(1 + i)^n$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
101	1.2868 3395	1.3994 8848	1.6549 0183	1.9563 8585	101
102	1.2900 5104	1.4041 5344	1.6631 7634	1.9694 2842	102
103	1.2932 7616	1.4088 3395	1.6714 9223	1.9825 5794	103
104	1.2965 0935	1.4135 3007	1.6798 4969	1.9957 7499	104
105	1.2997 5063	1.4182 4183	1.6882 4894	2.0090 8016	105
106	1.3030 0000	1.4229 6931	1.6966 9018	2.0224 7403	106
107	1.3062 5750	1.4277 1254	1.7051 7363	2.0359 5719	107
108	1.3095 2315	1.4324 7158	1.7136 9950	2.0495 3024	108
109	1.3127 9696	1.4372 4649	1.7222 6800	2.0631 9377	109
110	1.3160 7895	1.4420 3731	1.7308 7934	2.0769 4840	110
111	1.3193 6915	1.4468 4410	1.7395 3373	2.0907 9472	111
112	1.3226 6757	1.4516 6691	1.7482 3140	2.1047 3335	112
113	1.3259 7424	1.4565 0580	1.7569 7256	2.1187 6491	113
114	1.3292 8917	1.4613 6082	1.7657 5742	2.1328 9000	114
115	1.3326 1240	1.4662 3202	1.7745 8621	2.1471 0927	115
116	1.3359 4393	1.4711 1946	1.7834 5914	2.1614 2333	116
117	1.3392 8379	1.4760 2320	1.7923 7644	2.1758 3282	117
118	1.3426 3200	1.4809 4327	1.8013 3832	2.1903 3837	118
119	1.3459 8858	1.4858 7979	1.8103 4501	2.2049 4063	119
120	1.3493 5355	1.4908 3268	1.8193 9673	2.2196 4023	120
121	1.3527 2693	1.4958 0212	1.8284 9372	2.2344 3784	121
122	1.3561 0875	1.5007 8813	1.8376 3619	2.2493 3409	122
123	1.3594 9902	1.5057 9076	1.8468 2437	2.2643 2965	123
124	1.3628 9777	1.5108 1006	1.8560 5849	2.2794 2518	124
125	1.3663 0501	1.5158 4609	1.8653 3878	2.2946 2135	125
126	1.3697 2077	1.5208 9892	1.8746 6548	2.3099 1882	126
127	1.3731 4508	1.5259 6858	1.8840 3880	2.3253 1828	127
128	1.3765 7794	1.5310 5514	1.8934 5900	2.3408 2040	128
129	1.3800 1938	1.5361 5866	1.9029 2629	2.3564 2587	129
130	1.3834 6943	1.5412 7919	1.9124 4092	2.3721 3538	130
131	1.3869 2811	1.5464 1678	1.9220 0313	2.3879 4962	131
132	1.3903 9543	1.5515 7151	1.9316 1314	2.4038 6928	132
133	1.3938 7142	1.5567 4341	1.9412 7121	2.4198 9507	133
134	1.3973 5609	1.5619 3256	1.9509 7757	2.4360 2771	134
135	1.4008 4948	1.5671 3900	1.9607 3245	2.4522 6789	135
136	1.4043 5161	1.5723 6279	1.9705 3612	2.4686 1635	136
137	1.4078 6249	1.5776 0400	1.9803 8880	2.4850 7379	137
138	1.4113 8214	1.5828 6268	1.9902 9074	2.5016 4095	138
139	1.4149 1060	1.5881 3889	2.0002 4219	2.5183 1855	139
140	1.4184 4787	1.5934 3269	2.0102 4340	2.5351 0734	140
141	1.4219 9399	1.5987 4413	2.0202 9462	2.5520 0806	141
142	1.4255 4898	1.6040 7328	2.0303 9609	2.5690 2145	142
143	1.4291 1285	1.6094 2019	2.0405 4808	2.5861 4826	143
144	1.4326 8563	1.6147 8492	2.0507 5082	2.6033 8924	144
145	1.4362 6735	1.6201 6754	2.0610 0457	2.6207 4517	145
146	1.4398 5802	1.6255 6810	2.0713 0959	2.6382 1681	146
147	1.4434 5766	1.6309 8666	2.0816 6614	2.6558 0492	147
148	1.4470 6631	1.6364 2328	2.0920 7447	2.6735 1028	148
149	1.4506 8397	1.6418 7802	2.1025 3484	2.6913 3369	149
150	1.4543 1068	1.6473 5095	2.1130 4752	2.7092 7591	150

TABLE 3. $\log (1 + i)^n$

n	$\frac{1}{4}\%$	$\frac{1}{8}\%$	$\frac{1}{2}\%$	$\frac{3}{8}\%$	n
101	0.109 5225	0.145 9693	0.218 7722	0.291 4545	101
102	0.110 6069	0.147 4146	0.220 9383	0.294 3402	102
103	0.111 6913	0.148 8598	0.223 1044	0.297 2259	103
104	0.112 7757	0.150 3051	0.225 2704	0.300 1116	104
105	0.113 8600	0.151 7503	0.227 4365	0.302 9973	105
106	0.114 9444	0.153 1955	0.229 6025	0.305 8830	106
107	0.116 0288	0.154 6408	0.231 7686	0.308 7686	107
108	0.117 1132	0.156 0860	0.233 9347	0.311 6543	108
109	0.118 1976	0.157 5313	0.236 1007	0.314 5400	109
110	0.119 2819	0.158 9765	0.238 2668	0.317 4257	110
111	0.120 3663	0.160 4217	0.240 4328	0.320 3114	111
112	0.121 4507	0.161 8670	0.242 5989	0.323 1971	112
113	0.122 5351	0.163 3122	0.244 7650	0.326 0828	113
114	0.123 6195	0.164 7575	0.246 9310	0.328 9685	114
115	0.124 7038	0.166 2027	0.249 0971	0.331 8542	115
116	0.125 7882	0.167 6479	0.251 2632	0.334 7398	116
117	0.126 8726	0.169 0932	0.253 4292	0.337 6255	117
118	0.127 9570	0.170 5384	0.255 5953	0.340 5112	118
119	0.129 0414	0.171 9837	0.257 7613	0.343 3969	119
120	0.130 1258	0.173 4289	0.259 9274	0.346 2826	120
121	0.131 2101	0.174 8741	0.262 0935	0.249 1683	121
122	0.132 2945	0.176 3194	0.264 2595	0.352 0540	122
123	0.133 3789	0.177 7646	0.266 4256	0.354 9397	123
124	0.134 4633	0.179 2099	0.268 5917	0.357 8253	124
125	0.135 5477	0.180 6551	0.270 7577	0.360 7110	125
126	0.136 6320	0.182 1004	0.272 9238	0.363 5967	126
127	0.137 7164	0.183 5456	0.275 0898	0.366 4824	127
128	0.138 8008	0.184 9908	0.277 2559	0.369 3681	128
129	0.139 8852	0.186 4361	0.279 4220	0.372 2538	129
130	0.140 9696	0.187 8813	0.281 5880	0.375 1395	130
131	0.142 0540	0.189 3266	0.283 7541	0.378 0252	131
132	0.143 1383	0.190 7718	0.285 9201	0.380 9109	132
133	0.144 2227	0.192 2170	0.288 0862	0.383 7965	133
134	0.145 3071	0.193 6623	0.290 2523	0.386 6822	134
135	0.146 3915	0.195 1075	0.292 4183	0.389 5679	135
136	0.147 4758	0.196 5528	0.294 5844	0.392 4536	136
137	0.148 5602	0.197 9980	0.296 7505	0.395 3393	137
138	0.149 6446	0.199 4432	0.298 9165	0.398 2250	138
139	0.150 7290	0.200 8885	0.301 0826	0.401 1107	139
140	0.151 8134	0.202 3337	0.303 2486	0.403 9964	140
141	0.152 8978	0.203 7790	0.305 4147	0.406 8821	141
142	0.153 9821	0.205 2242	0.307 5808	0.409 7677	142
143	0.155 0665	0.206 6694	0.309 7468	0.412 6534	143
144	0.156 1509	0.208 1147	0.311 9129	0.415 5391	144
145	0.157 2353	0.209 5599	0.314 0790	0.418 4248	145
146	0.158 3197	0.211 0052	0.316 2450	0.421 3105	146
147	0.159 4041	0.212 4504	0.318 4111	0.424 1962	147
148	0.160 4884	0.213 8957	0.320 5771	0.427 0819	148
149	0.161 5728	0.215 3409	0.322 7432	0.429 9676	149
150	0.162 6572	0.216 7861	0.324 9093	0.432 8532	150

TABLE 3. $(1 + i)^n$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{3}{8}\%$	n
151	1.4579 4646	1.6528 4212	2.1236 1276	2.7273 3775	151
152	1.4615 9132	1.6583 5160	2.1342 3082	2.7455 2000	152
153	1.4652 4530	1.6638 7943	2.1449 0197	2.7638 2347	153
154	1.4689 0842	1.6694 2570	2.1556 2648	2.7822 4896	154
155	1.4725 8069	1.6749 9045	2.1664 0462	2.8007 9729	155
156	1.4762 6214	1.6805 7375	2.1772 3664	2.8194 6927	156
157	1.4799 5279	1.6861 7566	2.1881 2282	2.8382 6573	157
158	1.4836 5268	1.6917 9625	2.1990 6344	2.8571 8750	158
159	1.4873 6181	1.6974 3557	2.2100 5875	2.8762 3542	159
160	1.4910 8021	1.7030 9369	2.2211 0905	2.8954 1032	160
161	1.4948 0791	1.7087 7067	2.2322 1459	2.9147 1306	161
162	1.4985 4493	1.7144 6657	2.2433 7566	2.9341 4448	162
163	1.5022 9129	1.7201 8146	2.2545 9254	2.9537 0544	163
164	1.5060 4702	1.7259 1540	2.2658 6551	2.9733 9681	164
165	1.5098 1214	1.7316 6845	2.2771 9483	2.9932 1945	165
166	1.5135 8667	1.7374 4068	2.2885 8081	3.0131 7425	166
167	1.5173 7064	1.7432 3215	2.3000 2371	3.0332 6208	167
168	1.5211 6406	1.7490 4292	2.3115 2383	3.0534 3883	168
169	1.5249 6697	1.7548 7306	2.3230 8145	3.0738 4038	169
170	1.5287 7939	1.7607 2264	2.3346 9686	3.0943 3265	170
171	1.5326 0134	1.7665 9172	2.3463 7034	3.1149 6154	171
172	1.5364 3284	1.7724 8035	2.3581 0219	3.1357 2795	172
173	1.5402 7393	1.7783 8862	2.3698 9270	3.1566 3280	173
174	1.5441 2461	1.7843 1658	2.3817 4217	3.1776 7702	174
175	1.5479 8492	1.7902 6431	2.3936 5088	3.1988 6153	175
176	1.5518 5488	1.7962 3185	2.4056 1913	3.2201 8728	176
177	1.5557 3452	1.8022 1929	2.4176 4723	3.2416 5519	177
178	1.5596 2386	1.8082 2669	2.4297 3546	3.2632 6623	178
179	1.5635 2292	1.8142 5411	2.4418 8414	3.2850 2134	179
180	1.5674 3172	1.8203 0163	2.4540 9358	3.3069 2148	180
181	1.5713 5030	1.8263 6930	2.4663 6403	3.3289 6762	181
182	1.5752 7868	1.8324 5720	2.4786 9585	3.3511 6074	182
183	1.5792 1688	1.8385 6539	2.4910 8933	3.3735 0181	183
184	1.5831 6492	1.8446 9394	2.5035 4478	3.3959 9182	184
185	1.5871 2283	1.8508 4292	2.5160 6250	3.4186 3177	185
186	1.5910 9064	1.8570 1240	2.5286 4281	3.4414 2265	186
187	1.5950 6836	1.8632 0244	2.5412 8603	3.4643 6546	187
188	1.5990 5604	1.8694 1311	2.5539 9246	3.4874 6123	188
189	1.6030 5368	1.8756 4449	2.5667 6242	3.5107 1097	189
190	1.6070 6131	1.8818 9664	2.5795 9623	3.5341 1571	190
191	1.6110 7896	1.8881 6963	2.5924 9421	3.5576 7649	191
192	1.6151 0666	1.8944 6352	2.6054 5668	3.5813 9433	192
193	1.6191 4443	1.9007 7840	2.6184 8397	3.6052 7029	193
194	1.6231 9229	1.9071 1433	2.6315 7639	3.6293 0543	194
195	1.6272 5027	1.9134 7138	2.6447 3427	3.6535 0080	195
196	1.6313 1839	1.9198 4962	2.6579 5794	3.6778 5747	196
197	1.6353 9669	1.9262 4912	2.6712 4773	3.7023 7652	197
198	1.6394 8518	1.9326 6995	2.6846 0397	3.7270 5903	198
199	1.6435 8390	1.9391 1218	2.6980 2699	3.7519 0609	199
200	1.6476 9285	1.9455 7589	2.7115 1712	3.7769 1880	200

TABLE 3. $\log (1+i)^n$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
151	0.163 7416	0.218 2314	0.327 0753	0.435 7389	151
152	0.164 8260	0.219 6766	0.329 2414	0.438 6246	152
153	0.165 9103	0.221 1218	0.331 4074	0.441 5103	153
154	0.166 9947	0.222 5671	0.333 5735	0.444 3960	154
155	0.168 0791	0.224 0123	0.355 7396	0.447 2817	155
156	0.169 1635	0.225 4576	0.337 9056	0.450 1674	156
157	0.170 2479	0.226 9028	0.340 0717	0.453 0531	157
158	0.171 3322	0.228 3481	0.342 2377	0.455 9388	158
159	0.172 4166	0.229 7933	0.344 4038	0.458 8244	159
160	0.173 5010	0.231 2385	0.346 5699	0.461 7101	160
161	0.174 5854	0.232 6838	0.348 7359	0.464 5958	161
162	0.175 6698	0.234 1290	0.350 9020	0.467 4815	162
163	0.176 7542	0.235 5743	0.353 0681	0.470 3672	163
164	0.177 8385	0.237 0195	0.355 2341	0.473 2529	164
165	0.178 9229	0.238 4647	0.357 4002	0.476 1386	165
166	0.180 0073	0.239 9100	0.359 5662	0.479 0243	166
167	0.181 0917	0.241 3552	0.361 7323	0.481 9099	167
168	0.182 1761	0.242 8005	0.363 8984	0.484 7956	168
169	0.183 2604	0.244 2457	0.366 0644	0.487 6813	169
170	0.184 3448	0.245 6910	0.368 2305	0.490 5670	170
171	0.185 4292	0.247 1362	0.370 3966	0.493 4527	171
172	0.186 5136	0.248 5814	0.372 5626	0.496 3384	172
173	0.187 5980	0.250 0267	0.374 7287	0.499 2241	173
174	0.188 6823	0.251 4720	0.376 8947	0.502 1098	174
175	0.189 7667	0.252 9172	0.379 0608	0.504 9955	175
176	0.190 8511	0.254 3624	0.381 2269	0.507 8811	176
177	0.191 9355	0.255 8076	0.383 3929	0.510 7668	177
178	0.193 0199	0.257 2529	0.385 5590	0.513 6525	178
179	0.194 1043	0.258 6981	0.387 7250	0.516 5382	179
180	0.195 1886	0.260 1434	0.389 8911	0.519 4239	180
181	0.196 2730	0.261 5886	0.392 0572	0.522 3096	181
182	0.197 3574	0.263 0338	0.384 2232	0.525 1953	182
183	0.198 4418	0.264 4791	0.396 3893	0.528 0810	183
184	0.199 5262	0.265 9243	0.398 5554	0.530 9666	184
185	0.200 6105	0.267 3696	0.400 7214	0.533 8523	185
186	0.201 6949	0.268 8148	0.402 8875	0.536 7380	186
187	0.202 7793	0.270 2600	0.405 0535	0.539 6237	187
188	0.203 8637	0.271 7053	0.407 2196	0.542 5094	188
189	0.204 9481	0.273 1505	0.409 3857	0.545 3951	189
190	0.206 0324	0.274 5958	0.411 5517	0.548 2808	190
191	0.207 1168	0.276 0410	0.413 7178	0.551 1665	191
192	0.208 2012	0.277 4863	0.415 8838	0.554 0522	192
193	0.209 2856	0.278 9315	0.418 0499	0.556 9378	193
194	0.210 3700	0.280 3767	0.420 2160	0.559 8235	194
195	0.211 4544	0.281 8220	0.422 3820	0.562 7092	195
196	0.212 5387	0.283 2672	0.424 5481	0.565 5949	196
197	0.213 6231	0.284 7125	0.426 7142	0.568 4806	197
198	0.214 7075	0.286 1577	0.428 8802	0.571 3663	198
199	0.215 7919	0.287 6029	0.431 0463	0.574 2520	199
200	0.216 8763	0.289 0482	0.433 2123	0.577 1377	200

TABLE 3. $(1 + i)^n$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	$1\frac{1}{4}\%$	n
1	1.0075 0000	1.0087 5000	1.0100 0000	1.0125 0000	1
2	1.0150 5625	1.0175 7656	1.0201 0000	1.0251 5625	2
3	1.0226 6917	1.0264 8036	1.0303 0100	1.0379 7070	3
4	1.0303 3919	1.0354 6206	1.0406 0401	1.0509 4534	4
5	1.0380 6673	1.0445 2235	1.0510 1005	1.0640 8215	5
6	1.0458 5224	1.0536 6192	1.0615 2015	1.0773 8318	6
7	1.0536 9613	1.0628 8147	1.0721 3535	1.0908 5047	7
8	1.0615 9885	1.0721 8168	1.0828 5671	1.1044 8610	8
9	1.0695 6084	1.0815 6327	1.0936 8527	1.1182 9218	9
10	1.0775 8255	1.0910 2695	1.1046 2213	1.1322 7083	10
11	1.0856 6441	1.1005 7343	1.1156 6835	1.1464 2422	11
12	1.0938 0690	1.1102 0345	1.1268 2503	1.1607 5452	12
13	1.1020 1045	1.1199 1773	1.1380 9328	1.1752 6395	13
14	1.1102 7553	1.1297 1701	1.1494 7421	1.1899 5475	14
15	1.1186 0259	1.1396 0203	1.1609 6896	1.2048 2918	15
16	1.1269 9211	1.1495 7355	1.1725 7864	1.2198 8955	16
17	1.1354 4455	1.1596 3232	1.1843 0443	1.2351 3817	17
18	1.1439 6039	1.1697 7910	1.1961 4748	1.2505 7739	18
19	1.1525 4009	1.1800 1467	1.2081 0895	1.2662 0961	19
20	1.1611 8414	1.1903 3980	1.2201 9004	1.2820 3723	20
21	1.1698 9302	1.2007 5527	1.2323 9194	1.2980 6270	21
22	1.1786 6722	1.2112 6188	1.2447 1586	1.3142 8848	22
23	1.1875 0723	1.2218 6042	1.2571 6302	1.3307 1709	23
24	1.1964 1353	1.2325 5170	1.2697 3465	1.3473 5105	24
25	1.2053 8663	1.2433 3653	1.2824 3200	1.3641 9294	25
26	1.2144 2703	1.2542 1572	1.2952 5631	1.3812 4535	26
27	1.2235 3523	1.2651 9011	1.3082 0888	1.3985 1092	27
28	1.2327 1175	1.2762 6052	1.3212 9097	1.4159 9230	28
29	1.2419 5709	1.2874 2780	1.3345 0388	1.4336 9221	29
30	1.2512 7176	1.2986 9280	1.3478 4892	1.4516 1336	30
31	1.2606 5630	1.3100 5636	1.3613 2740	1.4697 5853	31
32	1.2701 1122	1.3215 1935	1.3749 4068	1.4881 3051	32
33	1.2796 3706	1.3330 8265	1.3886 9009	1.5067 3214	33
34	1.2892 3434	1.3447 4712	1.4025 7699	1.5255 6629	34
35	1.2989 0359	1.3565 1366	1.4166 0276	1.5446 3587	35
36	1.3086 4537	1.3683 8315	1.4307 6878	1.5639 4382	36
37	1.3184 6021	1.3803 5650	1.4450 7647	1.5834 9312	37
38	1.3283 4866	1.3924 3462	1.4595 2724	1.6032 8678	38
39	1.3383 1128	1.4046 1843	1.4741 2251	1.6233 2787	39
40	1.3483 4861	1.4169 0884	1.4888 6373	1.6436 1946	40
41	1.3584 6123	1.4293 0679	1.5037 5237	1.6641 6471	41
42	1.3686 4969	1.4418 1322	1.5187 8989	1.6849 6677	42
43	1.3789 1456	1.4544 2909	1.5339 7779	1.7060 2885	43
44	1.3892 5642	1.4671 5534	1.5493 1757	1.7273 5421	44
45	1.3996 7584	1.4799 9295	1.5648 1075	1.7489 4614	45
46	1.4101 7341	1.4929 4289	1.5804 5885	1.7708 0797	46
47	1.4207 4971	1.5060 0614	1.5962 6344	1.7929 4306	47
48	1.4314 0533	1.5191 8370	1.6122 2608	1.8153 5485	48
49	1.4421 4087	1.5324 7655	1.6283 4834	1.8380 4679	49
50	1.4529 5693	1.5456 8572	1.6446 3182	1.8610 2237	50

TABLE 3. $\log (1 + i)^n$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	$1\frac{1}{4}\%$	n
1	0.003 2451	0.003 7835	0.004 3214	0.005 3950	1
2	0.006 4901	0.007 5671	0.008 6427	0.010 7901	2
3	0.009 7352	0.011 3506	0.012 9641	0.016 1851	3
4	0.012 9802	0.015 1342	0.017 2855	0.021 5801	4
5	0.016 2253	0.018 9177	0.021 6069	0.026 9752	5
6	0.019 4703	0.022 7013	0.025 9282	0.032 3702	6
7	0.022 7154	0.026 4848	0.030 2496	0.037 7652	7
8	0.025 9604	0.030 2684	0.034 5710	0.043 1603	8
9	0.029 2055	0.034 0519	0.038 8924	0.048 5553	9
10	0.032 4505	0.037 8355	0.043 2137	0.053 9504	10
11	0.035 6956	0.041 6190	0.047 5350	0.059 3454	11
12	0.038 9407	0.045 4026	0.051 8565	0.064 7404	12
13	0.042 1857	0.049 1861	0.056 1779	0.070 1354	13
14	0.045 4308	0.052 9697	0.060 4992	0.075 5304	14
15	0.048 6758	0.056 7532	0.064 8206	0.080 9255	15
16	0.051 9209	0.060 5368	0.069 1420	0.086 3205	16
17	0.055 1659	0.064 3203	0.073 4634	0.091 7155	17
18	0.058 4110	0.068 1039	0.077 7847	0.097 1106	18
19	0.061 6560	0.071 8874	0.082 1061	0.102 5056	19
20	0.064 9011	0.075 6710	0.086 4275	0.107 9006	20
21	0.068 1462	0.079 4545	0.090 7488	0.113 2957	21
22	0.071 3912	0.083 2380	0.095 0702	0.118 6907	22
23	0.074 6363	0.087 0216	0.099 3916	0.124 0857	23
24	0.077 8813	0.090 8051	0.103 7130	0.129 4808	24
25	0.081 1264	0.094 5887	0.108 0343	0.134 8758	25
26	0.084 3714	0.098 3722	0.112 3557	0.140 2708	26
27	0.087 6165	0.102 1558	0.116 6771	0.145 6659	27
28	0.090 8615	0.105 9393	0.120 9985	0.151 0609	28
29	0.094 1066	0.109 7229	0.125 3198	0.156 4559	29
30	0.097 3516	0.113 5064	0.129 6412	0.161 8510	30
31	0.100 5967	0.117 2900	0.133 9626	0.167 2460	31
32	0.103 8418	0.121 0735	0.138 2840	0.172 6410	32
33	0.107 0868	0.124 8571	0.142 6053	0.178 0361	33
34	0.110 3319	0.128 6406	0.146 9267	0.183 4311	34
35	0.113 5769	0.132 4242	0.151 2481	0.188 8261	35
36	0.116 8220	0.136 2077	0.155 5695	0.194 2211	36
37	0.120 0670	0.139 9913	0.159 8908	0.199 6162	37
38	0.123 3121	0.143 7748	0.164 2122	0.205 0112	38
39	0.126 5571	0.147 5584	0.168 5336	0.210 4062	39
40	0.129 8022	0.151 3419	0.172 8550	0.215 8013	40
41	0.133 0472	0.155 1255	0.177 1763	0.221 1963	41
42	0.136 2923	0.158 9090	0.181 4977	0.226 5913	42
43	0.139 5374	0.162 6926	0.185 8191	0.231 9864	43
44	0.142 7824	0.166 4761	0.190 1404	0.237 3814	44
45	0.146 0275	0.170 2596	0.194 4618	0.242 7764	45
46	0.149 2725	0.174 0432	0.198 7832	0.248 1715	46
47	0.152 5176	0.177 8267	0.203 1046	0.253 5665	47
48	0.155 7626	0.181 6103	0.207 4259	0.258 9615	48
49	0.159 0077	0.185 3938	0.211 7473	0.264 3566	49
50	0.162 2527	0.189 1774	0.216 0687	0.269 7516	50

TABLE 3. $(1 + i)^n$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	1 $\frac{1}{4}\%$	n
51	1.4638 5411	1.5594 1222	1.6610 7814	1.8842 8515	51
52	1.4748 3301	1.5730 5708	1.6776 8892	1.9078 3872	52
53	1.4858 9426	1.5868 2133	1.6944 6581	1.9316 8670	53
54	1.4970 3847	1.6007 0602	1.7114 1047	1.9558 3279	54
55	1.5082 6626	1.6147 1219	1.7285 2457	1.9802 8070	55
56	1.5195 7825	1.6288 4093	1.7458 0982	2.0050 3420	56
57	1.5309 7509	1.6430 9328	1.7632 6792	2.0300 9713	57
58	1.5424 5740	1.6574 7035	1.7809 0060	2.0554 7335	58
59	1.5540 2583	1.6719 7322	1.7987 0960	2.0811 6676	59
60	1.5656 8103	1.6866 0298	1.8166 9670	2.1071 8135	60
61	1.5774 2363	1.7013 6076	1.8348 6367	2.1335 2111	61
62	1.5892 5431	1.7162 4766	1.8532 1230	2.1601 9013	62
63	1.6011 7372	1.7312 6483	1.8717 4443	2.1871 9250	63
64	1.6131 8252	1.7464 1340	1.8904 6187	2.2145 3241	64
65	1.6252 8139	1.7616 9452	1.9093 6649	2.2422 1407	65
66	1.6374 7100	1.7771 0934	1.9284 6015	2.2702 4174	66
67	1.6497 5203	1.7926 5905	1.9477 4475	2.2986 1976	67
68	1.6621 2517	1.8083 4482	1.9672 2220	2.3273 5251	68
69	1.6745 9111	1.8241 6783	1.9868 9442	2.3564 4442	69
70	1.6871 5055	1.8401 2930	2.0067 6337	2.3858 9997	70
71	1.6998 0418	1.8562 3043	2.0268 3100	2.4157 2372	71
72	1.7125 5271	1.8724 7245	2.0470 9931	2.4459 2027	72
73	1.7253 9685	1.8888 5658	2.0675 7031	2.4764 9427	73
74	1.7383 3733	1.9053 8408	2.0882 4601	2.5074 5045	74
75	1.7513 7486	1.9220 5619	2.1091 2847	2.5387 9358	75
76	1.7645 1017	1.9388 7418	2.1302 1975	2.5705 2850	76
77	1.7777 4400	1.9558 3933	2.1515 2195	2.6026 6011	77
78	1.7910 7708	1.9729 5292	2.1730 3717	2.6351 9336	78
79	1.8045 1015	1.9902 1626	2.1947 6754	2.6681 3327	79
80	1.8180 4398	2.0076 3066	2.2167 1522	2.7014 8494	80
81	1.8316 7931	2.0251 9742	2.2388 8237	2.7352 5350	81
82	1.8454 1691	2.0429 1790	2.2612 7119	2.7694 4417	82
83	1.8592 5753	2.0607 9343	2.2838 8390	2.8040 6222	83
84	1.8732 0196	2.0788 2537	2.3067 2274	2.8391 1300	84
85	1.8872 5098	2.0970 1510	2.3297 8997	2.8746 0191	85
86	1.9014 0536	2.1153 6398	2.3530 8787	2.9105 3444	86
87	1.9156 6590	2.1338 7341	2.3766 1875	2.9469 1612	87
88	1.9300 3339	2.1525 4481	2.4003 8494	2.9837 5257	88
89	1.9445 0865	2.1713 7957	2.4243 8879	3.0210 4948	89
90	1.9590 9246	2.1903 7914	2.4486 3267	3.0588 1260	90
91	1.9737 8565	2.2095 4496	2.4731 1900	3.0970 4775	91
92	1.9885 8905	2.2288 7848	2.4978 5019	3.1357 6085	92
93	2.0035 0346	2.2483 8117	2.5228 2869	3.1749 5786	93
94	2.0185 2974	2.2680 5450	2.5480 5698	3.2146 4483	94
95	2.0336 6871	2.2878 9998	2.5735 3755	3.2548 2789	95
96	2.0489 2123	2.3079 1910	2.5992 7293	3.2955 1324	96
97	2.0642 8814	2.3281 1340	2.6252 6565	3.3367 0716	97
98	2.0797 7030	2.3484 8439	2.6515 1831	3.3784 1600	98
99	2.0953 6858	2.3690 3363	2.6780 3349	3.4206 4620	99
100	2.1110 8384	2.3897 6267	2.7048 1383	3.4634 0427	100

TABLE 3. $\log (1 + i)^n$

n	$\frac{1}{4}\%$	$\frac{1}{2}\%$	1%	1 $\frac{1}{4}\%$	n
51	0.165 4978	0.192 9609	0.220 3901	0.275 1466	51
52	0.168 7428	0.196 7445	0.224 7114	0.280 5417	52
53	0.171 9879	0.200 5280	0.229 0328	0.285 9367	53
54	0.175 2330	0.204 3116	0.233 3542	0.291 3317	54
55	0.178 4780	0.208 0951	0.237 6756	0.296 7268	55
56	0.181 7231	0.211 8787	0.241 9969	0.302 1218	56
57	0.184 9681	0.215 6622	0.246 3183	0.307 5168	57
58	0.188 2132	0.219 4458	0.250 6397	0.312 9118	58
59	0.191 4582	0.223 2293	0.254 9611	0.318 3069	59
60	0.194 7033	0.227 0129	0.259 2824	0.323 7019	60
61	0.197 9483	0.230 7964	0.263 6038	0.329 0969	61
62	0.201 1934	0.234 5800	0.267 9252	0.334 4920	62
63	0.204 4385	0.238 3635	0.272 2465	0.339 8870	63
64	0.207 6835	0.242 1471	0.276 5679	0.345 2820	64
65	0.210 9286	0.245 9306	0.280 8893	0.350 6771	65
66	0.214 1736	0.249 7141	0.285 2107	0.356 0721	66
67	0.217 4187	0.253 4977	0.289 5320	0.361 4671	67
68	0.220 6637	0.257 2812	0.293 8534	0.366 8622	68
69	0.223 9088	0.261 0648	0.298 1748	0.372 2572	69
70	0.227 1538	0.264 8483	0.302 4962	0.377 6522	70
71	0.230 3989	0.268 6319	0.306 8175	0.383 0473	71
72	0.233 6439	0.272 4154	0.311 1389	0.388 4423	72
73	0.236 8890	0.276 1990	0.315 4603	0.393 8373	73
74	0.240 1341	0.279 9825	0.319 7817	0.399 2324	74
75	0.243 3791	0.283 7661	0.324 1030	0.404 6274	75
76	0.246 6242	0.287 5496	0.328 4244	0.410 0224	76
77	0.249 8692	0.291 3332	0.332 7458	0.415 4175	77
78	0.253 1143	0.295 1167	0.337 0672	0.420 8125	78
79	0.256 3593	0.298 9003	0.341 3887	0.426 2075	79
80	0.259 6044	0.302 6838	0.345 7099	0.431 6026	80
81	0.262 8494	0.306 4674	0.350 0313	0.436 9976	81
82	0.266 0945	0.310 2509	0.354 3527	0.442 3926	82
83	0.269 3395	0.314 0345	0.358 6740	0.447 7876	83
84	0.272 5846	0.317 8180	0.362 9954	0.453 1827	84
85	0.275 8297	0.321 6016	0.367 3168	0.458 5777	85
86	0.279 0747	0.325 3851	0.371 6381	0.463 9727	86
87	0.282 3198	0.329 1686	0.375 9595	0.469 3678	87
88	0.285 5648	0.332 9522	0.380 2809	0.474 7628	88
89	0.288 8099	0.336 7357	0.384 6023	0.480 1578	89
90	0.292 0549	0.340 5193	0.388 9237	0.485 5529	90
91	0.295 3000	0.344 3028	0.393 2450	0.490 9479	91
92	0.298 5450	0.348 0864	0.397 5664	0.496 3429	92
93	0.301 7901	0.351 8699	0.401 8878	0.501 7380	93
94	0.305 0352	0.355 6535	0.406 2091	0.507 1330	94
95	0.308 2802	0.359 4370	0.410 5305	0.512 5280	95
96	0.311 5253	0.363 2206	0.414 8519	0.517 9231	96
97	0.314 7703	0.367 0041	0.419 1733	0.523 3181	97
98	0.318 0154	0.370 7877	0.423 4946	0.528 7131	98
99	0.321 2604	0.374 5712	0.427 8160	0.534 1082	99
100	0.324 5055	0.378 3548	0.432 1374	0.539 5032	100

TABLE 3. $(1 + i)^n$

n	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	2%	$2\frac{1}{2}\%$	n
1	1.0150 0000	1.0175 0000	1.0200 0000	1.0250 0000	1
2	1.0302 2500	1.0353 0625	1.0404 0000	1.0506 2500	2
3	1.0456 7838	1.0534 2411	1.0612 0800	1.0768 9063	3
4	1.0613 6355	1.0718 5903	1.0824 3216	1.1038 1289	4
5	1.0772 8400	1.0906 1656	1.1040 8080	1.1314 0821	5
6	1.0934 4326	1.1097 0235	1.1261 6242	1.1596 9342	6
7	1.1098 4491	1.1291 2215	1.1486 8567	1.1886 8575	7
8	1.1264 9259	1.1488 8178	1.1716 5938	1.2184 0290	8
9	1.1433 8998	1.1689 8721	1.1950 9257	1.2488 6297	9
10	1.1605 4083	1.1894 4449	1.2189 9442	1.2800 8454	10
11	1.1779 4894	1.2102 5977	1.2433 7431	1.3120 8666	11
12	1.1956 1817	1.2314 3931	1.2682 4179	1.3448 8882	12
13	1.2135 5244	1.2529 8950	1.2936 0663	1.3785 1104	13
14	1.2317 5573	1.2749 1682	1.3194 7876	1.4129 7382	14
15	1.2502 3207	1.2972 2786	1.3458 6834	1.4482 9817	15
16	1.2689 8555	1.3199 2935	1.3727 8571	1.4845 0562	16
17	1.2880 2033	1.3430 2811	1.4002 4142	1.5216 1826	17
18	1.3073 4064	1.3665 3111	1.4282 4625	1.5596 5872	18
19	1.3269 5075	1.3904 4540	1.4568 1117	1.5986 5019	19
20	1.3468 5501	1.4147 7820	1.4859 4740	1.6386 1644	20
21	1.3670 5783	1.4395 3681	1.5156 6634	1.6795 8185	21
22	1.3875 6370	1.4647 2871	1.5459 7967	1.7215 7140	22
23	1.4083 7715	1.4903 6146	1.5768 9926	1.7646 1068	23
24	1.4295 0281	1.5164 4279	1.6084 3725	1.8087 2595	24
25	1.4509 4535	1.5429 8054	1.6406 0599	1.8539 4410	25
26	1.4727 0953	1.5699 8269	1.6734 1811	1.9002 9270	26
27	1.4948 0018	1.5974 5739	1.7068 8648	1.9478 0002	27
28	1.5172 2218	1.6254 1290	1.7410 2421	1.9964 9502	28
29	1.5399 8051	1.6538 5762	1.7758 4469	2.0464 0739	29
30	1.5630 8022	1.6828 0013	1.8113 6158	2.0975 6758	30
31	1.5865 2642	1.7122 4913	1.8475 8882	2.1500 0677	31
32	1.6103 2432	1.7422 1349	1.8845 4059	2.2037 5694	32
33	1.6344 7918	1.7727 0223	1.9222 3140	2.2588 5086	33
34	1.6589 9637	1.8037 2452	1.9606 7603	2.3153 2213	34
35	1.6838 8132	1.8352 8970	1.9998 8955	2.3732 0519	35
36	1.7091 3954	1.8674 0727	2.0398 8734	2.4325 3532	36
37	1.7347 7663	1.9000 8689	2.0806 8509	2.4933 4870	37
38	1.7607 9828	1.9333 3841	2.1222 9879	2.5556 8242	38
39	1.7872 1025	1.9671 7184	2.1647 4477	2.6195 7448	39
40	1.8140 1841	2.0015 9734	2.2080 3966	2.6850 6384	40
41	1.8412 2868	2.0366 2530	2.2522 0046	2.7521 9043	41
42	1.8688 4712	2.0722 6624	2.2972 4447	2.8209 9520	42
43	1.8968 7982	2.1085 3090	2.3431 8936	2.8915 2008	43
44	1.9253 3302	2.1454 3019	2.3900 5314	2.9638 0808	44
45	1.9542 1301	2.1829 7522	2.4378 5421	3.0379 0328	45
46	1.9835 2621	2.2211 7728	2.4866 1129	3.1138 5086	46
47	2.0132 7910	2.2600 4789	2.5363 4351	3.1916 9713	47
48	2.0434 7829	2.2995 9872	2.5870 7039	3.2714 8956	48
49	2.0741 3046	2.3398 4170	2.6388 1179	3.3532 7680	49
50	2.1052 4242	2.3807 8893	2.6915 8803	3.4371 0872	50

TABLE 3. $\log (1 + i)^n$

n	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	2%	$2\frac{1}{2}\%$	n
1	0.006 4660	0.007 5344	0.008 6002	0.010 7239	1
2	0.012 9321	0.015 0688	0.017 2003	0.021 4477	2
3	0.019 3981	0.022 6033	0.025 8005	0.032 1716	3
4	0.025 8642	0.030 1377	0.034 4007	0.042 8955	4
5	0.032 3302	0.037 6721	0.043 0009	0.053 6193	5
6	0.038 7963	0.045 2065	0.051 6010	0.064 3432	6
7	0.045 2623	0.052 7409	0.060 2012	0.075 0671	7
8	0.051 7283	0.060 2753	0.068 8014	0.085 7909	8
9	0.058 1944	0.067 8098	0.077 4015	0.096 5148	9
10	0.064 6604	0.075 3442	0.086 0017	0.107 2387	10
11	0.071 1265	0.082 8786	0.094 6019	0.117 9625	11
12	0.077 5925	0.090 4130	0.103 2021	0.128 6864	12
13	0.084 0585	0.097 9474	0.111 8022	0.139 4103	13
14	0.090 5246	0.105 4819	0.120 4024	0.150 1341	14
15	0.096 9906	0.113 0163	0.129 0026	0.160 8580	15
16	0.103 4567	0.120 5507	0.137 6027	0.171 5818	16
17	0.109 9227	0.128 0851	0.146 2029	0.182 3057	17
18	0.116 3888	0.135 6195	0.154 8031	0.193 0296	18
19	0.122 8548	0.143 1539	0.163 4033	0.203 7534	19
20	0.129 3208	0.150 6884	0.172 0034	0.214 4773	20
21	0.135 7869	0.158 2228	0.180 6036	0.225 2012	21
22	0.142 2529	0.165 7572	0.189 2038	0.235 9250	22
23	0.148 7190	0.173 2916	0.197 8040	0.246 6489	23
24	0.155 1850	0.180 8260	0.206 4041	0.257 3728	24
25	0.161 6511	0.188 3604	0.215 0043	0.268 0966	25
26	0.168 1171	0.195 8949	0.223 6045	0.278 8205	26
27	0.174 5831	0.203 4293	0.232 2046	0.289 5444	27
28	0.181 0492	0.210 9637	0.240 8048	0.300 2682	28
29	0.187 5152	0.218 4981	0.249 4050	0.310 9921	29
30	0.193 9813	0.226 0325	0.258 0052	0.321 7160	30
31	0.200 4473	0.233 5670	0.266 6053	0.332 4398	31
32	0.206 9134	0.241 1014	0.275 2055	0.343 1637	32
33	0.213 3794	0.248 6358	0.283 8057	0.353 8876	33
34	0.219 8454	0.256 1702	0.292 4058	0.364 6114	34
35	0.226 3115	0.263 7046	0.301 0060	0.375 3353	35
36	0.232 7775	0.271 2390	0.309 6062	0.386 0592	36
37	0.239 2436	0.278 7735	0.318 2064	0.396 7830	37
38	0.245 7096	0.286 3079	0.326 8065	0.407 5069	38
39	0.252 1756	0.293 8423	0.335 4067	0.418 2308	39
40	0.258 6417	0.301 3767	0.344 0069	0.428 9546	40
41	0.265 1077	0.308 9111	0.352 6070	0.439 6785	41
42	0.271 5738	0.316 4456	0.361 2072	0.450 4023	42
43	0.278 0398	0.323 9800	0.369 8074	0.461 1262	43
44	0.284 5059	0.331 5144	0.378 4076	0.471 8501	44
45	0.290 9719	0.339 0488	0.387 0077	0.482 5739	45
46	0.297 4379	0.346 5832	0.395 6079	0.493 2978	46
47	0.303 9040	0.354 1176	0.404 2081	0.504 0217	47
48	0.310 3700	0.361 6521	0.412 8082	0.514 7455	48
49	0.316 8361	0.369 1865	0.421 4084	0.525 4694	49
50	0.323 3021	0.376 7209	0.430 0086	0.536 1933	50

TABLE 3. $(1 + i)^n$

n	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	2%	$2\frac{1}{2}\%$	n
51	2.1368 2106	2.4224 5274	2.7454 1979	3.5230 3644	51
52	2.1688 7337	2.4648 4566	2.8003 2819	3.6111 1235	52
53	2.2014 0647	2.5071 8046	2.8563 3475	3.7013 9016	53
54	2.2344 2757	2.5518 7012	2.9134 6144	3.7939 2491	54
55	2.2679 4398	2.5965 2785	2.9717 3067	3.8887 7303	55
56	2.3019 6314	2.6419 6708	3.0311 6529	3.9859 9236	56
57	2.3364 9259	2.6882 0151	3.0917 8859	4.0856 4217	57
58	2.3715 3998	2.7352 4503	3.1536 2436	4.1877 8322	58
59	2.4071 1308	2.7831 1182	3.2166 9685	4.2924 7780	59
60	2.4432 1978	2.8318 1628	3.2810 3079	4.3997 8975	60
61	2.4798 6807	2.8813 7306	3.3466 5140	4.5097 8449	61
62	2.5170 6609	2.9317 9709	3.4135 8443	4.6225 2910	62
63	2.5548 2208	2.9831 0354	3.4818 5612	4.7380 9233	63
64	2.5931 4442	3.0343 0785	3.5514 9324	4.8565 4464	64
65	2.6320 4158	3.0884 2574	3.6225 2311	4.9779 5826	65
66	2.6715 2221	3.1424 7319	3.6949 7357	5.1024 0721	66
67	2.7115 9504	3.1974 6647	3.7688 7304	5.2299 6739	67
68	2.7522 6896	3.2534 2213	3.8442 5050	5.3607 1658	68
69	2.7935 5300	3.3103 5702	3.9211 3551	5.4947 3449	69
70	2.8354 5629	3.3682 8827	3.9995 5822	5.6321 0286	70
71	2.8779 8814	3.4272 3331	4.0795 4939	5.7729 0543	71
72	2.9211 5796	3.4872 0990	4.1611 4038	5.9172 2806	72
73	2.9649 7533	3.5482 3607	4.2443 6318	6.0651 5876	73
74	3.0094 4996	3.6103 3020	4.3292 5045	6.2167 8773	74
75	3.0545 9171	3.6735 1098	4.4158 3546	6.3722 0743	75
76	3.1004 1059	3.7377 9742	4.5041 5216	6.5315 1261	76
77	3.1469 1674	3.8032 0888	4.5942 3521	6.6948 0043	77
78	3.1941 2050	3.8697 6503	4.6861 1991	6.8621 7044	78
79	3.2420 3230	3.9374 8592	4.7798 4231	7.0337 2470	79
80	3.2906 6279	4.0063 9192	4.8754 3916	7.2095 6782	80
81	3.3400 2273	4.0765 0378	4.9729 4794	7.3898 0701	81
82	3.3901 2307	4.1478 4260	5.0724 0690	7.5745 5219	82
83	3.4409 7492	4.2204 2984	5.1738 5504	7.7639 1599	83
84	3.4925 8954	4.2942 8737	5.2773 3214	7.9580 1389	84
85	3.5449 7838	4.3694 3740	5.3828 7878	8.1569 6424	85
86	3.5981 5306	4.4459 0255	5.4905 3636	8.3608 8834	86
87	3.6521 2535	4.5237 0584	5.6003 4708	8.5699 1055	87
88	3.7069 0723	4.6028 7070	5.7123 5402	8.7841 5832	88
89	3.7625 1084	4.6834 2093	5.8266 0110	9.0037 6228	89
90	3.8189 4851	4.7653 8080	5.9431 3313	9.2288 5633	90
91	3.8762 3273	4.8487 7496	6.0619 9579	9.4595 7774	91
92	3.9343 7622	4.9336 2853	6.1832 3570	9.6960 6718	92
93	3.9933 9187	5.0199 6703	6.3069 0042	9.9384 6886	93
94	4.0532 9275	5.1078 1645	6.4330 3843	10.1869 3058	94
95	4.1140 9214	5.1972 0324	6.5616 9920	10.4416 0385	95
96	4.1758 0352	5.2881 5429	6.6929 3318	10.7026 4395	96
97	4.2384 4057	5.3806 9699	6.8267 9184	10.9702 1004	97
98	4.3020 1718	5.4748 5919	6.9633 2768	11.2444 6530	98
99	4.3665 4744	5.5706 6923	7.1025 9423	11.5255 7693	99
100	4.4320 4565	5.6681 5594	7.2446 4612	11.8137 1635	100

TABLE 3. $\log (1 + i)^n$

n	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	2%	$2\frac{1}{2}\%$	n
51	0.329 7682	0.384 2553	0.438 6088	0.546 9171	51
52	0.336 2342	0.391 7897	0.447 2089	0.557 6410	52
53	0.342 7002	0.399 3241	0.455 8091	0.568 3649	53
54	0.349 1663	0.406 8586	0.464 4093	0.579 0887	54
55	0.355 6323	0.414 3930	0.473 0094	0.589 8126	55
56	0.362 0984	0.421 9274	0.481 6096	0.600 5365	56
57	0.368 5644	0.429 4618	0.490 2098	0.611 2603	57
58	0.375 0304	0.436 9962	0.498 8100	0.621 9842	58
59	0.381 4965	0.444 5307	0.507 4101	0.632 7081	59
60	0.387 9625	0.452 0651	0.516 0103	0.643 4319	60
61	0.394 4286	0.459 5995	0.524 6105	0.654 1558	61
62	0.400 8946	0.467 1339	0.533 2107	0.664 8797	62
63	0.407 3607	0.474 6683	0.541 8108	0.675 6035	63
64	0.413 8267	0.482 2027	0.550 4110	0.686 3274	64
65	0.420 2927	0.489 7372	0.559 0112	0.697 0513	65
66	0.426 7588	0.497 2716	0.567 6113	0.707 7751	66
67	0.433 2248	0.504 8060	0.576 2115	0.718 4990	67
68	0.439 6909	0.512 3404	0.584 8117	0.729 2228	68
69	0.446 1569	0.519 8748	0.593 4119	0.739 9467	69
70	0.452 6230	0.527 4093	0.602 0120	0.750 6706	70
71	0.459 0890	0.534 9437	0.610 6122	0.761 3944	71
72	0.465 5550	0.542 4781	0.619 2124	0.772 1183	72
73	0.472 0211	0.550 0125	0.627 8125	0.782 8422	73
74	0.478 4871	0.557 5469	0.636 4127	0.793 5660	74
75	0.484 9532	0.565 0813	0.645 0129	0.804 2899	75
76	0.491 4192	0.572 6158	0.653 6131	0.815 0138	76
77	0.497 8852	0.580 1502	0.662 2132	0.825 7376	77
78	0.504 3513	0.587 6846	0.670 8134	0.836 4615	78
79	0.510 8173	0.595 2190	0.679 4136	0.847 1854	79
80	0.517 2834	0.602 7534	0.688 0137	0.857 9092	80
81	0.523 7494	0.610 2878	0.696 6139	0.868 6331	81
82	0.530 2155	0.617 8223	0.705 2141	0.879 3570	82
83	0.536 6815	0.625 3567	0.713 8143	0.890 0808	83
84	0.543 1475	0.632 8911	0.722 4144	0.900 8047	84
85	0.549 6136	0.640 4255	0.731 0146	0.911 5286	85
86	0.556 0796	0.647 9599	0.739 6148	0.922 2524	86
87	0.562 5457	0.655 4944	0.748 2149	0.932 9763	87
88	0.569 0117	0.663 0288	0.756 8151	0.943 7002	88
89	0.575 4778	0.670 5632	0.765 4153	0.954 4240	89
90	0.581 9438	0.678 0976	0.774 0155	0.965 1479	90
91	0.588 4098	0.685 6320	0.782 6156	0.975 8718	91
92	0.594 8759	0.693 1664	0.791 2158	0.986 5956	92
93	0.601 3419	0.700 7009	0.799 8160	0.997 3195	93
94	0.607 8080	0.708 2353	0.808 4161	1.008 0433	94
95	0.614 2740	0.715 7697	0.817 0163	1.018 7672	95
96	0.620 7401	0.723 3041	0.825 6165	1.029 4911	96
97	0.627 2061	0.730 8385	0.834 2167	1.040 2149	97
98	0.633 6721	0.738 3730	0.842 8168	1.050 9388	98
99	0.640 1382	0.745 9074	0.851 4170	1.061 6627	99
100	0.646 6042	0.753 4418	0.860 0172	1.072 3865	100

TABLE 3. $(1 + i)^n$

n	3%	3½%	4%	4½%	n
1	1.0300 0000	1.0350 0000	1.0400 0000	1.0450 0000	1
2	1.0609 0000	1.0712 2500	1.0816 0000	1.0920 2500	2
3	1.0927 2700	1.1087 1788	1.1248 6400	1.1411 6613	3
4	1.1255 0881	1.1475 2300	1.1698 5856	1.1925 1860	4
5	1.1592 7407	1.1876 8631	1.2166 5290	1.2461 8194	5
6	1.1940 5230	1.2292 5533	1.2653 1902	1.3022 6012	6
7	1.2298 7387	1.2722 7926	1.3159 3178	1.3608 6183	7
8	1.2667 7008	1.3168 0904	1.3685 6905	1.4221 0061	8
9	1.3047 7318	1.3628 9735	1.4233 1181	1.4860 9514	9
10	1.3439 1638	1.4105 9876	1.4802 4428	1.5529 6942	10
11	1.3842 3387	1.4599 6972	1.5394 5406	1.6228 5305	11
12	1.4257 6089	1.5110 6866	1.6010 3222	1.6958 8143	12
13	1.4685 3371	1.5639 5606	1.6650 7351	1.7721 9610	13
14	1.5125 8972	1.6186 9452	1.7316 7645	1.8519 4492	14
15	1.5579 6742	1.6753 4883	1.8009 4351	1.9352 8244	15
16	1.6047 0644	1.7339 8604	1.8729 8128	2.0223 7015	16
17	1.6528 4763	1.7946 7555	1.9479 0050	2.1133 7681	17
18	1.7024 3306	1.8574 8920	2.0258 1652	2.2084 7877	18
19	1.7535 0605	1.9225 0132	2.1068 4918	2.3078 6031	19
20	1.8061 1123	1.9897 8886	2.1911 2314	2.4117 1402	20
21	1.8602 9457	2.0594 3147	2.2787 6807	2.5202 4116	21
22	1.9161 0341	2.1315 1158	2.3699 1879	2.6336 5201	22
23	1.9735 8651	2.2061 1448	2.4647 1554	2.7521 6635	23
24	2.0327 9411	2.2833 2849	2.5633 0416	2.8760 1383	24
25	2.0937 7793	2.3632 4498	2.6658 3633	3.0054 3446	25
26	2.1565 9127	2.4459 5856	2.7724 6978	3.1406 7901	26
27	2.2212 8901	2.5315 6711	2.8833 6858	3.2820 0956	27
28	2.2879 2768	2.6201 7196	2.9987 0332	3.4296 9999	28
29	2.3565 6551	2.7118 7798	3.1186 5145	3.5840 3649	29
30	2.4272 6247	2.8067 9370	3.2433 9751	3.7453 1813	30
31	2.5000 8035	2.9050 3148	3.3731 3341	3.9138 5745	31
32	2.5750 8276	3.0067 0759	3.5080 5875	4.0899 8104	32
33	2.6523 3524	3.1119 4235	3.6483 8110	4.2740 3018	33
34	2.7319 0530	3.2208 6033	3.7943 1634	4.4663 6154	34
35	2.8138 6245	3.3335 9045	3.9460 8899	4.6673 4781	35
36	2.8982 7833	3.4502 6611	4.1039 3255	4.8773 7846	36
37	2.9852 2668	3.5710 2543	4.2680 8986	5.0968 6049	37
38	3.0747 8348	3.6960 1132	4.4388 1345	5.3262 1921	38
39	3.1670 2698	3.8253 7171	4.6163 6599	5.5658 9908	39
40	3.2620 3779	3.9592 5972	4.8010 2063	5.8163 6454	40
41	3.3598 9893	4.0978 3381	4.9930 6145	6.0781 0094	41
42	3.4606 9589	4.2412 5799	5.1927 8391	6.3516 1548	42
43	3.5645 1677	4.3897 0202	5.4004 9527	6.6374 3818	43
44	3.6714 5227	4.5433 4160	5.6165 1508	6.9361 2290	44
45	3.7815 9584	4.7023 5855	5.8411 7568	7.2482 4843	45
46	3.8950 4372	4.8669 4110	6.0748 2271	7.5744 1961	46
47	4.0118 9503	5.0372 8404	6.3178 1562	7.9152 6849	47
48	4.1322 5188	5.2135 8898	6.5705 2824	8.2714 5557	48
49	4.2562 1944	5.3960 6459	6.8333 4937	8.6436 7107	49
50	4.3839 0602	5.5849 2686	7.1066 8335	9.0326 3627	50

TABLE 3. $\log (1 + i)^n$

n	3%	3½%	4%	4½%	n
1	0.012 8372	0.014 9403	0.017 0333	0.019 1163	1
2	0.025 6744	0.029 8807	0.034 0667	0.038 2326	2
3	0.038 5117	0.044 8210	0.051 1000	0.057 3489	3
4	0.051 3489	0.059 7614	0.068 1334	0.076 4652	4
5	0.064 1861	0.074 7017	0.085 1667	0.095 5815	5
6	0.077 0233	0.089 6421	0.102 2000	0.114 6977	6
7	0.089 8606	0.104 5824	0.119 2334	0.133 8140	7
8	0.102 6978	0.119 5228	0.136 2667	0.152 9303	8
9	0.115 5350	0.134 4631	0.153 3001	0.172 0466	9
10	0.128 3722	0.149 4035	0.170 3334	0.191 1629	10
11	0.141 2095	0.164 3438	0.187 3667	0.210 2792	11
12	0.154 0467	0.179 2842	0.204 4001	0.229 3955	12
13	0.166 8839	0.194 2245	0.221 4334	0.248 5118	13
14	0.179 7211	0.209 1649	0.238 4668	0.267 6281	14
15	0.192 5584	0.224 1052	0.255 5001	0.286 7444	15
16	0.205 3956	0.239 0456	0.272 5334	0.305 8606	16
17	0.218 2328	0.253 9859	0.289 5668	0.324 9769	17
18	0.231 0700	0.268 9263	0.306 6001	0.344 0932	18
19	0.243 9073	0.283 8666	0.323 6334	0.363 2095	19
20	0.256 7445	0.298 8070	0.340 6668	0.382 3258	20
21	0.269 5817	0.313 7473	0.357 7001	0.401 4421	21
22	0.282 4189	0.328 6877	0.374 7335	0.420 5584	22
23	0.295 2562	0.343 6280	0.391 7668	0.439 6747	23
24	0.308 0934	0.358 5684	0.408 8001	0.458 7910	24
25	0.320 9306	0.373 5087	0.425 8335	0.477 9073	25
26	0.333 7678	0.388 4491	0.442 8668	0.497 0236	26
27	0.346 6051	0.403 3894	0.459 9002	0.516 1398	27
28	0.359 4423	0.418 3298	0.476 9335	0.535 2561	28
29	0.372 2795	0.433 2701	0.493 9668	0.554 3724	29
30	0.385 1167	0.448 2105	0.511 0002	0.573 4887	30
31	0.397 9540	0.463 1508	0.528 0335	0.592 6050	31
32	0.410 7912	0.478 0912	0.545 0669	0.611 7213	32
33	0.423 6284	0.493 0315	0.562 1002	0.630 8376	33
34	0.436 4656	0.507 9719	0.579 1335	0.649 9539	34
35	0.449 3029	0.522 9122	0.596 1669	0.669 0702	35
36	0.462 1401	0.537 8526	0.613 2002	0.688 1865	36
37	0.474 9773	0.552 7929	0.630 2336	0.707 3027	37
38	0.487 8145	0.567 7333	0.647 2669	0.726 4190	38
39	0.500 6518	0.582 6736	0.664 3002	0.745 5353	39
40	0.513 4890	0.597 6140	0.681 3336	0.764 6516	40
41	0.526 3262	0.612 5543	0.698 3669	0.783 7679	41
42	0.539 1634	0.627 4947	0.715 4003	0.802 8842	42
43	0.552 0007	0.642 4350	0.732 4336	0.822 0005	43
44	0.564 8379	0.657 3754	0.749 4669	0.841 1168	44
45	0.577 6751	0.672 3157	0.766 5003	0.860 2331	45
46	0.590 5123	0.687 2561	0.783 5336	0.879 3494	46
47	0.603 3496	0.702 1964	0.800 5669	0.898 4656	47
48	0.616 1868	0.717 1368	0.817 6003	0.917 5819	48
49	0.629 0240	0.732 0771	0.834 6336	0.936 6982	49
50	0.641 8612	0.747 0175	0.851 6670	0.955 8145	50

TABLE 3. $(1 + i)^n$

n	3%	3½%	4%	4½%	n
51	4.5154 2320	5.7803 9930	7.3909 5068	9.4391 0490	51
52	4.6508 8590	5.9827 1327	7.6865 8871	9.8638 6463	52
53	4.7904 1247	6.1921 0824	7.9940 5226	10.3077 3853	53
54	4.9341 2485	6.4088 3202	8.3138 1435	10.7715 8677	54
55	5.0821 4859	6.6331 4114	8.6463 6692	11.2563 0817	55
56	5.2346 1305	6.8653 0108	8.9922 2160	11.7628 4204	56
57	5.3916 5144	7.1055 8662	9.3519 1046	12.2921 6993	57
58	5.5534 0098	7.3542 8215	9.7259 8688	12.8453 1758	58
59	5.7200 0301	7.6116 8203	10.1150 2635	13.4233 5687	59
60	5.8916 0310	7.8780 9090	10.5196 2741	14.0274 0793	60
61	6.0683 5120	8.1538 2408	10.9404 1250	14.6586 4129	61
62	6.2504 0173	8.4392 0793	11.3780 2900	15.3182 8014	62
63	6.4379 1379	8.7345 8020	11.8331 5016	16.0076 0275	63
64	6.6310 5120	9.0402 9051	12.3064 7617	16.7279 4487	64
65	6.8299 8273	9.3567 0068	12.7987 3522	17.4807 0239	65
66	7.0348 8222	9.6841 8520	13.3106 8463	18.2673 3400	66
67	7.2459 2868	10.0231 3168	13.8431 1201	19.0893 6403	67
68	7.4633 0654	10.3739 4129	14.3968 3649	19.9483 8541	68
69	7.6872 0574	10.7370 2924	14.9727 0995	20.8460 6276	69
70	7.9178 2191	11.1128 2526	15.5716 1835	21.7841 3558	70
71	8.1553 5657	11.5017 7414	16.1944 8308	22.7644 2168	71
72	8.4000 1727	11.9043 3624	16.8422 6241	23.7888 2066	72
73	8.6520 1778	12.3209 8801	17.5159 5290	24.8593 1759	73
74	8.9115 7832	12.7522 2259	18.2165 9102	25.9779 8688	74
75	9.1789 2567	13.1985 5038	18.9452 5466	27.1469 9629	75
76	9.4542 9344	13.6604 9964	19.7030 6485	28.3686 1112	76
77	9.7379 2224	14.1386 1713	20.4911 8744	29.6451 9862	77
78	10.0300 5991	14.6334 6873	21.3108 3494	30.9792 3256	78
79	10.3309 6171	15.1456 4013	22.1632 6834	32.3792 9802	79
80	10.6408 9056	15.6757 3754	23.0497 9907	33.8300 9643	80
81	10.9601 1727	16.2243 8035	23.9717 9103	35.3524 5077	81
82	11.2889 2079	16.7922 4195	24.9306 6267	36.9433 1106	82
83	11.6275 8842	17.3799 7041	25.9278 8918	38.6057 6006	83
84	11.9764 1607	17.9882 6938	26.9650 0475	40.3430 1926	84
85	12.3357 0855	18.6178 5881	28.0436 0494	42.1584 5513	85
86	12.7057 7981	19.2694 8387	29.1653 4914	44.0555 8561	86
87	13.0869 5320	19.9439 1580	30.3319 6310	46.0380 8696	87
88	13.4795 6180	20.6419 5285	31.5452 4163	48.1098 0087	88
89	13.8839 4865	21.3644 2120	32.8070 5129	50.2747 4191	89
90	14.3004 6711	22.1121 7595	34.1193 3334	52.5371 0530	90
91	14.7294 8112	22.8861 0210	35.4841 0668	54.9012 7503	91
92	15.1713 6556	23.6871 1568	36.9034 7094	57.3718 3241	92
93	15.6265 0652	24.5161 6473	38.3796 0978	59.9535 6487	93
94	16.0953 0172	25.3742 3049	39.9147 9417	62.6514 7529	94
95	16.5781 6077	26.2623 2856	41.5113 8594	65.4707 9168	95
96	17.0755 0559	27.1815 1006	43.1718 4138	68.4169 7730	96
97	17.5877 7076	28.1328 6291	44.8987 1503	71.4957 4128	97
98	18.1154 0388	29.1175 1311	46.6946 6363	74.7130 4964	98
99	18.6588 6600	30.1366 2607	48.5624 5018	78.0751 3687	99
100	19.2186 3198	31.1914 0798	50.5049 4818	81.5885 1803	100

TABLE 3. $\log (1 + i)^n$

n	3%	3½%	4%	4½%	n
51	0.654 6985	0.761 9578	0.868 7003	0.974 9308	51
52	0.667 5357	0.776 8982	0.885 7336	0.994 0471	52
53	0.680 3729	0.791 8385	0.902 7670	1.013 1634	53
54	0.693 2101	0.806 7789	0.919 8003	1.032 2797	54
55	0.706 0474	0.821 7192	0.936 8337	1.051 3960	55
56	0.718 8846	0.836 6596	0.953 8670	1.070 5123	56
57	0.731 7218	0.851 5999	0.970 9003	1.089 6286	57
58	0.744 5590	0.866 5403	0.987 9337	1.108 7448	58
59	0.757 3963	0.881 4806	1.004 9670	1.127 8611	59
60	0.770 2335	0.896 4210	1.022 0004	1.146 9774	60
61	0.783 0707	0.911 3613	1.039 0337	1.166 0937	61
62	0.795 9079	0.926 3017	1.056 0670	1.185 2100	62
63	0.808 7452	0.941 2420	1.073 1004	1.204 3263	63
64	0.821 5824	0.956 1824	1.090 1337	1.223 4426	64
65	0.834 4196	0.971 1227	1.107 1671	1.242 5589	65
66	0.847 2568	0.986 0631	1.124 2004	1.261 6752	66
67	0.860 0941	1.001 0034	1.141 2337	1.280 7915	67
68	0.872 9313	1.015 9438	1.158 2671	1.299 9077	68
69	0.885 7685	1.030 8841	1.175 3004	1.319 0240	69
70	0.898 6057	1.045 8245	1.192 3338	1.338 1403	70
71	0.911 4430	1.060 7648	1.209 3671	1.357 2566	71
72	0.924 2802	1.075 7052	1.226 4004	1.376 3729	72
73	0.937 1174	1.090 6455	1.243 4338	1.395 4892	73
74	0.949 9546	1.105 5859	1.260 4671	1.414 6055	74
75	0.962 7919	1.120 5262	1.277 5004	1.433 7218	75
76	0.975 6291	1.135 4666	1.294 5338	1.452 8381	76
77	0.988 4663	1.150 4069	1.311 5671	1.471 9544	77
78	1.001 3035	1.165 3473	1.328 6005	1.491 0707	78
79	1.014 1408	1.180 2876	1.345 6338	1.510 1869	79
80	1.026 9780	1.195 2280	1.362 6671	1.529 3032	80
81	1.039 8152	1.210 1683	1.379 7005	1.548 4195	81
82	1.052 6524	1.225 1087	1.396 7338	1.567 5358	82
83	1.065 4897	1.240 0490	1.413 7672	1.586 6521	83
84	1.078 3269	1.254 9894	1.430 8005	1.605 7684	84
85	1.091 1641	1.269 9297	1.447 8338	1.624 8847	85
86	1.104 0013	1.284 8701	1.464 8672	1.644 0010	86
87	1.116 8385	1.299 8104	1.481 9005	1.663 1173	87
88	1.129 6758	1.314 7508	1.498 9339	1.682 2336	88
89	1.142 5130	1.329 6911	1.515 9672	1.701 3498	89
90	1.155 3502	1.344 6315	1.533 0005	1.720 4661	90
91	1.168 1874	1.359 5718	1.550 0339	1.739 5824	91
92	1.181 0247	1.374 5122	1.567 0672	1.758 6987	92
93	1.193 8619	1.389 4525	1.584 1006	1.777 8150	93
94	1.206 6991	1.404 3929	1.601 1339	1.796 9313	94
95	1.219 5363	1.419 3332	1.618 1672	1.816 0476	95
96	1.232 3736	1.434 2736	1.635 2006	1.835 1639	96
97	1.245 2108	1.449 2139	1.652 2339	1.854 2802	97
98	1.258 0480	1.464 1543	1.669 2673	1.873 3965	98
99	1.270 8852	1.479 0946	1.686 3006	1.892 5127	99
100	1.283 7225	1.494 0350	1.703 3339	1.911 6290	100

TABLE 3. $(1 + i)^n$

n	5%	5½%	6%	7%	n
1	1.0500 0000	1.0550 0000	1.0600 0000	1.0700 0000	1
2	1.1025 0000	1.1130 2500	1.1236 0000	1.1449 0000	2
3	1.1576 2500	1.1742 4138	1.1910 1600	1.2250 4300	3
4	1.2155 0625	1.2388 2465	1.2624 7696	1.3107 9601	4
5	1.2762 8156	1.3069 6001	1.3382 2558	1.4025 5173	5
6	1.3400 9564	1.3788 4281	1.4185 1911	1.5007 3035	6
7	1.4071 0042	1.4546 7916	1.5036 3026	1.6057 8148	7
8	1.4774 5544	1.5346 8651	1.5938 4807	1.7181 8618	8
9	1.5513 2822	1.6190 9427	1.6894 7896	1.8384 5921	9
10	1.6288 9463	1.7081 4446	1.7908 4770	1.9671 5136	10
11	1.7103 3936	1.8020 9240	1.8982 9856	2.1048 5195	11
12	1.7958 5633	1.9012 0749	2.0121 9647	2.2521 9159	12
13	1.8856 4914	2.0057 7390	2.1329 2826	2.4098 4500	13
14	1.9799 3160	2.1160 9146	2.2609 0396	2.5785 3415	14
15	2.0789 2818	2.2324 7649	2.3965 5819	2.7590 3154	15
16	2.1828 7459	2.3552 6270	2.5403 5168	2.9521 6375	16
17	2.2920 1832	2.4848 0215	2.6927 7279	3.1588 1521	17
18	2.4066 1923	2.6214 6627	2.8543 3915	3.3799 3228	18
19	2.5269 5020	2.7656 4691	3.0255 9950	3.6165 2754	19
20	2.6532 9771	2.9177 5749	3.2071 3547	3.8696 8446	20
21	2.7859 6259	3.0782 3415	3.3995 6360	4.1405 6237	21
22	2.9252 6072	3.2475 3703	3.6035 3742	4.4304 0174	22
23	3.0715 2376	3.4261 5157	3.8197 4966	4.7405 2986	23
24	3.2250 9994	3.6145 8990	4.0489 3464	5.0723 6695	24
25	3.3863 5494	3.8133 9235	4.2918 7072	5.4274 3264	25
26	3.5556 7269	4.0231 2893	4.5493 8296	5.8073 5292	26
27	3.7334 5632	4.2444 0102	4.8223 4594	6.2138 6763	27
28	3.9201 2914	4.4778 4307	5.1116 8670	6.6488 3836	28
29	4.1161 3560	4.7241 2444	5.4183 8790	7.1142 5705	29
30	4.3219 4238	4.9839 5129	5.7434 9117	7.6122 5504	30
31	4.5380 3949	5.2580 6861	6.0881 0064	8.1451 1290	31
32	4.7649 4147	5.5472 6238	6.4533 8668	8.7152 7080	32
33	5.0031 8854	5.8523 6181	6.8405 8988	9.3253 3975	33
34	5.2533 4797	6.1742 4171	7.2510 2528	9.9781 1354	34
35	5.5160 1537	6.5138 2501	7.6860 8679	10.6765 8148	35
36	5.7918 1614	6.8720 8538	8.1472 5200	11.4239 4219	36
37	6.0814 0694	7.2500 5008	8.6360 8712	12.2236 1814	37
38	6.3854 7729	7.6488 0283	9.1542 5235	13.0792 7141	38
39	6.7047 5115	8.0694 8699	9.7035 0749	13.9948 2041	39
40	7.0399 8871	8.5133 0877	10.2857 1794	14.9744 5784	40
41	7.3919 8815	8.9815 4076	10.9028 6101	16.0226 6989	41
42	7.7615 8756	9.4755 2550	11.5570 3267	17.1442 5678	42
43	8.1496 6693	9.9966 7940	12.2504 5463	18.3443 5475	43
44	8.5571 5028	10.5464 9677	12.9854 8191	19.6284 5959	44
45	8.9850 0779	11.1265 5409	13.7646 1083	21.0024 5176	45
46	9.4342 5818	11.7385 1456	14.5904 8748	22.4726 2338	46
47	9.9059 7109	12.3841 3287	15.4659 1673	24.0457 0702	47
48	10.4012 6965	13.0652 6017	16.3938 7173	25.7289 0651	48
49	10.9213 3313	13.7838 4948	17.3775 0403	27.5299 2997	49
50	11.4673 9979	14.5419 6120	18.4201 5427	29.4570 2506	50

TABLE 3. $\log (1 + i)^n$

n	5%	5½%	6%	7%	n
1	0.021 1893	0.023 2525	0.025 3059	0.029 3838	1
2	0.042 3786	0.046 5049	0.050 6117	0.058 7676	2
3	0.063 5679	0.069 7574	0.075 9176	0.088 1513	3
4	0.084 7572	0.093 0098	0.101 2235	0.117 5351	4
5	0.105 9465	0.116 2623	0.126 5293	0.146 9189	5
6	0.127 1358	0.139 5148	0.151 8352	0.176 3027	6
7	0.148 3251	0.162 7672	0.177 1411	0.205 6864	7
8	0.169 5144	0.186 0197	0.202 4469	0.235 0702	8
9	0.190 7037	0.209 2721	0.227 7528	0.264 4540	9
10	0.211 8930	0.232 5246	0.253 0587	0.293 8378	10
11	0.233 0823	0.255 7771	0.278 3645	0.323 2216	11
12	0.254 2716	0.279 0295	0.303 6704	0.352 6053	12
13	0.275 4609	0.302 2820	0.328 9762	0.381 9891	13
14	0.296 6502	0.325 5344	0.354 2821	0.411 3729	14
15	0.317 8395	0.348 7869	0.379 5880	0.440 7567	15
16	0.339 0288	0.372 0394	0.404 8938	0.470 1404	16
17	0.360 2181	0.395 2918	0.430 1997	0.499 5242	17
18	0.381 4074	0.418 5443	0.455 5056	0.528 9080	18
19	0.402 5967	0.441 7967	0.480 8114	0.558 2918	19
20	0.423 7860	0.465 0492	0.506 1173	0.587 6756	20
21	0.444 9753	0.488 3017	0.531 4232	0.617 0593	21
22	0.466 1646	0.511 5541	0.556 7290	0.646 4431	22
23	0.487 3539	0.534 8066	0.582 0349	0.675 8269	23
24	0.508 5432	0.558 0590	0.607 3408	0.705 2107	24
25	0.529 7325	0.581 3115	0.632 6466	0.734 5944	25
26	0.550 9218	0.604 5639	0.657 9525	0.763 9782	26
27	0.572 1111	0.627 8164	0.683 2584	0.793 3620	27
28	0.593 3004	0.651 0689	0.708 5642	0.822 7458	28
29	0.614 4897	0.674 3213	0.733 8701	0.852 1296	29
30	0.635 6790	0.697 5738	0.759 1760	0.881 5133	30
31	0.656 8683	0.720 8262	0.784 4818	0.910 8971	31
32	0.678 0576	0.744 0787	0.809 7877	0.940 2809	32
33	0.699 2469	0.767 3312	0.835 0936	0.969 6647	33
34	0.720 4362	0.790 5836	0.860 3994	0.999 0484	34
35	0.741 6255	0.813 8361	0.885 7053	1.028 4322	35
36	0.762 8148	0.837 0885	0.911 0112	1.057 8160	36
37	0.784 0041	0.860 3410	0.936 3170	1.087 1998	37
38	0.805 1934	0.883 5935	0.961 6229	1.116 5836	38
39	0.826 3827	0.906 8459	0.986 9287	1.145 9673	39
40	0.847 5720	0.930 0984	1.012 2346	1.175 3511	40
41	0.868 7613	0.953 3508	1.037 5405	1.204 7349	41
42	0.889 9506	0.976 6033	1.062 8463	1.234 1187	42
43	0.911 1399	0.999 8558	1.088 1522	1.263 5024	43
44	0.932 3292	1.023 1082	1.113 4581	1.292 8862	44
45	0.953 5185	1.046 3607	1.138 7639	1.322 2700	45
46	0.974 7078	1.069 6131	1.164 0698	1.351 6538	46
47	0.995 8971	1.092 8656	1.189 3757	1.381 0376	47
48	1.017 0864	1.116 1181	1.214 6815	1.410 4213	48
49	1.038 2757	1.139 3705	1.239 9874	1.439 8051	49
50	1.059 4650	1.162 6230	1.265 2933	1.469 1889	50

TABLE 3. $(1 + i)^n$

n	5%	5½%	6%	7%	n
51	12.0407 6978	15.3417 6907	19.5253 6353	31.5190 1682	51
52	12.6428 0826	16.1855 6637	20.6968 8534	33.7253 4799	52
53	13.2749 4868	17.0757 7252	21.9386 9846	36.0861 2235	53
54	13.9386 9611	18.0149 4001	23.2550 2037	38.6121 5092	54
55	14.6356 3092	19.0057 6171	24.6503 2159	41.3150 0148	55
56	15.3674 1246	20.0510 7860	26.1293 4089	44.2070 5159	56
57	16.1357 8309	21.1538 8793	27.6971 0134	47.3015 4520	57
58	16.9425 7224	22.3173 5176	29.3589 2742	50.6126 5336	58
59	17.7897 0085	23.5448 0611	31.1204 6307	54.1555 3910	59
60	18.6791 8589	24.8397 7045	32.9876 9085	57.9464 2683	60
61	19.6131 4519	26.2059 5782	34.9669 5230	62.0026 7671	61
62	20.5938 0245	27.6472 8550	37.0649 6944	66.3428 6408	62
63	21.6234 9257	29.1678 8620	39.2888 6761	70.9868 6457	63
64	22.7046 6720	30.7721 1994	41.6461 9967	75.9559 4509	64
65	23.8399 0056	32.4645 8654	44.1449 7165	81.2728 6124	65
66	25.0318 9559	34.2501 3880	46.7936 6994	86.9619 6153	66
67	26.2834 9037	36.1338 9643	49.6012 9014	93.0492 9884	67
68	27.5976 6488	38.1212 6074	52.5773 6755	99.5627 4976	68
69	28.9775 4813	40.2179 3008	55.7320 0960	106.5321 4224	69
70	30.4264 2554	42.4299 1623	59.0759 3018	113.9893 9220	70
71	31.9477 4681	44.7635 6163	62.6204 8599	121.9686 4965	71
72	33.5451 3415	47.2255 5751	66.3777 1515	130.5064 5513	72
73	35.2223 9086	49.8229 6318	70.3603 7806	139.6419 0699	73
74	36.9835 1040	52.5632 2615	74.5820 0074	149.4168 4047	74
75	38.8326 8592	55.4542 0359	79.0569 2079	159.8760 1931	75
76	40.7743 2022	58.5041 8479	83.8003 3603	171.0673 4066	76
77	42.8130 3623	61.7219 1495	88.8283 5620	183.0420 5451	77
78	44.9536 8804	65.1166 2027	94.1580 5757	195.8549 9832	78
79	47.2013 7244	68.6980 3439	99.8075 4102	209.5648 4820	79
80	49.5614 4107	72.4764 2628	105.7959 9348	224.2343 8758	80
81	52.0395 1312	76.4626 2973	112.1437 5309	239.9307 9471	81
82	54.6414 8878	80.6680 7436	118.8723 7828	256.7259 5034	82
83	57.3735 6322	85.1048 1845	126.0047 2097	274.6967 6686	83
84	60.2422 4138	89.7855 8347	133.5650 0423	293.9255 4054	84
85	63.2543 5344	94.7237 9056	141.5789 0449	314.5003 2838	85
86	66.4170 7112	99.9335 9904	150.0736 3875	336.5153 5137	86
87	69.7379 2467	105.4299 4698	159.0780 5708	360.0714 2596	87
88	73.2248 2091	111.2285 9407	168.6227 4050	385.2764 2578	88
89	76.8860 6195	117.3461 6674	178.7401 0493	412.2457 7558	89
90	80.7303 6505	123.8002 0591	189.4645 1123	441.1029 7988	90
91	84.7668 8330	130.6092 1724	200.8323 8190	471.9801 8847	91
92	89.0052 2747	137.7927 2419	212.8823 2482	505.0188 0166	92
93	93.4554 8884	145.3713 2402	225.6552 6431	540.3701 1778	93
94	98.1282 6328	153.3667 4684	239.1945 8017	578.1960 2602	94
95	103.0346 7645	161.8019 1791	253.5462 5498	618.6697 4784	95
96	108.1864 1027	170.7010 2340	268.7590 3028	661.9766 3019	96
97	113.5957 3078	180.0895 7969	284.8845 7209	708.3149 9430	97
98	119.2755 1732	189.9945 0657	301.9776 4642	757.8970 4390	98
99	125.2392 9319	200.4442 0443	320.0963 0520	810.9498 3698	99
100	131.5012 5785	211.4686 3567	339.3020 8351	867.7163 2557	100

TABLE 3. $\log (1 + i)^n$

n	5%	5½%	6%	7%	n
51	1.080 6543	1.185 8754	1.290 5991	1.498 5727	51
52	1.101 8436	1.209 1279	1.315 9050	1.527 9564	52
53	1.123 0329	1.232 3804	1.341 2109	1.557 3402	53
54	1.144 2222	1.255 6328	1.366 5167	1.586 7240	54
55	1.165 4115	1.278 8853	1.391 8226	1.616 1078	55
56	1.186 6007	1.302 1377	1.417 1285	1.645 4916	56
57	1.207 7900	1.325 3902	1.442 4343	1.674 8753	57
58	1.228 9793	1.348 6427	1.467 7402	1.704 2591	58
59	1.250 1686	1.371 8951	1.493 0461	1.733 6429	59
60	1.271 3579	1.395 1476	1.518 3519	1.763 0267	60
61	1.292 5472	1.418 4000	1.543 6578	1.792 4104	61
62	1.313 7365	1.441 6525	1.568 9636	1.821 7942	62
63	1.334 9258	1.464 9050	1.594 2695	1.851 1780	63
64	1.356 1151	1.488 1574	1.619 5754	1.880 5618	64
65	1.377 3044	1.511 4099	1.644 8812	1.909 9456	65
66	1.398 4937	1.534 6623	1.670 1871	1.939 3293	66
67	1.419 6830	1.557 9148	1.695 4930	1.968 7131	67
68	1.440 8723	1.581 1673	1.720 7988	1.998 0969	68
69	1.462 0616	1.604 4197	1.746 1047	2.027 4807	69
70	1.483 2509	1.627 6722	1.771 4106	2.056 8644	70
71	1.504 4402	1.650 9246	1.796 7164	2.086 2482	71
72	1.525 6295	1.674 1771	1.822 0223	2.115 6320	72
73	1.546 8188	1.697 4296	1.847 3282	2.145 0158	73
74	1.568 0081	1.720 6820	1.872 6340	2.174 3995	74
75	1.589 1974	1.743 9345	1.897 9399	2.203 7833	75
76	1.610 3867	1.767 1869	1.923 2458	2.233 1671	76
77	1.631 5760	1.790 4394	1.948 5516	2.262 5509	77
78	1.652 7653	1.813 6918	1.973 8575	2.291 9347	78
79	1.673 9546	1.836 9443	1.999 1634	2.321 3184	79
80	1.695 1439	1.860 1968	2.024 4692	2.350 7022	80
81	1.716 3332	1.883 4492	2.049 7751	2.380 0860	81
82	1.737 5225	1.906 7017	2.075 0810	2.409 4698	82
83	1.758 7118	1.929 9541	2.100 3868	2.438 8535	83
84	1.779 9011	1.953 2066	2.125 6927	2.463 2373	84
85	1.801 0904	1.976 4591	2.150 9986	2.497 6211	85
86	1.822 2797	1.999 7115	2.176 3044	2.527 0049	86
87	1.843 4690	2.022 9640	2.201 6103	2.556 3887	87
88	1.864 6583	2.046 2164	2.226 9161	2.585 7724	88
89	1.885 8476	2.069 4689	2.252 2220	2.615 1562	89
90	1.907 0369	2.092 7214	2.277 5279	2.644 5400	90
91	1.928 2262	2.115 9738	2.302 8337	2.673 9238	91
92	1.949 4155	2.139 2263	2.328 1396	2.703 3075	92
93	1.970 6048	2.162 4787	2.353 4455	2.732 6913	93
94	1.991 7941	2.185 7312	2.378 7513	2.762 0751	94
95	2.012 9834	2.208 9837	2.404 0572	2.791 4589	95
96	2.034 1727	2.232 2361	2.429 3631	2.820 8427	96
97	2.055 3620	2.255 4886	2.454 6689	2.850 2264	97
98	2.076 5513	2.278 7410	2.479 9748	2.879 6102	98
99	2.097 7406	2.301 9935	2.505 2807	2.908 9940	99
100	2.118 9299	2.325 2460	2.530 5865	2.938 3778	100



TABLE 4

$$(1 + i)^{-n}$$

TABLE 4. $(1 + i)^{-n}$

n	$\frac{1}{4}\%$	$\frac{3}{8}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
1	0.9975 0623	0.9966 7774	0.9950 2488	0.9933 7748	1
2	0.9950 1869	0.9933 6652	0.9900 7450	0.9867 9882	2
3	0.9925 3734	0.9900 6630	0.9851 4876	0.9802 6373	3
4	0.9900 6219	0.9867 7704	0.9802 4752	0.9737 7192	4
5	0.9875 9321	0.9834 9871	0.9753 7067	0.9673 2310	5
6	0.9851 3038	0.9802 3127	0.9705 1808	0.9609 1699	6
7	0.9826 7370	0.9769 7469	0.9656 8963	0.9545 5330	7
8	0.9802 2314	0.9737 2893	0.9608 8520	0.9482 3175	8
9	0.9777 7869	0.9704 9395	0.9561 0468	0.9419 5207	9
10	0.9753 4034	0.9672 6972	0.9513 4794	0.9357 1398	10
11	0.9729 0807	0.9640 5620	0.9466 1489	0.9295 1720	11
12	0.9704 8187	0.9608 5335	0.9419 0534	0.9233 6145	12
13	0.9680 6171	0.9576 6115	0.9372 1924	0.9172 4648	13
14	0.9656 4759	0.9544 7955	0.9325 5646	0.9111 7200	14
15	0.9632 3949	0.9513 0852	0.9279 1688	0.9051 3775	15
16	0.9608 3740	0.9481 4803	0.9233 0037	0.8991 4346	16
17	0.9584 4130	0.9449 9803	0.9187 0684	0.8931 8886	17
18	0.9560 5117	0.9418 5851	0.9141 3616	0.8872 7371	18
19	0.9536 6700	0.9387 2941	0.9095 8822	0.8813 9772	19
20	0.9512 8878	0.9356 1071	0.9050 6290	0.8755 6065	20
21	0.9489 1649	0.9325 0236	0.9005 6010	0.8697 6224	21
22	0.9465 5011	0.9294 0435	0.8960 7971	0.8640 0222	22
23	0.9441 8964	0.9263 1663	0.8916 2160	0.8582 8035	23
24	0.9418 3505	0.9232 3916	0.8871 8567	0.8525 9638	24
25	0.9394 8634	0.9201 7192	0.8827 7181	0.8469 5004	25
26	0.9371 4348	0.9171 1487	0.8783 7991	0.8413 4110	26
27	0.9348 0646	0.9140 6798	0.8740 0986	0.8357 6931	27
28	0.9324 7527	0.9110 3121	0.8696 6155	0.8302 3441	28
29	0.9301 4990	0.9080 0453	0.8653 3488	0.8247 3617	29
30	0.9278 3032	0.9049 8790	0.8610 2973	0.8192 7434	30
31	0.9255 1653	0.9019 8130	0.8567 4600	0.8138 4868	31
32	0.9232 0851	0.8989 8468	0.8524 8358	0.8084 5896	32
33	0.9209 0624	0.8959 9802	0.8482 4237	0.8031 0492	33
34	0.9186 0972	0.8930 2128	0.8440 2226	0.7977 8635	34
35	0.9163 1892	0.8900 5444	0.8398 2314	0.7925 0299	35
36	0.9140 3384	0.8870 9745	0.8356 4492	0.7872 5463	36
37	0.9117 5445	0.8841 5028	0.8314 8748	0.7820 4102	37
38	0.9094 8075	0.8812 1290	0.8273 5073	0.7768 6194	38
39	0.9072 1272	0.8782 8528	0.8232 3455	0.7717 1716	39
40	0.9049 5034	0.8753 6739	0.8191 3886	0.7666 0645	40
41	0.9026 9361	0.8724 5920	0.8150 6354	0.7615 2959	41
42	0.9004 4250	0.8695 6066	0.8110 0850	0.7564 8635	42
43	0.8981 9701	0.8666 7175	0.8069 7363	0.7514 7650	43
44	0.8959 5712	0.8637 9245	0.8029 5884	0.7464 9984	44
45	0.8937 2281	0.8609 2270	0.7989 6402	0.7415 5613	45
46	0.8914 9407	0.8580 6249	0.7949 8907	0.7366 4516	46
47	0.8892 7090	0.8552 1179	0.7910 3390	0.7317 6672	47
48	0.8870 5326	0.8523 7055	0.7870 9841	0.7269 2058	48
49	0.8848 4116	0.8495 3876	0.7831 8250	0.7221 0654	49
50	0.8826 3457	0.8467 1637	0.7792 8607	0.7173 2437	50

TABLE 4. $(1 + i)^{-n}$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
51	0.8804 3349	0.8439 0336	0.7754 0902	0.7125 7388	51
52	0.8782 3790	0.8410 9969	0.7715 5127	0.7078 5485	52
53	0.8760 4778	0.8383 0534	0.7677 1270	0.7031 6707	53
54	0.8738 6312	0.8355 2027	0.7638 9324	0.6985 1033	54
55	0.8716 8391	0.8327 4446	0.7600 9277	0.6938 8444	55
56	0.8695 1013	0.8299 7787	0.7563 1122	0.6892 8918	56
57	0.8673 4178	0.8272 2047	0.7525 4847	0.6847 2435	57
58	0.8651 7883	0.8244 7222	0.7488 0445	0.6801 8975	58
59	0.8630 2128	0.8217 3311	0.7450 7906	0.6756 8518	59
60	0.8608 6911	0.8190 0310	0.7413 7220	0.6712 1044	60
61	0.8587 2230	0.8162 8216	0.7376 8378	0.6667 6534	61
62	0.8565 8085	0.8135 7026	0.7340 1371	0.6623 4968	62
63	0.8544 4474	0.8108 6737	0.7303 6190	0.6579 6326	63
64	0.8523 1395	0.8081 7346	0.7267 2826	0.6536 0588	64
65	0.8501 8848	0.8054 8850	0.7231 1269	0.6492 7737	65
66	0.8480 6831	0.8028 1246	0.7195 1512	0.6449 7752	66
67	0.8459 5343	0.8001 4531	0.7159 3544	0.6407 0614	67
68	0.8438 4382	0.7974 8702	0.7123 7357	0.6364 6306	68
69	0.8417 3947	0.7948 3756	0.7088 2943	0.6322 4807	69
70	0.8396 4037	0.7921 9690	0.7053 0291	0.6280 6100	70
71	0.8375 4650	0.7895 6502	0.7017 9394	0.6239 0165	71
72	0.8354 5786	0.7869 4188	0.6983 0243	0.6197 6985	72
73	0.8333 7442	0.7843 2745	0.6948 2829	0.6156 6541	73
74	0.8312 9618	0.7817 2171	0.6913 7143	0.6115 8816	74
75	0.8292 2312	0.7791 2463	0.6879 3177	0.6075 3791	75
76	0.8271 5523	0.7765 3618	0.6845 0923	0.6035 1448	76
77	0.8250 9250	0.7739 5632	0.6811 0371	0.5995 1769	77
78	0.8230 3491	0.7713 8504	0.6777 1513	0.5955 4738	78
79	0.8209 8246	0.7688 2230	0.6743 4342	0.5916 0336	79
80	0.8189 3512	0.7662 6807	0.6709 8847	0.5876 8545	80
81	0.8168 9289	0.7637 2233	0.6676 5022	0.5837 9350	81
82	0.8148 5575	0.7611 8505	0.6643 2858	0.5799 2732	82
83	0.8128 2369	0.7586 5619	0.6610 2346	0.5760 8674	83
84	0.8107 9670	0.7561 3574	0.6577 3479	0.5722 7159	84
85	0.8087 7476	0.7536 2366	0.6544 6248	0.5684 8171	85
86	0.8067 5787	0.7511 1993	0.6512 0644	0.5647 1693	86
87	0.8047 4600	0.7486 2451	0.6479 6661	0.5609 7709	87
88	0.8027 3915	0.7461 3739	0.6447 4290	0.5572 6201	88
89	0.8007 3731	0.7436 5853	0.6415 3522	0.5535 7153	89
90	0.7987 4046	0.7411 8790	0.6383 4350	0.5499 0549	90
91	0.7967 4859	0.7387 2548	0.6351 6766	0.5462 6374	91
92	0.7947 6168	0.7362 7125	0.6320 0763	0.5426 4610	92
93	0.7927 7973	0.7338 2516	0.6288 6331	0.5390 5241	93
94	0.7908 0273	0.7313 8720	0.6257 3464	0.5354 8253	94
95	0.7888 3065	0.7289 5735	0.6226 2153	0.5319 3629	95
96	0.7868 6349	0.7265 3556	0.6195 2391	0.5284 1353	96
97	0.7849 0124	0.7241 2182	0.6164 4170	0.5249 1410	97
98	0.7829 4388	0.7217 1610	0.6133 7483	0.5214 3785	98
99	0.7809 9140	0.7193 1837	0.6103 2321	0.5179 8462	99
100	0.7790 4379	0.7169 2861	0.6072 8678	0.5145 5426	100

TABLE 4. $(1 + i)^{-n}$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
101	0.7771 0104	0.7145 4681	0.6042 6545	0.5111 4660	101
102	0.7751 6313	0.7121 7290	0.6012 5915	0.5077 6152	102
104	0.7732 3006	0.7098 0688	0.5982 6781	0.5043 9886	103
104	0.7713 0180	0.7074 4872	0.5952 9136	0.5010 5847	104
105	0.7693 7836	0.7050 9839	0.5923 2971	0.4977 4020	105
106	0.7674 5971	0.7027 5587	0.5893 8279	0.4944 4391	106
107	0.7655 4584	0.7004 2114	0.5864 5054	0.4911 6945	107
108	0.7636 3675	0.6980 9416	0.5835 3288	0.4879 1667	108
109	0.7617 3242	0.6957 7491	0.5806 2973	0.4846 8543	109
110	0.7598 3284	0.6934 6336	0.5777 4102	0.4814 7559	110
111	0.7579 3799	0.6911 5950	0.5748 6669	0.4782 8701	111
112	0.7560 4787	0.6888 6329	0.5720 0666	0.4751 1955	112
113	0.7541 6247	0.6865 7470	0.5691 6085	0.4719 7306	113
114	0.7522 8176	0.6842 9372	0.5663 2921	0.4688 4741	114
115	0.7504 0575	0.6820 2032	0.5635 1165	0.4657 4246	115
116	0.7485 3441	0.6797 5448	0.5607 0811	0.4626 5808	116
117	0.7466 6774	0.6774 9616	0.5579 1852	0.4595 9411	117
118	0.7448 0573	0.6752 4534	0.5551 4280	0.4565 5044	118
119	0.7429 4836	0.6730 0200	0.5523 8090	0.4535 2693	119
120	0.7410 9562	0.6707 6611	0.5496 3273	0.4505 2344	120
121	0.7392 4750	0.6685 3765	0.5468 9824	0.4475 3984	121
122	0.7374 0399	0.6663 1660	0.5441 7736	0.4445 7600	122
123	0.7355 6508	0.6641 0292	0.5414 7001	0.4416 3179	123
124	0.7337 3075	0.6618 9660	0.5387 7612	0.4387 0708	124
125	0.7319 0100	0.6596 9761	0.5360 9565	0.4358 0173	125
126	0.7300 7581	0.6575 0592	0.5334 2850	0.4329 1563	126
127	0.7282 5517	0.6553 2152	0.5307 7463	0.4300 4864	127
128	0.7264 3907	0.6531 4437	0.5281 3396	0.4272 0063	128
129	0.7246 2750	0.6509 7445	0.5255 0643	0.4243 7149	129
130	0.7228 2045	0.6488 1175	0.5228 9197	0.4215 6108	130
131	0.7210 1791	0.6466 5623	0.5202 9052	0.4187 6929	131
132	0.7192 1986	0.6445 0787	0.5177 0201	0.4159 9598	132
133	0.7174 2629	0.6423 6665	0.5151 2637	0.4132 4104	133
134	0.7156 3720	0.6402 3254	0.5125 6356	0.4105 0434	134
135	0.7138 5257	0.6381 0552	0.5100 1349	0.4077 8577	135
136	0.7120 7239	0.6359 8557	0.5074 7611	0.4050 8520	136
137	0.7102 9664	0.6338 7266	0.5049 5135	0.4024 0252	137
138	0.7085 2533	0.6317 6677	0.5024 3916	0.3997 3760	138
139	0.7067 5843	0.6296 6788	0.4999 3946	0.3970 9033	139
140	0.7049 9595	0.6275 7596	0.4974 5220	0.3944 6059	140
141	0.7032 3785	0.6254 9099	0.4949 7731	0.3918 4827	141
142	0.7014 8414	0.6234 1295	0.4925 1474	0.3892 5325	142
143	0.6997 3480	0.6213 4181	0.4900 6442	0.3866 7541	143
144	0.6979 8983	0.6192 7755	0.4876 2628	0.3841 1465	144
145	0.6962 4921	0.6172 2015	0.4852 0028	0.3815 7084	145
146	0.6945 1292	0.6151 6958	0.4827 8635	0.3790 4389	146
147	0.6927 8097	0.6131 2583	0.4803 8443	0.3765 3366	147
148	0.6910 5334	0.6110 8887	0.4779 9446	0.3740 4006	148
149	0.6893 3001	0.6090 5867	0.4756 1637	0.3715 6297	149
150	0.6876 1098	0.6070 3522	0.4732 5012	0.3691 0229	150

TABLE 4. $(1 + i)^{-n}$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
151	0.6858 9624	0.6050 1849	0.4708 9565	0.3666 5791	151
152	0.6841 8578	0.6030 0847	0.4685 5288	0.3642 2971	152
153	0.6824 7958	0.6010 0512	0.4662 2177	0.3618 1759	153
154	0.6807 7764	0.5990 0842	0.4639 0226	0.3594 2145	154
155	0.6790 7994	0.5970 1836	0.4615 9429	0.3570 4117	155
156	0.6773 8647	0.5950 3491	0.4592 9780	0.3546 7666	156
157	0.6756 9723	0.5930 5805	0.4570 1274	0.3523 2781	157
158	0.6740 1220	0.5910 8776	0.4547 3904	0.3499 9451	158
159	0.6723 3137	0.5891 2401	0.4524 7666	0.3476 7667	159
160	0.6706 5473	0.5871 6679	0.4502 2553	0.3453 7417	160
161	0.6689 8228	0.5852 1607	0.4479 8560	0.3430 8693	161
162	0.6673 1399	0.5832 7183	0.4457 5682	0.3408 1483	162
163	0.6656 4987	0.5813 3405	0.4435 3912	0.3385 5778	163
164	0.6639 8989	0.5794 0271	0.4413 3246	0.3363 1567	164
165	0.6623 3406	0.5774 7778	0.4391 3678	0.3340 8841	165
166	0.6606 8235	0.5755 5925	0.4369 5202	0.3318 7591	166
167	0.6590 3476	0.5736 4710	0.4347 7813	0.3296 7805	167
168	0.6573 9129	0.5717 4129	0.4326 1505	0.3274 9476	168
169	0.6557 5191	0.5698 4182	0.4304 6274	0.3253 2592	169
170	0.6541 1661	0.5679 4866	0.4283 2113	0.3231 7144	170
171	0.6524 8540	0.5660 6178	0.4261 9018	0.3210 3123	171
172	0.6508 5826	0.5641 8118	0.4240 6983	0.3189 0520	172
173	0.6492 3517	0.5623 0682	0.4219 6003	0.3167 9324	173
174	0.6476 1613	0.5604 3870	0.4198 6073	0.3146 9527	174
175	0.6460 0112	0.5585 7677	0.4177 7187	0.3126 1120	175
176	0.6443 9015	0.5567 2104	0.4156 9340	0.3105 4093	176
177	0.6427 8319	0.5548 7147	0.4136 2528	0.3084 8436	177
178	0.6411 8024	0.5530 2804	0.4115 6744	0.3064 4142	178
179	0.6395 8129	0.5511 9074	0.4095 1984	0.3044 1201	179
180	0.6379 8632	0.5493 5954	0.4074 8243	0.3023 9603	180
181	0.6363 9533	0.5475 3442	0.4054 5515	0.3003 9341	181
182	0.6348 0831	0.5457 1537	0.4034 3796	0.2984 0405	182
183	0.6332 2525	0.5439 0237	0.4014 3081	0.2964 2786	183
184	0.6316 4613	0.5420 9538	0.3994 3364	0.2944 6477	184
185	0.6300 7096	0.5402 9440	0.3974 4641	0.2925 1467	185
186	0.6284 9971	0.5384 9940	0.3954 6906	0.2905 7748	186
187	0.6269 3238	0.5367 1037	0.3935 0155	0.2886 5313	187
188	0.6253 6895	0.5349 2728	0.3915 4383	0.2867 4152	188
189	0.6238 0943	0.5331 5011	0.3895 9586	0.2848 4257	189
190	0.6222 5380	0.5313 7885	0.3876 5757	0.2829 5619	190
191	0.6207 0204	0.5296 1347	0.3857 2892	0.2810 8231	191
192	0.6191 5416	0.5278 5396	0.3838 0987	0.2792 2084	192
193	0.6176 1013	0.5261 0029	0.3819 0037	0.2773 7170	193
194	0.6160 6996	0.5243 5245	0.3800 0037	0.2755 3480	194
195	0.6145 3362	0.5226 1041	0.3781 0982	0.2737 1006	195
196	0.6130 0112	0.5208 7417	0.3762 2868	0.2718 9741	196
197	0.6114 7244	0.5191 4369	0.3743 5689	0.2700 9677	197
198	0.6099 4757	0.5174 1896	0.3724 9442	0.2683 0805	198
199	0.6084 2650	0.5156 9996	0.3706 4121	0.2665 3117	199
200	0.6069 0923	0.5139 8667	0.3687 9723	0.2647 6607	200

TABLE 4. $(1 + i)^{-n}$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	1 $\frac{1}{4}\%$	n
1	0.9925 5583	0.9913 2590	0.9900 9901	0.9876 5432	1
2	0.9851 6708	0.9827 2704	0.9802 9605	0.9754 6106	2
3	0.9778 3333	0.9742 0276	0.9705 9015	0.9634 1833	3
4	0.9705 5417	0.9657 5243	0.9609 8034	0.9515 2428	4
5	0.9633 2920	0.9573 7539	0.9514 6569	0.9397 7706	5
6	0.9561 5802	0.9490 7102	0.9420 4524	0.9281 7488	6
7	0.9490 4022	0.9408 3868	0.9327 1805	0.9167 1593	7
8	0.9419 7540	0.9326 7775	0.9234 8322	0.9053 9845	8
9	0.9349 6318	0.9245 8761	0.9143 3982	0.8942 2069	9
10	0.9280 0315	0.9165 6765	0.9052 8695	0.8831 8093	10
11	0.9210 9494	0.9086 1724	0.8963 2372	0.8722 7746	11
12	0.9142 3815	0.9007 3581	0.8874 4923	0.8615 0860	12
13	0.9074 3241	0.8929 2273	0.8786 6260	0.8508 7269	13
14	0.9006 7733	0.8851 7743	0.8699 6297	0.8403 6809	14
15	0.8939 7254	0.8774 9931	0.8613 4947	0.8299 9318	15
16	0.8873 1766	0.8698 8779	0.8528 2126	0.8197 4635	16
17	0.8807 1231	0.8623 4230	0.8443 7749	0.8096 2602	17
18	0.8741 5614	0.8548 6225	0.8360 1731	0.7996 3064	18
19	0.8676 4878	0.8474 4709	0.8277 3992	0.7897 5866	19
20	0.8611 8985	0.8400 9624	0.8195 4447	0.7800 0855	20
21	0.8547 7901	0.8328 0917	0.8114 3017	0.7703 7881	21
22	0.8484 1589	0.8255 8530	0.8033 9621	0.7608 6796	22
23	0.8421 0014	0.8184 2409	0.7954 4179	0.7514 7453	23
24	0.8358 3140	0.8113 2499	0.7875 6613	0.7421 9707	24
25	0.8296 0933	0.8042 8748	0.7797 6844	0.7330 3414	25
26	0.8234 3358	0.7973 1101	0.7720 4796	0.7239 8434	26
27	0.8173 0380	0.7903 9505	0.7644 0392	0.7150 4626	27
28	0.8112 1966	0.7835 3908	0.7568 3557	0.7062 1853	28
29	0.8051 8080	0.7767 4258	0.7493 4215	0.6974 9978	29
30	0.7991 8690	0.7700 0504	0.7419 2292	0.6888 8867	30
31	0.7932 3762	0.7633 2594	0.7345 7715	0.6803 8387	31
32	0.7873 3262	0.7567 0477	0.7273 0411	0.6719 8407	32
33	0.7814 7158	0.7501 4104	0.7201 0307	0.6636 8797	33
34	0.7756 5418	0.7436 3424	0.7129 7334	0.6554 9429	34
35	0.7698 8008	0.7371 8388	0.7059 1420	0.6474 0177	35
36	0.7641 4896	0.7307 8947	0.6989 2495	0.6394 0916	36
37	0.7584 6051	0.7244 5053	0.6920 0490	0.6315 1522	37
38	0.7528 1440	0.7181 6657	0.6851 5337	0.6237 1873	38
39	0.7472 1032	0.7119 3712	0.6783 6967	0.6160 1850	39
40	0.7416 4796	0.7057 6171	0.6716 5314	0.6084 1334	40
41	0.7361 2701	0.6996 3986	0.6650 0311	0.6009 0206	41
42	0.7306 4716	0.6935 7111	0.6584 1892	0.5934 8352	42
43	0.7252 0809	0.6875 5500	0.6518 9992	0.5861 5656	43
44	0.7198 0952	0.6815 9108	0.6454 4546	0.5789 2006	44
45	0.7144 5114	0.6756 7889	0.6390 5492	0.5717 7290	45
46	0.7091 3264	0.6698 1798	0.6327 2764	0.5647 1397	46
47	0.7038 5374	0.6640 0792	0.6264 6301	0.5577 4219	47
48	0.6986 1414	0.6582 4824	0.6202 6041	0.5508 5649	48
49	0.6934 1353	0.6525 3853	0.6141 1921	0.5440 5579	49
50	0.6882 5165	0.6468 7835	0.6080 3882	0.5373 3905	50

TABLE 4. $(1 + i)^{-n}$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	1 $\frac{1}{4}\%$	n
51	0.6831 2819	0.6412 6726	0.6020 1864	0.5307 0524	51
52	0.6780 4286	0.6357 0484	0.5960 5806	0.5241 5332	52
53	0.6729 9540	0.6301 9067	0.5901 5649	0.5176 8229	53
54	0.6679 8551	0.6247 2433	0.5843 1336	0.5112 9115	54
55	0.6630 1291	0.6193 0541	0.5785 2808	0.5049 7892	55
56	0.6580 7733	0.6139 3349	0.5728 0008	0.4987 4461	56
57	0.6531 7849	0.6086 0817	0.5671 2879	0.4925 8727	57
58	0.6483 1612	0.6033 2904	0.5615 1365	0.4865 0594	58
59	0.6434 8995	0.5980 9571	0.5559 5411	0.4804 9970	59
60	0.6386 9970	0.5929 0776	0.5504 4962	0.4745 6760	60
61	0.6339 4511	0.5877 6482	0.5449 9962	0.4687 0874	61
62	0.6292 2592	0.5826 6649	0.5396 0358	0.4629 2222	62
63	0.6245 4185	0.5776 1238	0.5342 6097	0.4572 0713	63
64	0.6198 9266	0.5726 0211	0.5289 7126	0.4515 6259	64
65	0.6152 7807	0.5676 3530	0.5237 3392	0.4459 8775	65
66	0.6106 9784	0.5627 1158	0.5185 4844	0.4404 8173	66
67	0.6061 5170	0.5578 3056	0.5134 1429	0.4350 4368	67
68	0.6016 3940	0.5529 9188	0.5083 3099	0.4296 7277	68
69	0.5971 6070	0.5481 9517	0.5032 9801	0.4243 6817	69
70	0.5927 1533	0.5434 4007	0.4983 1486	0.4191 2905	70
71	0.5883 0306	0.5387 2622	0.4933 8105	0.4139 5462	71
72	0.5839 2363	0.5340 5325	0.4884 9609	0.4088 4407	72
73	0.5795 7681	0.5294 2082	0.4836 5949	0.4037 9661	73
74	0.5752 6234	0.5248 2857	0.4788 7078	0.3988 1147	74
75	0.5709 7999	0.5202 7615	0.4741 2949	0.3938 8787	75
76	0.5667 2952	0.5157 6322	0.4694 3514	0.3890 2506	76
77	0.5625 1069	0.5112 8944	0.4647 8726	0.3842 2228	77
78	0.5583 2326	0.5068 5447	0.4601 8541	0.3794 7879	78
79	0.5541 6701	0.5024 5796	0.4556 2912	0.3747 9387	79
80	0.5500 4170	0.4980 9959	0.4511 1794	0.3701 6679	80
81	0.5459 4710	0.4937 7902	0.4466 5142	0.3655 9683	81
82	0.5418 8297	0.4894 9593	0.4422 2913	0.3610 8329	82
83	0.5378 4911	0.4852 4999	0.4378 5063	0.3566 2547	83
84	0.5338 4527	0.4810 4089	0.4335 1547	0.3522 2268	84
85	0.5298 7123	0.4768 6829	0.4292 2324	0.3478 7426	85
86	0.5259 2678	0.4727 3188	0.4249 7350	0.3435 7951	86
87	0.5220 1169	0.4686 3136	0.4207 6585	0.3393 3779	87
88	0.5181 2575	0.4645 6640	0.4165 9985	0.3351 4843	88
89	0.5142 6873	0.4605 3671	0.4124 7510	0.3310 1080	89
90	0.5104 4043	0.4565 4197	0.4083 9119	0.3269 2425	90
91	0.5066 4063	0.4525 8187	0.4043 4771	0.3228 8814	91
92	0.5028 6911	0.4486 5613	0.4003 4427	0.3189 0187	92
93	0.4991 2567	0.4447 6444	0.3963 8046	0.3149 6481	93
94	0.4954 1009	0.4409 0651	0.3924 5590	0.3110 7636	94
95	0.4917 2217	0.4370 8204	0.3885 7020	0.3072 3591	95
96	0.4880 6171	0.4332 9075	0.3847 2297	0.3034 4287	96
97	0.4844 2850	0.4295 3234	0.3809 1383	0.2996 9666	97
98	0.4808 2233	0.4258 0654	0.3771 4241	0.2959 9670	98
99	0.4772 4301	0.4221 1305	0.3734 0832	0.2923 4242	99
100	0.4736 9033	0.4184 5159	0.3697 1121	0.2887 3326	100

TABLE 4. $(1 + i)^{-n}$

n	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	2%	$2\frac{1}{2}\%$	n
1	0.9852 2167	0.9828 0098	0.9803 9216	0.9756 0976	1
2	0.9706 6175	0.9658 9777	0.9611 6878	0.9518 1440	2
3	0.9563 1699	0.9492 8528	0.9423 2233	0.9285 9941	3
4	0.9421 8423	0.9329 5851	0.9238 4543	0.9059 5064	4
5	0.9282 6033	0.9169 1254	0.9057 3081	0.8838 5429	5
6	0.9145 4219	0.9011 4254	0.8879 7138	0.8622 9687	6
7	0.9010 2679	0.8856 4378	0.8705 6018	0.8412 6524	7
8	0.8877 1112	0.8704 1157	0.8534 9037	0.8207 4657	8
9	0.8745 9224	0.8554 4135	0.8367 5527	0.8007 2836	9
10	0.8616 6723	0.8407 2860	0.8203 4830	0.7811 9840	10
11	0.8489 3323	0.8262 6889	0.8042 6304	0.7621 4478	11
12	0.8363 8742	0.8120 5788	0.7884 9318	0.7435 5589	12
13	0.8240 2702	0.7980 9128	0.7730 3253	0.7254 2038	13
14	0.8118 4928	0.7843 6490	0.7578 7502	0.7077 2720	14
15	0.7998 5150	0.7708 7459	0.7430 1473	0.6904 6556	15
16	0.7880 3104	0.7576 1631	0.7284 4581	0.6736 2493	16
17	0.7763 8526	0.7445 8605	0.7141 6256	0.6571 9506	17
18	0.7649 1159	0.7317 7990	0.7001 5937	0.6411 6591	18
19	0.7536 0747	0.7191 9401	0.6864 3076	0.6255 2772	19
20	0.7424 7042	0.7068 2458	0.6729 7133	0.6102 7094	20
21	0.7314 9795	0.6946 6789	0.6597 7582	0.5953 8629	21
22	0.7206 8763	0.6827 2028	0.6468 3904	0.5808 6467	22
23	0.7100 3708	0.6709 7817	0.6341 5592	0.5666 9724	23
24	0.6995 4392	0.6594 3800	0.6217 2149	0.5528 7535	24
25	0.6892 0583	0.6480 9632	0.6095 3087	0.5393 9059	25
26	0.6790 2052	0.6369 4970	0.5975 7928	0.5262 3472	26
27	0.6689 8574	0.6259 9479	0.5858 6204	0.5133 9973	27
28	0.6590 9925	0.6152 2829	0.5743 7455	0.5008 7778	28
29	0.6493 5887	0.6046 4697	0.5631 1231	0.4886 6125	29
30	0.6397 6243	0.5942 4764	0.5520 7089	0.4767 4269	30
31	0.6303 0781	0.5840 2716	0.5412 4597	0.4651 1481	31
32	0.6209 9292	0.5739 8247	0.5306 3330	0.4537 7055	32
33	0.6118 1568	0.5641 1053	0.5202 2873	0.4427 0298	33
34	0.6027 7407	0.5544 0839	0.5100 2817	0.4319 0534	34
35	0.5938 6608	0.5448 7311	0.5000 2761	0.4213 7107	35
36	0.5850 8974	0.5355 0183	0.4902 2315	0.4110 9372	36
37	0.5764 4309	0.5262 9172	0.4806 1093	0.4010 6705	37
38	0.5679 2423	0.5172 4002	0.4711 8719	0.3912 8492	38
39	0.5595 3126	0.5083 4400	0.4619 4822	0.3817 4139	39
40	0.5512 6232	0.4996 0098	0.4528 9042	0.3724 3062	40
41	0.5431 1559	0.4910 0834	0.4440 1021	0.3633 4695	41
42	0.5350 8925	0.4825 6348	0.4353 0413	0.3544 8483	42
43	0.5271 8153	0.4742 6386	0.4267 6875	0.3458 3886	43
44	0.5193 9067	0.4661 0699	0.4184 0074	0.3374 0376	44
45	0.5117 1494	0.4580 9040	0.4101 9680	0.3291 7440	45
46	0.5041 5265	0.4502 1170	0.4021 5373	0.3211 4576	46
47	0.4967 0212	0.4424 6850	0.3942 6836	0.3133 1294	47
48	0.4893 6170	0.4348 5848	0.3865 3761	0.3056 7116	48
49	0.4821 2975	0.4273 7934	0.3789 5844	0.2982 1576	49
50	0.4750 0468	0.4200 2883	0.3715 2788	0.2909 4221	50

TABLE 4. $(1 + i)^{-n}$

n	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	2%	$2\frac{1}{2}\%$	n
51	0.4679 8491	0.4128 0475	0.3642 4302	0.2838 4606	51
52	0.4610 6887	0.4057 0492	0.3571 0100	0.2769 2298	52
53	0.4542 5505	0.3987 2719	0.3500 9902	0.2701 6876	53
54	0.4475 4192	0.3918 6947	0.3432 3433	0.2635 7928	54
55	0.4409 2800	0.3851 2970	0.3365 0425	0.2571 5052	55
56	0.4344 1182	0.3785 0585	0.3299 0613	0.2508 7855	56
57	0.4279 9194	0.3719 9592	0.3234 3738	0.2447 5956	57
58	0.4216 6694	0.3655 9796	0.3170 9547	0.2387 8982	58
59	0.4154 3541	0.3593 1003	0.3108 7791	0.2329 6568	59
60	0.4092 9597	0.3531 3025	0.3047 8227	0.2272 8359	60
61	0.4032 4726	0.3470 5676	0.2988 0614	0.2217 4009	61
62	0.3972 8794	0.3410 8772	0.2929 4720	0.2163 3179	62
63	0.3914 1669	0.3352 2135	0.2872 0314	0.2110 5541	63
64	0.3856 3221	0.3294 5587	0.2815 7170	0.2059 0771	64
65	0.3799 3321	0.3237 8956	0.2760 5069	0.2008 8557	65
66	0.3743 1843	0.3182 2069	0.2706 3793	0.1959 8593	66
67	0.3687 8663	0.3127 4761	0.2653 3130	0.1912 0578	67
68	0.3633 3658	0.3073 6866	0.2601 2873	0.1865 4223	68
69	0.3579 6708	0.3020 8222	0.2550 2817	0.1819 9241	69
70	0.3526 7692	0.2968 8670	0.2500 2761	0.1775 5358	70
71	0.3474 6495	0.2917 8054	0.2451 2511	0.1732 2300	71
72	0.3423 3000	0.2867 6221	0.2403 1874	0.1689 9805	72
73	0.3372 7093	0.2818 3018	0.2356 0661	0.1648 7615	73
74	0.3322 8663	0.2769 8298	0.2309 8687	0.1608 5478	74
75	0.3273 7599	0.2722 1914	0.2264 5771	0.1569 3149	75
76	0.3225 3793	0.2675 3724	0.2220 1737	0.1531 0389	76
77	0.3177 7136	0.2629 3586	0.2176 6408	0.1493 6965	77
78	0.3130 7523	0.2584 1362	0.2133 9616	0.1457 2649	78
79	0.3084 4850	0.2539 6916	0.2092 1192	0.1421 7218	79
80	0.3038 9015	0.2496 0114	0.2051 0973	0.1387 0457	80
81	0.2993 9916	0.2453 0825	0.2010 8797	0.1353 2153	81
82	0.2949 7454	0.2410 8919	0.1971 4507	0.1320 2101	82
83	0.2906 1531	0.2369 4269	0.1932 7948	0.1288 0098	83
84	0.2863 2050	0.2328 6751	0.1894 8968	0.1256 5949	84
85	0.2820 8917	0.2288 6242	0.1857 7420	0.1225 9463	85
86	0.2779 2036	0.2249 2621	0.1821 3157	0.1196 0452	86
87	0.2738 1316	0.2210 5770	0.1785 6036	0.1166 8733	87
88	0.2697 6666	0.2172 5572	0.1750 5918	0.1138 4130	88
89	0.2657 7997	0.2135 1914	0.1716 2665	0.1110 6468	89
90	0.2618 5218	0.2098 4682	0.1682 6142	0.1083 5579	90
91	0.2579 8245	0.2062 3766	0.1649 6217	0.1057 1296	91
92	0.2541 6990	0.2026 9057	0.1617 2762	0.1031 3460	92
93	0.2504 1369	0.1992 0450	0.1585 5649	0.1006 1912	93
94	0.2467 1300	0.1957 7837	0.1554 4754	0.0981 6500	94
95	0.2430 6699	0.1924 1118	0.1523 9955	0.0957 7073	95
96	0.2394 7487	0.1891 0190	0.1494 1132	0.0934 3486	96
97	0.2359 3583	0.1858 4953	0.1464 8169	0.0911 5596	97
98	0.2324 4909	0.1826 5310	0.1436 0950	0.0889 3264	98
99	0.2290 1389	0.1795 1165	0.1407 9363	0.0867 6355	99
100	0.2256 2944	0.1764 2422	0.1380 3297	0.0846 4737	100

TABLE 4. $(1 + i)^{-n}$

n	3%	3½%	4%	4½%	n
1	0.9708 7379	0.9661 8357	0.9615 3846	0.9569 3780	1
2	0.9425 9591	0.9335 1070	0.9245 5621	0.9157 2995	2
3	0.9151 4166	0.9019 4271	0.8889 9636	0.8762 9660	3
4	0.8884 8705	0.8714 4223	0.8548 0419	0.8385 6134	4
5	0.8626 0878	0.8419 7317	0.8219 2711	0.8024 5105	5
6	0.8374 8426	0.8135 0064	0.7903 1453	0.7678 9574	6
7	0.8130 9151	0.7859 9096	0.7599 1781	0.7348 2846	7
8	0.7894 0923	0.7594 1156	0.7306 9021	0.7031 8513	8
9	0.7664 1673	0.7337 3097	0.7025 8674	0.6729 0443	9
10	0.7440 9391	0.7089 1881	0.6755 6417	0.6439 2768	10
11	0.7224 2128	0.6849 4571	0.6495 8093	0.6161 9874	11
12	0.7013 7988	0.6617 8330	0.6245 9705	0.5896 6386	12
13	0.6809 5134	0.6394 0415	0.6005 7409	0.5642 7164	13
14	0.6611 1781	0.6177 8179	0.5774 7508	0.5399 7286	14
15	0.6418 6195	0.5968 9062	0.5552 6450	0.5167 2044	15
16	0.6231 6694	0.5767 0591	0.5339 0818	0.4944 6932	16
17	0.6050 1645	0.5572 0378	0.5133 7325	0.4731 7639	17
18	0.5873 9461	0.5383 6114	0.4936 2812	0.4528 0037	18
19	0.5702 8603	0.5201 5569	0.4746 4242	0.4333 0179	19
20	0.5536 7575	0.5025 6588	0.4563 8695	0.4146 4286	20
21	0.5375 4928	0.4855 7090	0.4388 3360	0.3967 8743	21
22	0.5218 9250	0.4691 5063	0.4219 5539	0.3797 0089	22
23	0.5066 9175	0.4532 8563	0.4057 2633	0.3633 5013	23
24	0.4919 3374	0.4379 5713	0.3901 2147	0.3477 0347	24
25	0.4776 0557	0.4231 4699	0.3751 1680	0.3327 3060	25
26	0.4636 9473	0.4088 3767	0.3606 8923	0.3184 0248	26
27	0.4501 8906	0.3950 1224	0.3468 1657	0.3046 9137	27
28	0.4370 7675	0.3816 5434	0.3334 7747	0.2915 7069	28
29	0.4243 4636	0.3687 4815	0.3206 5141	0.2790 1502	29
30	0.4119 8676	0.3562 7841	0.3083 1867	0.2670 0002	30
31	0.3999 8715	0.3442 3035	0.2964 6026	0.2555 0241	31
32	0.3883 3703	0.3325 8971	0.2850 5794	0.2444 9991	32
33	0.3770 2625	0.3213 4271	0.2740 9417	0.2339 7121	33
34	0.3660 4490	0.3104 7605	0.2635 5209	0.2238 9589	34
35	0.3553 8340	0.2999 7686	0.2534 1547	0.2142 5444	35
36	0.3450 3243	0.2898 3272	0.2436 6872	0.2050 2817	36
37	0.3349 8294	0.2800 3161	0.2342 9685	0.1961 9921	37
38	0.3252 2615	0.2705 6194	0.2252 8543	0.1877 5044	38
39	0.3157 5355	0.2614 1250	0.2166 2061	0.1796 6549	39
40	0.3065 5684	0.2525 7247	0.2082 8904	0.1719 2870	40
41	0.2976 2800	0.2440 3137	0.2002 7793	0.1645 2507	41
42	0.2889 5922	0.2357 7910	0.1925 7493	0.1574 4026	42
43	0.2805 4294	0.2278 0590	0.1851 6820	0.1506 6054	43
44	0.2723 7178	0.2201 0231	0.1780 4635	0.1441 7276	44
45	0.2644 3862	0.2126 5924	0.1711 9841	0.1379 6437	45
46	0.2567 3653	0.2054 6787	0.1646 1386	0.1320 2332	46
47	0.2492 5876	0.1985 1968	0.1582 8256	0.1263 3810	47
48	0.2419 9880	0.1918 0645	0.1521 9476	0.1208 9771	48
49	0.2349 5029	0.1853 2024	0.1463 4112	0.1156 9158	49
50	0.2281 0708	0.1790 5337	0.1407 1262	0.1107 0965	50

TABLE 4. $(1 + i)^{-n}$

n	3%	3½%	4%	4½%	n
51	0.2214 6318	0.1729 9843	0.1353 0059	0.1059 4225	51
52	0.2150 1280	0.1671 4824	0.1300 9672	0.1013 8014	52
53	0.2087 5029	0.1614 9589	0.1250 9300	0.0970 1449	53
54	0.2026 7019	0.1560 3467	0.1202 8173	0.0928 3683	54
55	0.1967 6717	0.1507 5814	0.1156 5551	0.0888 3907	55
56	0.1910 3609	0.1456 6004	0.1112 0722	0.0850 1347	56
57	0.1854 7193	0.1407 3433	0.1069 3002	0.0813 5260	57
58	0.1800 6984	0.1359 7520	0.1028 1733	0.0778 4938	58
59	0.1748 2508	0.1313 7701	0.0988 6282	0.0744 9701	59
60	0.1697 3309	0.1269 3431	0.0950 6040	0.0712 8901	60
61	0.1647 8941	0.1226 4184	0.0914 0423	0.0682 1915	61
62	0.1599 8972	0.1184 9453	0.0878 8868	0.0652 8148	62
63	0.1553 2982	0.1144 8747	0.0845 0835	0.0624 7032	63
64	0.1508 0565	0.1106 1591	0.0812 5803	0.0597 8021	64
65	0.1464 1325	0.1068 7528	0.0781 3272	0.0572 0594	65
66	0.1421 4879	0.1032 6114	0.0751 2762	0.0547 4253	66
67	0.1380 0853	0.0997 6922	0.0722 3809	0.0523 8519	67
68	0.1339 8887	0.0963 9538	0.0694 5970	0.0501 2937	68
69	0.1300 8628	0.0931 3563	0.0667 8818	0.0479 7069	69
70	0.1262 9736	0.0899 8612	0.0642 1940	0.0459 0497	70
71	0.1226 1880	0.0869 4311	0.0617 4942	0.0439 2820	71
72	0.1190 4737	0.0840 0300	0.0593 7445	0.0420 3655	72
73	0.1155 7998	0.0811 6232	0.0570 9081	0.0402 2637	73
74	0.1122 1357	0.0784 1770	0.0548 9501	0.0384 9413	74
75	0.1089 4521	0.0757 6590	0.0527 8367	0.0368 3649	75
76	0.1057 7205	0.0732 0376	0.0507 5353	0.0352 5023	76
77	0.1026 9131	0.0707 2827	0.0488 0147	0.0337 3228	77
78	0.0997 0030	0.0683 3650	0.0469 2449	0.0322 7969	78
79	0.0967 9641	0.0660 2560	0.0451 1970	0.0308 8965	79
80	0.0939 7710	0.0637 9285	0.0433 8433	0.0295 5948	80
81	0.0912 3990	0.0616 3561	0.0417 1570	0.0282 8658	81
82	0.0885 8243	0.0595 5131	0.0401 1125	0.0270 6850	82
83	0.0860 0236	0.0575 3750	0.0385 6851	0.0259 0287	83
84	0.0834 9743	0.0555 9178	0.0370 8510	0.0247 8744	84
85	0.0810 6547	0.0537 1187	0.0356 5875	0.0237 2003	85
86	0.0787 0434	0.0518 9553	0.0342 8726	0.0226 9860	86
87	0.0764 1198	0.0501 4060	0.0329 6852	0.0217 2115	87
88	0.0741 8639	0.0484 4503	0.0317 0050	0.0207 8579	88
89	0.0720 2562	0.0468 0679	0.0304 8125	0.0198 9070	89
90	0.0699 2779	0.0452 2395	0.0293 0890	0.0190 3417	90
91	0.0678 9105	0.0436 9464	0.0281 8163	0.0182 1451	91
92	0.0659 1364	0.0422 1704	0.0270 9772	0.0174 3016	92
93	0.0639 9383	0.0407 8941	0.0260 5550	0.0166 7958	93
94	0.0621 2993	0.0394 1006	0.0250 5337	0.0159 6132	94
95	0.0603 2032	0.0380 7735	0.0240 8978	0.0152 7399	95
96	0.0585 6342	0.0367 8971	0.0231 6325	0.0146 1626	96
97	0.0568 5769	0.0355 4562	0.0222 7235	0.0139 8685	97
98	0.0552 0164	0.0343 4359	0.0214 1572	0.0133 8454	98
99	0.0535 9383	0.0331 8221	0.0205 9204	0.0128 0817	99
100	0.0520 3284	0.0320 6011	0.0198 0004	0.0122 5663	100

TABLE 4. $(1 + i)^{-n}$

n	5%	5½%	6%	7%	n
1	0.9523 8095	0.9478 6730	0.9433 9623	0.9345 7944	1
2	0.9070 2948	0.8984 5242	0.8899 9644	0.8734 3873	2
3	0.8638 3760	0.8516 1366	0.8396 1928	0.8162 9788	3
4	0.8227 0247	0.8072 1674	0.7920 9366	0.7628 9521	4
5	0.7835 2617	0.7651 3435	0.7472 5817	0.7129 8618	5
6	0.7462 1540	0.7252 4583	0.7049 6054	0.6663 4222	6
7	0.7106 8133	0.6874 3681	0.6650 5711	0.6227 4974	7
8	0.6768 3936	0.6515 9887	0.6274 1237	0.5820 0910	8
9	0.6446 0892	0.6176 2926	0.5918 9846	0.5439 3374	9
10	0.6139 1325	0.5854 3058	0.5583 9478	0.5083 4929	10
11	0.5846 7929	0.5549 1050	0.5267 8753	0.4750 9280	11
12	0.5568 3742	0.5259 8152	0.4969 6936	0.4440 1196	12
13	0.5303 2135	0.4985 6068	0.4688 3902	0.4149 6445	13
14	0.5050 6795	0.4725 6937	0.4423 0096	0.3878 1724	14
15	0.4810 1710	0.4479 3305	0.4172 6506	0.3624 4602	15
16	0.4581 1152	0.4245 8109	0.3936 4628	0.3387 3460	16
17	0.4362 9669	0.4024 4653	0.3713 6442	0.3165 7439	17
18	0.4155 2065	0.3814 6590	0.3503 4379	0.2958 6392	18
19	0.3957 3396	0.3615 7906	0.3305 1301	0.2765 0832	19
20	0.3768 8948	0.3427 2896	0.3118 0473	0.2584 1900	20
21	0.3589 4236	0.3248 6158	0.2941 5540	0.2415 1309	21
22	0.3418 4987	0.3079 2567	0.2775 0510	0.2257 1317	22
23	0.3255 7131	0.2918 7267	0.2617 9726	0.2109 4688	23
24	0.3100 6791	0.2766 5656	0.2469 7855	0.1971 4662	24
25	0.2953 0277	0.2622 3370	0.2329 9863	0.1842 4918	25
26	0.2812 4073	0.2485 6275	0.2198 1003	0.1721 9549	26
27	0.2678 4832	0.2356 0450	0.2073 6795	0.1609 3037	27
28	0.2550 9364	0.2233 2181	0.1956 3014	0.1504 0221	28
29	0.2429 4632	0.2116 7944	0.1845 5674	0.1405 6282	29
30	0.2313 7745	0.2006 4402	0.1741 1013	0.1313 6712	30
31	0.2203 5947	0.1901 8390	0.1642 5484	0.1227 7301	31
32	0.2098 6617	0.1802 6910	0.1549 5740	0.1147 4113	32
33	0.1998 7254	0.1708 7119	0.1461 8622	0.1072 3470	33
34	0.1903 5480	0.1619 6321	0.1379 1153	0.1002 1934	34
35	0.1812 9029	0.1535 1963	0.1301 0522	0.0936 6294	35
36	0.1726 5741	0.1455 1624	0.1227 4077	0.0875 3546	36
37	0.1644 3563	0.1379 3008	0.1157 9318	0.0818 0884	37
38	0.1566 0536	0.1307 3941	0.1092 3885	0.0764 5686	38
39	0.1491 4797	0.1239 2362	0.1030 5552	0.0714 5501	39
40	0.1420 4568	0.1174 6314	0.0972 2219	0.0667 8038	40
41	0.1352 8160	0.1113 3947	0.0917 1905	0.0624 1157	41
42	0.1288 3962	0.1055 3504	0.0865 2740	0.0583 2857	42
43	0.1227 0440	0.1000 3322	0.0816 2962	0.0545 1268	43
44	0.1168 6133	0.0948 1822	0.0770 0908	0.0509 4643	44
45	0.1112 9651	0.0898 7509	0.0726 5007	0.0476 1349	45
46	0.1059 9668	0.0851 8965	0.0685 3781	0.0444 9859	46
47	0.1009 4921	0.0807 4849	0.0646 5831	0.0415 8747	47
48	0.0961 4211	0.0765 3885	0.0609 9840	0.0388 6679	48
49	0.0915 6391	0.0725 4867	0.0575 4566	0.0363 2410	49
50	0.0872 0373	0.0687 6652	0.0542 8836	0.0339 4776	50

TABLE 4. $(1 + i)^{-n}$

n	5%	5½%	6%	7%	n
51	0.0830 5117	0.0651 8153	0.0512 1544	0.0317 2688	51
52	0.0790 9635	0.0617 8344	0.0483 1645	0.0296 5129	52
53	0.0753 2986	0.0585 6250	0.0455 8156	0.0277 1148	53
54	0.0717 4272	0.0555 0948	0.0430 0147	0.0258 9858	54
55	0.0683 2640	0.0526 1562	0.0405 6742	0.0242 0428	55
56	0.0650 7276	0.0498 7263	0.0382 7115	0.0226 2083	56
57	0.0619 7406	0.0472 7263	0.0361 0486	0.0211 4096	57
58	0.0590 2291	0.0448 0818	0.0340 6119	0.0197 5791	58
59	0.0562 1230	0.0424 7221	0.0321 3320	0.0184 6533	59
60	0.0535 3552	0.0402 5802	0.0303 1434	0.0172 5732	60
61	0.0509 8621	0.0381 5926	0.0285 9843	0.0161 2834	61
62	0.0485 5830	0.0361 6992	0.0269 7965	0.0150 7321	62
63	0.0462 4600	0.0342 8428	0.0254 5250	0.0140 8711	63
64	0.0440 4381	0.0324 9695	0.0240 1179	0.0131 6553	64
65	0.0419 4648	0.0308 0279	0.0226 5264	0.0123 0423	65
66	0.0399 4903	0.0291 9696	0.0213 7041	0.0114 9928	66
67	0.0380 4670	0.0276 7485	0.0201 6077	0.0107 4699	67
68	0.0362 3495	0.0262 3208	0.0190 1959	0.0100 4392	68
69	0.0345 0948	0.0248 6453	0.0179 4301	0.0093 8684	69
70	0.0328 6617	0.0235 6828	0.0169 2737	0.0087 7275	70
71	0.0313 0111	0.0223 3960	0.0159 6921	0.0081 9883	71
72	0.0298 1058	0.0211 7498	0.0150 6530	0.0076 6246	72
73	0.0283 9103	0.0200 7107	0.0142 1254	0.0071 6117	73
74	0.0270 3908	0.0190 2471	0.0134 0806	0.0066 9269	74
75	0.0257 5150	0.0180 3290	0.0126 4911	0.0062 5485	75
76	0.0245 2524	0.0170 9279	0.0119 3313	0.0058 4565	76
77	0.0233 5737	0.0162 0170	0.0112 5767	0.0054 6323	77
78	0.0222 4512	0.0153 5706	0.0106 2044	0.0051 0582	78
79	0.0211 8582	0.0145 5646	0.0100 1928	0.0047 7179	79
80	0.0201 7698	0.0137 9759	0.0094 5215	0.0044 5962	80
81	0.0192 1617	0.0130 7828	0.0089 1713	0.0041 6787	81
82	0.0183 0111	0.0123 9648	0.0084 1238	0.0038 9520	82
83	0.0174 2963	0.0117 5022	0.0079 3621	0.0036 4038	83
84	0.0165 9965	0.0111 3765	0.0074 8699	0.0034 0222	84
85	0.0158 0919	0.0105 5701	0.0070 6320	0.0031 7965	85
86	0.0150 5637	0.0100 0664	0.0066 6340	0.0029 7163	86
87	0.0143 3940	0.0094 8497	0.0062 8622	0.0027 7723	87
88	0.0136 5657	0.0089 9049	0.0059 3040	0.0025 9554	88
89	0.0130 0626	0.0085 2180	0.0055 9472	0.0024 2574	89
90	0.0123 8691	0.0080 7753	0.0052 7803	0.0022 6704	90
91	0.0117 9706	0.0076 5643	0.0049 7928	0.0021 1873	91
92	0.0112 3530	0.0072 5728	0.0046 9743	0.0019 8012	92
93	0.0107 0028	0.0068 7894	0.0044 3154	0.0018 5058	93
94	0.0101 9074	0.0065 2032	0.0041 8070	0.0017 2952	94
95	0.0097 0547	0.0061 8040	0.0039 4405	0.0016 1637	95
96	0.0092 4331	0.0058 5820	0.0037 2081	0.0015 1063	96
97	0.0088 0315	0.0055 5279	0.0035 1019	0.0014 1180	97
98	0.0083 8395	0.0052 6331	0.0033 1150	0.0013 1944	98
99	0.0079 8471	0.0049 8892	0.0031 2406	0.0012 3312	99
100	0.0076 0449	0.0047 2883	0.0029 4723	0.0011 5245	100

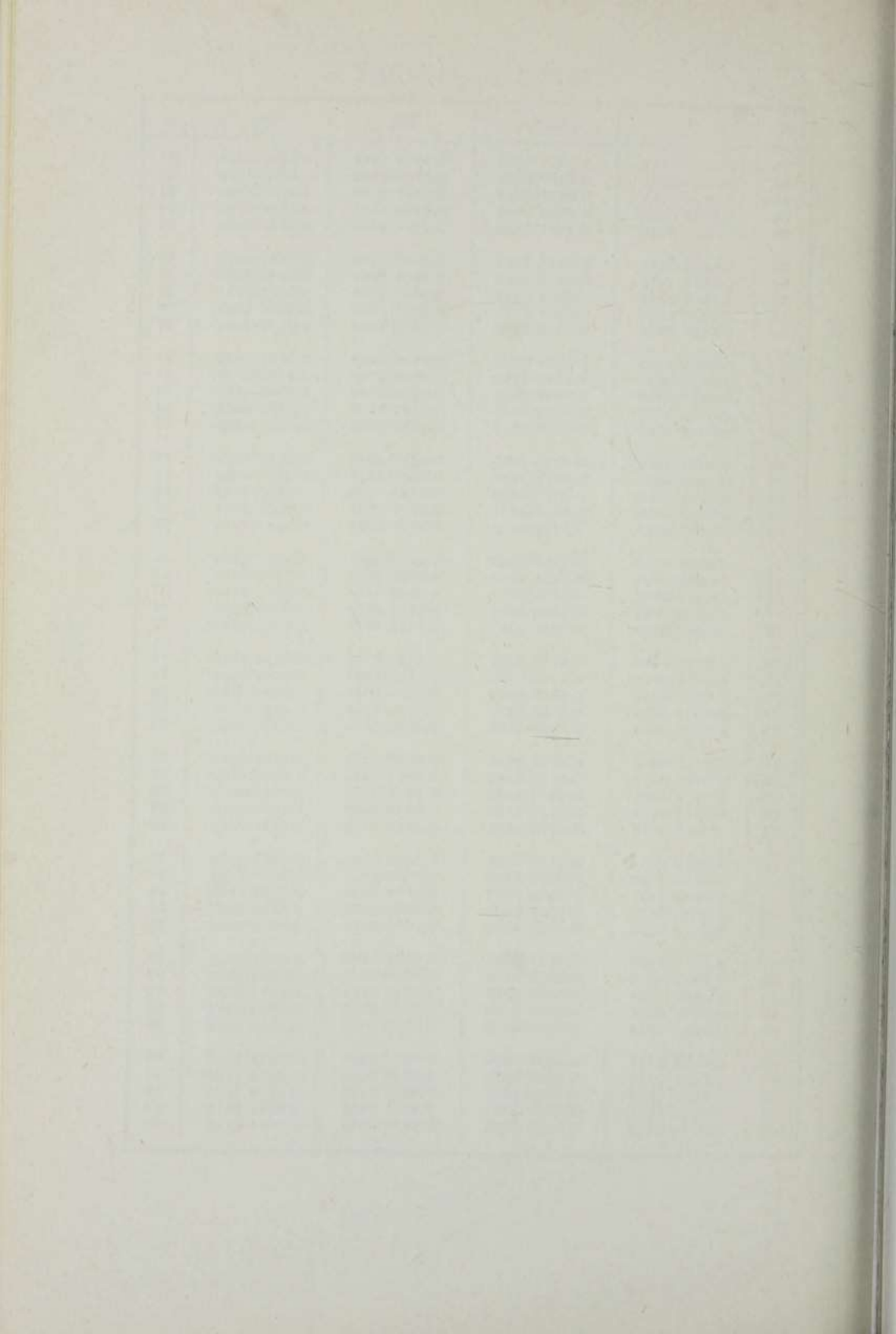


TABLE 5

$(1 + i)^{1/p}$ and $\log (1 + i)^{1/p}$

TABLE 5. $(1 + i)^{1/p}$

p	1/4%	7/24%	1/3%	5/12%	p
2	1.0012 4922	1.0014 5727	1.0016 6528	1.0020 8117	2
3	1.0008 3264	1.0009 7128	1.0011 0988	1.0013 8696	3
4	1.0006 2441	1.0007 2837	1.0008 3229	1.0010 4004	4
6	1.0004 1623	1.0004 8552	1.0005 5479	1.0006 9324	6
12	1.0002 0890	1.0002 4273	1.0002 7735	1.0003 4656	12
p	1/2%	7/12%	5/8%	2/3%	p
2	1.0024 9688	1.0029 1243	1.0031 2013	1.0033 2780	2
3	1.0016 6390	1.0019 4068	1.0020 7901	1.0022 1730	3
4	1.0012 4766	1.0014 5515	1.0015 5885	1.0016 6252	4
6	1.0008 3160	1.0009 6987	1.0010 3896	1.0011 0804	6
12	1.0004 1571	1.0004 8482	1.0005 1935	1.0005 5387	12
p	3/4%	7/8%	1%	1 1/8%	p
2	1.0037 4299	1.0043 6547	1.0049 8756	1.0056 0927	2
3	1.0024 9378	1.0029 0820	1.0033 2228	1.0037 3602	3
4	1.0018 6975	1.0021 8036	1.0024 9068	1.0028 0081	4
6	1.0012 4611	1.0014 5304	1.0016 5977	1.0018 6627	6
12	1.0006 2286	1.0007 2626	1.0008 2954	1.0009 3270	12
p	1 1/4%	1 3/8%	1 1/2%	1 3/4%	p
2	1.0062 3059	1.0068 5153	1.0074 7208	1.0087 1205	2
3	1.0041 4943	1.0045 6249	1.0049 7521	1.0057 9963	3
4	1.0031 1046	1.0034 1992	1.0037 2909	1.0043 4658	4
6	1.0020 7257	1.0022 7865	1.0024 8452	1.0028 9562	6
12	1.0010 3575	1.0011 3868	1.0012 4149	1.0014 4677	12
p	2%	2 1/4%	2 1/2%	2 3/4%	p
2	1.0099 5049	1.0111 8742	1.0124 2284	1.0136 5675	2
3	1.0066 2271	1.0074 4444	1.0082 6484	1.0090 8390	3
4	1.0049 6293	1.0055 7815	1.0061 9225	1.0068 0522	4
6	1.0033 0589	1.0037 1532	1.0041 2392	1.0045 3168	6
12	1.0016 5158	1.0018 5594	1.0020 5984	1.0022 6328	12
p	3%	3 1/2%	4%	4 1/2%	p
2	1.0148 8916	1.0173 4950	1.0198 0390	1.0222 5242	2
3	1.0099 0163	1.0115 3314	1.0131 5941	1.0147 8046	3
4	1.0074 1707	1.0086 3745	1.0098 5341	1.0110 6499	4
6	1.0049 3862	1.0057 5004	1.0065 5820	1.0073 6312	6
12	1.0024 6627	1.0028 7090	1.0032 7374	1.0036 7481	12
p	5%	5 1/2%	6%	6 1/2%	p
2	1.0246 9508	1.0271 3193	1.0295 6302	1.0319 8837	2
3	1.0163 9636	1.0180 0713	1.0196 1282	1.0212 1347	3
4	1.0122 7224	1.0134 7518	1.0146 7385	1.0158 6828	4
6	1.0081 6485	1.0089 6340	1.0097 5880	1.0105 5107	6
12	1.0040 7412	1.0044 7170	1.0048 6755	1.0052 6169	12
p	7%	7 1/2%	8%	8 1/2%	p
2	1.0344 0804	1.0368 2207	1.0392 3048	1.0416 3333	2
3	1.0228 0912	1.0243 9981	1.0259 8557	1.0275 6644	3
4	1.0170 5853	1.0182 4460	1.0194 2655	1.0206 0440	4
6	1.0113 4026	1.0121 2638	1.0129 0946	1.0136 8952	6
12	1.0056 5415	1.0060 4492	1.0064 3403	1.0068 2149	12

TABLE 5. $\log (1+i)^{1/p}$

p	1/4%	7/24%	1/3%	5/12%	p
2	0.000 5422	0.000 6324	0.000 7226	0.000 9029	2
3	0.000 3615	0.000 4216	0.000 4817	0.000 6019	3
4	0.000 2711	0.000 3162	0.000 3613	0.000 4515	4
6	0.000 1807	0.000 2108	0.000 2409	0.000 3010	6
12	0.000 0904	0.000 1054	0.000 1204	0.000 1505	12
p	1/2%	7/12%	5/8%	2/3%	p
2	0.001 0830	0.001 2630	0.001 3529	0.001 4428	2
3	0.000 7220	0.000 8420	0.000 9020	0.000 9619	3
4	0.000 5415	0.000 6315	0.000 6765	0.000 7214	4
6	0.000 3610	0.000 4210	0.000 4510	0.000 4809	6
12	0.000 1805	0.000 2105	0.000 2255	0.000 2405	12
p	3/4%	7/8%	1%	1 1/8%	p
2	0.001 6225	0.001 8918	0.002 1607	0.002 4293	2
3	0.001 0817	0.001 2612	0.001 4405	0.001 6195	3
4	0.000 8113	0.000 9459	0.001 0803	0.001 2146	4
6	0.000 5408	0.000 6306	0.000 7202	0.000 8098	6
12	0.000 2704	0.000 3153	0.000 3601	0.000 4049	12
p	1 1/4%	1 3/8%	1 1/2%	1 3/4%	p
2	0.002 6975	0.002 9654	0.003 2330	0.003 7672	2
3	0.001 7983	0.001 9770	0.002 1553	0.002 5115	3
4	0.001 3488	0.001 4827	0.001 6165	0.001 8836	4
6	0.000 8992	0.000 9885	0.001 0777	0.001 2557	6
12	0.000 4496	0.000 4942	0.000 5388	0.000 6279	12
p	2%	2 1/4%	2 1/2%	2 3/4%	p
2	0.004 3001	0.004 8317	0.005 3619	0.005 8909	2
3	0.002 8667	0.003 2211	0.003 5746	0.003 9273	3
4	0.002 1500	0.002 4158	0.002 6810	0.002 9455	4
6	0.001 4334	0.001 6106	0.001 7873	0.001 9636	6
12	0.000 7167	0.000 8053	0.000 8937	0.000 9818	12
p	3%	3 1/2%	4%	4 1/2%	p
2	0.006 4186	0.007 4702	0.008 5167	0.009 5581	2
3	0.004 2791	0.004 9801	0.005 6778	0.006 3721	3
4	0.003 2093	0.003 7351	0.004 2583	0.004 7791	4
6	0.002 1395	0.002 4901	0.002 8389	0.003 1860	6
12	0.001 0698	0.001 2450	0.001 4194	0.001 5930	12
p	5%	5 1/2%	6%	6 1/2%	p
2	0.010 5946	0.011 6262	0.012 6529	0.013 6748	2
3	0.007 0631	0.007 7508	0.008 4353	0.009 1165	3
4	0.005 2973	0.005 8131	0.006 3265	0.006 8374	4
6	0.003 5315	0.003 8754	0.004 2176	0.004 5583	6
12	0.001 7658	0.001 9377	0.002 1088	0.002 2792	12
p	7%	7 1/2%	8%	8 1/2%	p
2	0.014 6919	0.015 7042	0.016 7118	0.017 7149	2
3	0.009 7946	0.010 4695	0.011 1413	0.011 8099	3
4	0.007 3459	0.007 8521	0.008 3559	0.008 8575	4
6	0.004 8973	0.005 2347	0.005 5706	0.005 9049	6
12	0.002 4486	0.002 6174	0.002 7853	0.002 9525	12

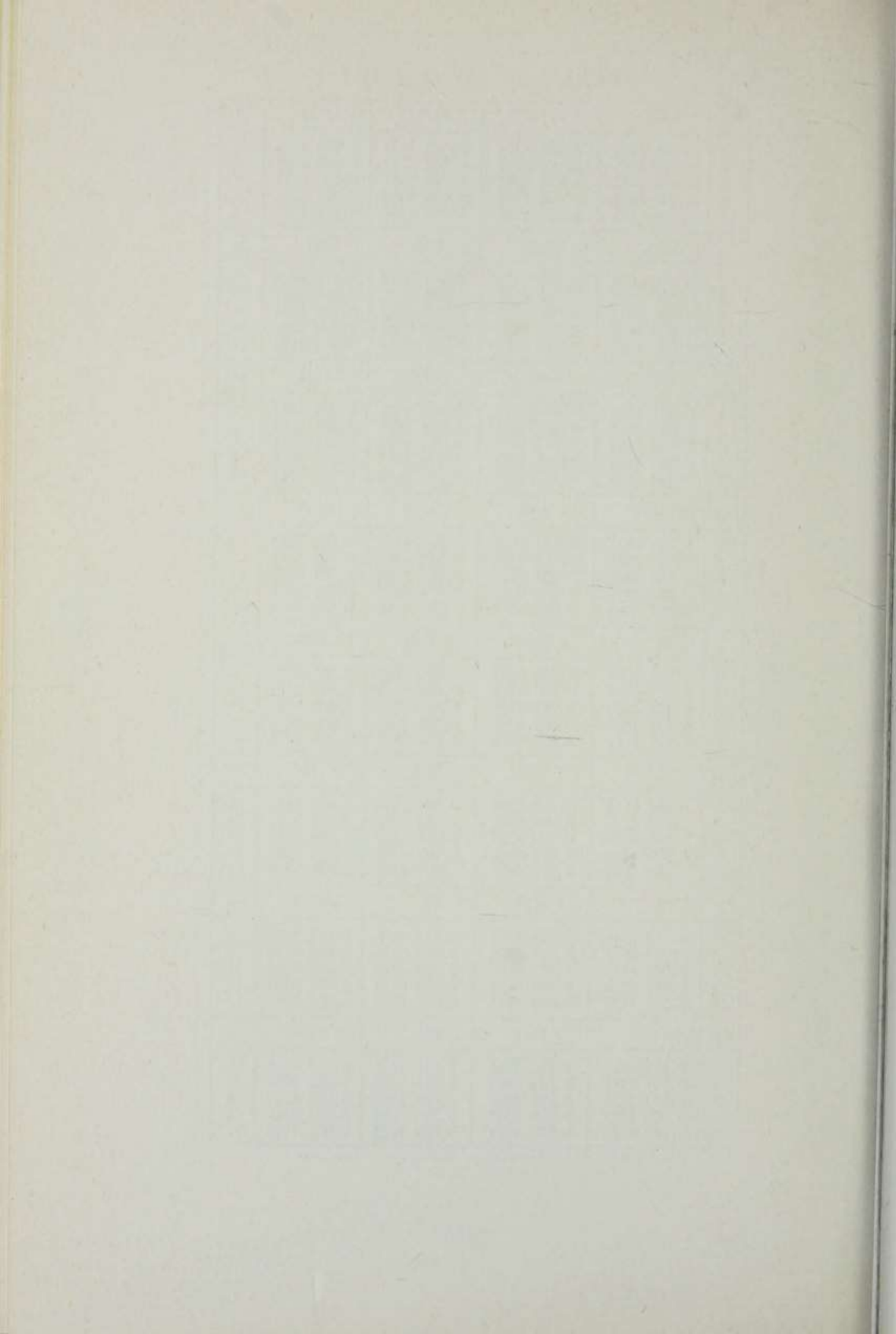


TABLE 6

$s_{\bar{m}i}$ and $\log s_{\bar{m}i}$

TABLE 6. $s_{ni} = \frac{(1+i)^n - 1}{i}$

n	$\frac{1}{4}\%$	$\frac{3}{8}\%$	$\frac{1}{2}\%$	$\frac{5}{8}\%$	n
1	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	1
2	2.0025 0000	2.0033 3333	2.0050 0000	2.0066 6667	2
3	3.0075 0625	3.0100 1111	3.0150 2500	3.0200 4444	3
4	4.0150 2502	4.0200 4448	4.0301 0013	4.0401 7807	4
5	5.0250 6258	5.0334 4463	5.0502 5063	5.0671 1259	5
6	6.0376 2523	6.0502 2278	6.0755 0188	6.1008 9335	6
7	7.0527 1930	7.0703 9019	7.1058 7939	7.1415 6597	7
8	8.0703 5110	8.0939 5816	8.1414 0879	8.1891 7641	8
9	9.0905 2697	9.1209 3802	9.1821 1583	9.2437 7092	9
10	10.1132 5329	10.1513 4114	10.2280 2641	10.3053 9606	10
11	11.1385 3642	11.1851 7895	11.2791 6654	11.3740 9870	11
12	12.1663 8277	12.2224 6288	12.3355 6237	12.4499 2602	12
13	13.1967 9872	13.2632 0442	13.3972 4018	13.5329 2553	13
14	14.2297 9072	14.3074 1510	14.4642 2639	14.6231 4503	14
15	15.2653 6520	15.3551 0648	15.5365 4752	15.7206 3266	15
16	16.3035 2861	16.4062 9017	16.6142 3026	16.8254 3688	16
17	17.3442 8743	17.4609 7781	17.6973 0141	17.9376 0646	17
18	18.3876 4815	18.5191 8107	18.7857 8791	19.0571 9051	18
19	19.4336 1727	19.5809 1167	19.8797 1685	20.1842 3844	19
20	20.4822 0131	20.6461 8137	20.9791 1544	21.3188 0003	20
21	21.5334 0682	21.7150 0198	22.0840 1101	22.4609 2536	21
22	22.5872 4033	22.7873 8532	23.1944 3107	23.6106 6487	22
23	23.6437 0843	23.8633 4327	24.3104 0322	24.7680 6930	23
24	24.7028 1770	24.9428 8775	25.4319 5524	25.9331 8976	24
25	25.7645 7475	26.0260 3071	26.5591 1502	27.1060 7769	25
26	26.8289 8619	27.1127 8414	27.6919 1059	28.2867 8488	26
27	27.8960 5865	28.2031 6009	28.8303 7015	29.4753 6344	27
28	28.9657 9880	29.2971 7062	29.9745 2200	30.6718 6586	28
29	30.0382 1330	30.3948 2786	31.1243 9461	31.8763 4497	29
30	31.1133 0883	31.4961 4395	32.2800 1658	33.0888 5394	30
31	32.1910 9210	32.6011 3110	33.4414 1666	34.3094 4630	31
32	33.2715 6983	33.7098 0154	34.6086 2375	35.5381 7594	32
33	34.3547 4876	34.8221 6754	35.7816 6686	36.7750 9711	33
34	35.4406 3563	35.9382 4143	36.9605 7520	38.0202 6443	34
35	36.5292 3722	37.0580 3557	38.1453 7807	39.2737 3286	35
36	37.6205 6031	38.1815 6236	39.3361 0496	40.5355 5774	36
37	38.7146 1171	39.3088 3423	40.5327 8549	41.8057 9479	37
38	39.8113 9824	40.4398 6368	41.7354 4942	43.0845 0009	38
39	40.9109 2673	41.5746 6322	42.9441 2666	44.3717 3009	39
40	42.0132 0405	42.7132 4543	44.1588 4730	45.6675 4163	40
41	43.1182 3706	43.8556 2292	45.3796 4153	46.9719 9191	41
42	44.2260 3265	45.0018 0833	46.6065 3974	48.2851 3852	42
43	45.3365 9774	46.1518 1436	47.8395 7244	49.6070 3944	43
44	46.4499 3923	47.3056 5374	49.0787 7030	50.9377 5304	44
45	47.5660 6408	48.4633 3925	50.3241 6415	52.2773 3806	45
46	48.6849 7924	49.6248 8371	51.5757 8497	53.6258 5365	46
47	49.8066 9169	50.7902 9999	52.8336 6390	54.9833 5934	47
48	50.9312 0842	51.9596 0099	54.0978 3222	56.3499 1507	48
49	52.0585 3644	53.1327 9966	55.3683 2138	57.7255 8117	49
50	53.1886 8278	54.3099 0899	56.6451 6299	59.1104 1837	50

TABLE 6. $\log s_{\overline{m}i}$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
1	0.000 0000	0.000 0000	0.000 0000	0.000 0000	1
2	0.301 5725	0.301 7532	0.302 1144	0.302 4752	2
3	0.478 2065	0.478 5681	0.479 2910	0.480 0133	3
4	0.603 6882	0.604 2309	0.605 3159	0.606 4005	4
5	0.701 1415	0.701 8653	0.703 3130	0.704 7606	5
6	0.780 8661	0.781 7713	0.783 5821	0.785 3934	6
7	0.848 3566	0.849 4434	0.851 6178	0.853 7934	7
8	0.906 8925	0.908 1609	0.910 6996	0.913 2402	8
9	0.958 5891	0.960 0395	0.962 9427	0.965 8491	9
10	1.004 8909	1.006 5234	1.009 7918	1.013 0647	10
11	1.046 8281	1.048 6429	1.052 2770	1.055 9170	11
12	1.085 1615	1.087 1587	1.091 1591	1.095 1668	12
13	1.120 4686	1.122 6485	1.127 0153	1.131 3917	13
14	1.153 1985	1.155 5612	1.160 2952	1.165 0408	14
15	1.183 7072	1.186 2528	1.191 3545	1.196 4700	15
16	1.212 2816	1.215 0104	1.220 4802	1.225 9664	16
17	1.239 1565	1.242 0686	1.247 9071	1.253 7645	17
18	1.264 5262	1.267 6218	1.273 8294	1.280 0588	18
19	1.288 5537	1.291 8329	1.298 4102	1.305 0123	19
20	1.311 3767	1.314 8397	1.321 7872	1.328 7627	20
21	1.333 1127	1.336 7598	1.344 0779	1.351 4277	21
22	1.353 8632	1.357 6945	1.365 3838	1.373 1082	22
23	1.373 7156	1.377 7312	1.385 7922	1.393 8921	23
24	1.392 7465	1.396 9468	1.405 3797	1.413 8560	24
25	1.411 0230	1.415 4079	1.424 2136	1.433 0667	25
26	1.428 6043	1.433 1741	1.442 3529	1.451 5836	26
27	1.445 5428	1.450 2978	1.459 8502	1.469 4591	27
28	1.461 8855	1.466 8257	1.476 7523	1.486 7402	28
29	1.477 6741	1.482 7997	1.493 1009	1.503 4685	29
30	1.492 9462	1.498 2574	1.508 9337	1.519 6817	30
31	1.507 7357	1.513 2326	1.524 2847	1.535 4137	31
32	1.522 0733	1.527 7562	1.539 1844	1.550 6951	32
33	1.535 9868	1.541 8558	1.553 6606	1.565 5538	33
34	1.549 5015	1.555 5568	1.567 7387	1.580 0151	34
35	1.562 6406	1.568 8824	1.581 4419	1.594 1022	35
36	1.575 4252	1.581 8537	1.594 7914	1.607 8362	36
37	1.587 8749	1.594 4902	1.607 8064	1.621 2365	37
38	1.600 0074	1.606 8096	1.620 5051	1.634 3210	38
39	1.611 8393	1.618 8288	1.632 9038	1.647 1064	39
40	1.623 3858	1.630 5626	1.645 0177	1.659 6076	40
41	1.634 6610	1.642 0253	1.656 8610	1.671 8390	41
42	1.645 6780	1.653 2300	1.668 4468	1.683 8134	42
43	1.656 4489	1.664 1888	1.679 7873	1.695 5433	43
44	1.666 9851	1.674 9130	1.690 8937	1.707 0398	44
45	1.677 2972	1.685 4134	1.701 7766	1.718 3135	45
46	1.687 3950	1.695 6995	1.712 4459	1.729 3742	46
47	1.697 2877	1.705 7808	1.722 9107	1.740 2312	47
48	1.706 9840	1.715 6658	1.733 1799	1.750 8932	48
49	1.716 4919	1.725 3627	1.743 2613	1.761 3683	49
50	1.725 8192	1.734 8791	1.753 1628	1.771 6641	50

TABLE 6. $s_{ni} = \frac{(1+i)^n - 1}{i}$

n	$\frac{1}{2}\%$	$\frac{3}{4}\%$	1%	$1\frac{1}{2}\%$	n
51	54.3216 5449	55.4909 4202	57.9283 8880	60.5044 8783	51
52	55.4574 5862	56.6759 1183	59.2180 3073	61.9078 5108	52
53	56.5961 0227	57.8648 3154	60.5141 2090	63.3205 7009	53
54	57.7375 9252	59.0577 1431	61.8166 9150	64.7427 0722	54
55	58.8819 3650	60.2545 7336	63.1257 7496	66.1743 2527	55
56	60.0291 4135	61.4554 2194	64.4414 0384	67.6154 8744	56
57	61.1792 1420	62.6602 7334	65.7636 1086	69.0662 5736	57
58	62.3321 6223	63.8691 4092	67.0924 2891	70.5266 9907	58
59	63.4879 9264	65.0820 3806	68.4278 9105	71.9968 7706	59
60	64.6467 1262	66.2989 7818	69.7700 3051	73.4768 5625	60
61	65.8083 2940	67.5199 7478	71.1188 8066	74.9667 0195	61
62	66.9728 5023	68.7450 4136	72.4744 7507	76.4664 7997	62
63	68.1402 8235	69.9741 9150	73.8368 4744	77.9762 5650	63
64	69.3106 3306	71.2074 3880	75.2060 3168	79.4960 9821	64
65	70.4839 0964	72.4447 9693	76.5820 6184	81.0260 7220	65
66	71.6601 1942	73.6862 7959	77.9649 7215	82.5662 4601	66
67	72.8392 6971	74.9319 0052	79.3547 9701	84.1166 8765	67
68	74.0213 6789	76.1816 7352	80.7515 7099	85.6774 6557	68
69	75.2064 2131	77.4356 1243	82.1553 2885	87.2486 4867	69
70	76.3944 3736	78.6937 3114	83.5661 0549	88.8303 0633	70
71	77.5854 2345	79.9560 4358	84.9839 3602	90.4225 0837	71
72	78.7793 8701	81.2225 6372	86.4088 5570	92.0253 2510	72
73	79.9763 3548	82.4933 0560	87.8408 9998	93.6388 2726	73
74	81.1762 7632	83.7682 8329	89.2801 0448	95.2630 8611	74
75	82.3792 1701	85.0475 1090	90.7265 0500	96.8981 7335	75
76	83.5851 6505	86.3310 0260	92.1801 3752	98.5441 6118	76
77	84.7941 2797	87.6187 7261	93.6410 3821	100.2011 2225	77
78	86.0061 1329	88.9108 3519	95.1092 4340	101.8691 2973	78
79	87.2211 2857	90.2072 0464	96.5847 8962	103.5482 5726	79
80	88.4391 8139	91.5078 9532	98.0677 1357	105.2385 7898	80
81	89.6602 7934	92.8129 2164	99.5580 5214	106.9401 6950	81
82	90.8844 3004	94.1222 9804	101.0558 4240	108.6531 0397	82
83	92.1116 4112	95.4360 3904	102.5611 2161	110.3774 5799	83
84	93.3419 2022	96.7541 5917	104.0739 2722	112.1133 0771	84
85	94.5752 7502	98.0766 7303	105.5942 9685	113.8607 2977	85
86	95.8117 1321	99.4035 9527	107.1222 6834	115.6198 0130	86
87	97.0512 4249	100.7349 4059	108.6578 7968	117.3905 9997	87
88	98.2938 7060	102.0707 2373	110.2011 6908	119.1732 0397	88
89	99.5396 0527	103.4109 5947	111.7521 7492	120.9676 9200	89
90	100.7884 5429	104.7556 6267	113.3109 3580	122.7741 4328	90
91	102.0404 2542	106.1048 4821	114.8774 9048	124.5926 3757	91
92	103.2955 2649	107.4585 3104	116.4518 7793	126.4232 5515	92
93	104.5537 6530	108.8167 2614	118.0341 3732	128.2660 7685	93
94	105.8151 4972	110.1794 4856	119.6243 0800	130.1211 8403	94
95	107.0796 8759	111.5467 1339	121.2224 2954	131.9886 5859	95
96	108.3473 8661	112.9185 3577	122.8285 4169	133.8685 8298	96
97	109.6182 5528	114.2949 3089	124.4426 8440	135.7610 4020	97
98	110.8923 0091	115.6759 1399	126.0648 9782	137.6661 1380	98
99	112.1693 3167	117.0615 0037	127.6952 2231	139.5838 8790	99
100	113.4499 5550	118.4517 0537	129.3336 9842	141.5144 4715	100

TABLE 6. $\log s_{\bar{m}i}$

n	$\frac{1}{4}\%$	$\frac{1}{8}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
51	1.734 9730	1.744 2221	1.762 8914	1.781 7876	51
52	1.743 9600	1.753 3985	1.772 4539	1.791 7458	52
53	1.752 7865	1.762 4147	1.781 8568	1.801 5448	53
54	1.761 4586	1.771 2766	1.791 1057	1.811 1908	54
55	1.769 9821	1.779 9900	1.800 2067	1.820 6895	55
56	1.778 3621	1.788 5602	1.809 1650	1.830 0462	56
57	1.786 6039	1.796 9923	1.817 9856	1.839 2659	57
58	1.794 7122	1.805 2911	1.826 6735	1.848 3536	58
59	1.802 6916	1.813 4612	1.835 2331	1.857 3137	59
60	1.810 5465	1.821 5069	1.843 6689	1.866 1505	60
61	1.818 2809	1.829 4322	1.851 9849	1.874 8684	61
62	1.825 8988	1.837 2414	1.860 1850	1.883 4711	62
63	1.833 4039	1.844 9379	1.868 2731	1.891 9624	63
64	1.840 7999	1.852 5254	1.876 2527	1.900 3458	64
65	1.848 0900	1.860 0072	1.884 1271	1.908 6248	65
66	1.855 2775	1.867 3867	1.891 8995	1.916 8025	66
67	1.862 3656	1.874 6667	1.899 5732	1.924 8822	67
68	1.869 3571	1.881 8505	1.907 1510	1.932 8666	68
69	1.876 2549	1.888 9407	1.914 6357	1.940 7587	69
70	1.883 0617	1.895 9401	1.922 0301	1.948 5612	70
71	1.889 7802	1.902 8513	1.929 3369	1.956 2765	71
72	1.896 4126	1.909 6767	1.936 5583	1.963 9073	72
73	1.902 9615	1.916 4187	1.943 6968	1.971 4560	73
74	1.909 4291	1.923 0796	1.950 7547	1.978 9247	74
75	1.915 8177	1.929 6616	1.957 7342	1.986 3156	75
76	1.922 1292	1.936 1668	1.964 6373	1.993 6309	76
77	1.928 3658	1.942 5972	1.971 4662	2.000 8726	77
78	1.934 5294	1.948 9547	1.978 2227	2.008 0426	78
79	1.940 6217	1.955 2412	1.984 9088	2.015 1428	79
80	1.946 6447	1.961 4586	1.991 5260	2.022 1749	80
81	1.952 6001	1.967 6084	1.998 0764	2.029 1409	81
82	1.958 4895	1.973 6925	2.004 5614	2.036 0421	82
83	1.964 3145	1.979 7124	2.010 9827	2.042 8804	83
84	1.970 0766	1.985 6696	2.017 3420	2.049 6572	84
85	1.975 7776	1.991 5657	2.023 6405	2.056 3740	85
86	1.981 4186	1.997 4021	2.029 8798	2.063 0322	86
87	1.987 0011	2.003 1801	2.036 0612	2.069 6333	87
88	1.992 5264	2.008 9012	2.042 1862	2.076 1786	88
89	1.997 9959	2.014 5665	2.048 2559	2.082 6694	89
90	2.003 4107	2.020 1775	2.054 2718	2.089 1069	90
91	2.008 7722	2.025 7352	2.060 2349	2.095 4924	91
92	2.014 0815	2.031 2409	2.066 1465	2.101 8270	92
93	2.019 3397	2.036 6955	2.072 0076	2.108 1118	93
94	2.024 5479	2.042 1006	2.077 8194	2.114 3480	94
95	2.029 7071	2.047 4568	2.083 5830	2.120 5366	95
96	2.034 8184	2.052 7653	2.089 2993	2.126 6787	96
97	2.039 8829	2.058 0269	2.094 9694	2.132 7751	97
98	2.044 9014	2.063 2429	2.100 5942	2.138 8270	98
99	2.049 8749	2.068 4141	2.106 1746	2.144 8353	99
100	2.054 8043	2.073 5414	2.111 7117	2.150 8008	100

TABLE 6. $s_{ni} = \frac{(1+i)^n - 1}{i}$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
101	114.7335 8038	119.8465 4439	130.9803 6692	143.4578 7680	101
102	116.0204 1434	121.2460 3287	132.6352 6875	145.4142 6264	102
103	117.3104 6537	122.6501 8632	134.2984 4509	147.3836 9106	103
104	118.6037 4153	124.0590 2027	135.9699 3732	149.3662 4900	104
105	119.9002 5089	125.4725 5034	137.6497 8701	151.3620 2399	105
106	121.2000 0152	126.8907 9217	139.3380 3594	153.3711 0415	106
107	122.5030 0152	128.3137 6148	141.0347 2612	155.3935 7818	107
108	123.8092 5902	129.7414 7402	142.7398 9975	157.4295 3537	108
109	125.1187 8217	131.1739 4560	144.4535 9925	159.4790 6560	109
110	126.4315 7913	132.6111 9208	146.1758 6725	161.5422 5937	110
111	127.7476 5807	134.0532 2939	147.9067 4658	163.6192 0777	111
112	129.0670 2722	135.5000 7349	149.6462 8032	165.7100 0249	112
113	130.3896 9479	136.9517 4040	151.3945 1172	167.8147 3584	113
114	131.7156 6902	138.4082 4620	153.1514 8428	169.9335 0074	114
115	133.0449 5820	139.8696 0702	154.9172 4170	172.0663 9075	115
116	134.3775 7059	141.3358 3905	156.6918 2791	174.2135 0002	116
117	135.7135 1452	142.8069 5851	158.4752 8704	176.3749 2335	117
118	137.0527 9830	144.2829 8170	160.2676 6348	178.5507 5618	118
119	138.3954 3030	145.7639 2498	162.0690 0180	180.7410 9455	119
120	139.7414 1888	147.2498 0477	163.8793 4681	182.9460 3518	120
121	141.0907 7242	148.7406 3745	165.6987 4354	185.1656 7542	121
122	142.4434 9935	150.2364 3958	167.5272 3726	187.4001 1325	122
123	143.7996 0810	151.7372 2771	169.3648 7344	189.6494 4734	123
124	145.1591 0712	153.2430 1847	171.2116 9781	191.9137 7699	124
125	146.5220 0489	154.7538 2853	173.0677 5630	194.1932 0217	125
126	147.8883 0990	156.2696 7463	174.9330 9508	196.4878 2352	126
127	149.2580 3068	157.7905 7354	176.8077 6056	198.7977 4234	127
128	150.6311 7575	159.3165 4212	178.6917 9936	201.1230 6062	128
129	152.0077 5369	160.8475 9726	180.5852 5836	203.4638 8103	129
130	153.3877 7308	162.3837 5592	182.4881 8465	205.8203 0690	130
131	154.7712 4251	163.9250 3510	184.4006 2557	208.1924 4228	131
132	156.1581 7062	165.4714 5189	186.3226 2870	210.5803 9189	132
133	157.5485 6604	167.0230 2339	188.2542 4184	212.9842 6117	133
134	158.9424 3746	168.5797 6680	190.1955 1305	215.4041 5625	134
135	160.3397 9355	170.1416 9936	192.1464 9062	217.8401 8396	135
136	161.7406 4304	171.7088 3836	194.1072 2307	220.2924 5185	136
137	163.1449 9464	173.2812 0115	196.0777 5919	222.7610 6820	137
138	164.5528 5713	174.8588 0516	198.0581 4798	225.2461 4198	138
139	165.9642 3927	176.4416 6784	200.0484 3872	227.7477 8293	139
140	167.3791 4987	178.0298 0673	202.0486 8092	230.2661 0148	140
141	168.7975 9775	179.6232 3942	204.0589 2432	232.8012 0883	141
142	170.2195 9174	181.2219 8355	206.0792 1894	235.3532 1688	142
143	171.6451 4072	182.8260 5683	208.1096 1504	237.9222 3833	143
144	173.0742 5357	184.4354 7702	210.1501 6311	240.5083 8659	144
145	174.5069 3921	186.0502 6194	212.2009 1393	243.1117 7583	145
146	175.9432 0655	187.6704 2948	214.2619 1850	245.7325 2100	146
147	177.3830 6457	189.2959 9758	216.3332 2809	248.3707 3781	147
148	178.8265 2223	190.9269 8424	218.4148 9423	251.0265 4273	148
149	180.2735 8854	192.5634 0752	220.5069 6870	253.7000 5301	149
150	181.7242 7251	194.2052 8554	222.6095 0354	256.3913 8670	150

TABLE 6. $\log s_{\bar{m}i}$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
101	2.059 6905	2.078 6255	2.117 2062	2.156 7244	101
102	2.064 5344	2.083 6675	2.122 6590	2.162 6070	102
103	2.069 3367	2.088 6683	2.128 0710	2.168 4494	103
104	2.074 0984	2.093 6284	2.133 4429	2.174 2525	104
105	2.078 8201	2.098 5487	2.138 7755	2.180 0169	105
106	2.083 5026	2.103 4301	2.144 0697	2.185 7435	106
107	2.088 1467	2.108 2732	2.149 3261	2.181 4331	107
108	2.092 7531	2.113 0788	2.154 5454	2.197 0862	108
109	2.097 3225	2.117 8476	2.159 7283	2.202 7037	109
110	2.101 8555	2.122 5802	2.164 8757	2.208 2862	110
111	2.106 3529	2.127 2773	2.169 9880	2.213 8343	111
112	2.110 8153	2.131 9395	2.175 0659	2.219 3487	112
113	2.115 2433	2.136 5675	2.180 1101	2.224 8301	113
114	2.119 6374	2.141 1620	2.185 1212	2.230 2790	114
115	2.123 9984	2.145 7234	2.190 0998	2.235 6961	115
116	2.128 3268	2.150 2523	2.195 0464	2.241 0818	116
117	2.132 6231	2.154 7494	2.199 9616	2.246 4368	117
118	2.136 8879	2.159 2151	2.204 8459	2.251 7617	118
119	2.141 1218	2.163 6501	2.209 7000	2.257 0569	119
120	2.145 3251	2.168 0547	2.214 5243	2.262 3230	120
121	2.149 4986	2.172 4297	2.219 3192	2.267 5605	121
122	2.153 6427	2.176 7753	2.224 0855	2.272 7699	122
123	2.157 7577	2.181 0922	2.228 8233	2.277 9515	123
124	2.161 8443	2.185 3807	2.233 5334	2.283 1061	124
125	2.165 9028	2.189 6414	2.238 2162	2.288 2340	125
126	2.169 9338	2.193 8747	2.242 8720	2.293 3357	126
127	2.173 9377	2.198 0811	2.247 5013	2.298 4115	127
128	2.177 9149	2.202 2609	2.252 1046	2.303 4619	128
129	2.181 8658	2.206 4146	2.256 6823	2.308 4873	129
130	2.185 7907	2.210 5426	2.261 2348	2.313 4882	130
131	2.189 6903	2.214 6453	2.265 7624	2.318 4650	131
132	2.193 5647	2.218 7231	2.270 2656	2.323 4179	132
133	2.197 4145	2.222 7764	2.274 7448	2.328 3475	133
134	2.201 2399	2.226 8054	2.279 2003	2.333 2541	134
135	2.205 0413	2.230 8108	2.283 6325	2.338 1380	135
136	2.208 8191	2.234 7927	2.288 0417	2.342 9996	136
137	2.212 5738	2.238 7514	2.292 4283	2.347 8393	137
138	2.216 3055	2.242 6875	2.296 7927	2.352 6574	138
139	2.220 0145	2.246 6011	2.301 1352	2.357 4542	139
140	2.223 7014	2.250 4927	2.305 4561	2.362 2300	140
141	2.227 3662	2.254 3625	2.309 7556	2.366 9853	141
142	2.231 0096	2.258 2109	2.314 0342	2.371 7202	142
143	2.234 6315	2.262 0381	2.318 2921	2.376 4351	143
144	2.238 2325	2.265 8445	2.322 5297	2.381 1302	144
145	2.241 8127	2.269 6303	2.326 7473	2.385 8060	145
146	2.245 3725	2.273 3959	2.330 9450	2.390 4627	146
147	2.248 9122	2.277 1414	2.335 1233	2.395 1004	147
148	2.252 4319	2.280 8673	2.339 2822	2.399 7196	148
149	2.255 9321	2.284 5738	2.343 4223	2.404 3206	149
150	2.259 4129	2.288 2610	2.347 5437	2.408 9034	150

TABLE 6. $s_{ni} = \frac{(1+i)^n - 1}{i}$

n	$\frac{1}{2}\%$	$\frac{3}{8}\%$	$\frac{1}{2}\%$	$\frac{3}{8}\%$	n
151	183.1785 8319	195.8526 3650	224.7225 5106	259.1006 6261	151
152	184.6365 2965	197.5054 7862	226.8461 6382	261.8280 0036	152
153	186.0981 2097	199.1638 3021	228.9803 9464	264.5735 2036	153
154	187.5633 6627	200.8277 0965	231.1252 9661	267.3373 4383	154
155	189.0322 7469	202.4971 3534	233.2809 2309	270.1195 9279	155
156	190.5048 5538	204.1721 2580	235.4473 2771	272.9203 9008	156
157	191.9811 1752	205.8526 9955	237.6245 6435	275.7398 5934	157
158	193.4610 7031	207.5388 7521	239.8126 8717	278.5781 2507	158
159	194.9447 2298	209.2306 7146	242.0117 5060	281.4353 1257	159
160	196.4320 8479	210.9281 0704	244.2218 0936	284.3115 4799	160
161	197.9231 6500	212.6312 0073	246.4429 1840	287.2069 5831	161
162	199.4179 7292	214.3399 7139	248.6751 3300	290.1216 7136	162
163	200.9165 1785	216.0544 3797	250.9185 0866	293.0558 1584	163
164	202.4188 0914	217.7746 1942	253.1731 0121	296.0095 2128	164
165	203.9248 5617	219.5005 3482	255.4389 6671	298.9829 1809	165
166	205.4346 6831	221.2322 0327	257.7161 6154	301.9761 3754	166
167	206.9482 5498	222.9696 4395	260.0047 4235	304.9893 1179	167
168	208.4656 2562	224.7128 7610	262.3047 6606	308.0225 7387	168
169	209.9867 8968	226.4619 1902	264.6162 8989	311.0760 5770	169
170	211.5117 5665	228.2167 9208	266.9393 7134	314.1498 9808	170
171	213.0405 3605	229.9775 1472	269.2740 6820	317.2442 3073	171
172	214.5731 3739	231.7441 0643	271.6204 3854	320.3591 9227	172
173	216.1095 7023	233.5165 8679	273.9785 4073	323.4949 2022	173
174	217.6498 4415	235.2949 7541	276.3484 3344	326.6515 5302	174
175	219.1939 6876	237.0792 8200	278.7301 7561	329.8292 3004	175
176	220.7419 5369	238.8695 5630	281.1238 2648	333.0280 9158	176
177	222.2938 0857	240.6657 8816	283.5294 4562	336.2482 7885	177
178	223.8495 4309	242.4680 0745	285.9470 9284	339.4899 3405	178
179	225.4091 6695	244.2762 3414	288.3768 2831	342.7532 0027	179
180	226.9726 8987	246.0904 8826	290.8187 1245	346.0382 2161	180
181	228.5401 2159	247.9107 8988	293.2728 0601	349.3451 4309	181
182	230.1114 7190	249.7371 5918	295.7391 7004	352.6741 1071	182
183	231.6867 5058	251.5696 1638	298.2178 6589	356.0252 7144	183
184	233.2659 6745	253.4081 8177	300.7089 5522	359.3987 7325	184
185	234.8491 3237	255.2528 7571	303.2125 0000	362.7947 6508	185
186	236.4362 5520	257.1037 1863	305.7285 6250	366.2133 9684	186
187	238.0273 4584	258.9607 3102	308.2572 0531	369.6548 1949	187
188	239.6224 1420	260.8239 3346	310.7984 9134	373.1191 8495	188
189	241.2214 7024	262.6933 4657	313.3524 8379	376.6066 4618	189
190	242.8245 2392	264.5689 9106	315.9192 4621	380.1173 5716	190
191	244.4315 8523	266.4508 8769	318.4988 4244	383.6514 7287	191
192	246.0426 6419	268.3390 5732	321.0913 3666	387.2091 4936	192
193	247.6577 7085	270.2335 2084	323.6967 9334	390.7905 4369	193
194	249.2769 1528	272.1342 9925	326.3152 7731	394.3958 1398	194
195	250.9001 0756	274.0414 1358	328.9468 5369	398.0251 1941	195
196	252.5273 5783	275.9548 8495	331.5915 8796	401.6786 2020	196
197	254.1586 7623	277.8747 3457	334.2495 4590	405.3564 7767	197
198	255.7940 7292	279.8009 8369	336.9207 9363	409.0588 5419	198
199	257.4335 5810	281.7336 5363	339.6053 9760	412.7859 1322	199
200	259.0771 4200	283.6727 6581	342.3034 2450	416.5378 1930	200

TABLE 6. $\log s_{\overline{m}i}$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
151	2.262 8747	2.291 9295	2.351 6466	2.413 4685	151
152	2.266 3176	2.295 5792	2.355 7314	2.418 0161	152
153	2.269 7420	2.299 2104	2.359 7983	2.422 5464	153
154	2.273 1480	2.302 8237	2.363 8475	2.427 0596	154
155	2.276 5359	2.306 4189	2.367 8792	2.431 5560	155
156	2.279 9061	2.309 9964	2.371 8938	2.436 0360	156
157	2.283 2585	2.313 5566	2.375 8913	2.440 4996	157
158	2.286 5936	2.317 0995	2.379 8722	2.444 9470	158
159	2.289 9115	2.320 6254	2.383 8364	2.449 3786	159
160	2.293 2124	2.324 1345	2.387 7844	2.453 7945	160
161	2.296 4966	2.327 6270	2.391 7163	2.458 1950	161
162	2.299 7643	2.331 1031	2.395 6324	2.462 5802	162
163	2.303 0157	2.334 5632	2.399 5327	2.466 9503	163
164	2.306 2509	2.338 0073	2.403 4175	2.471 3057	164
165	2.309 4702	2.341 4356	2.407 2871	2.475 6464	165
166	2.312 6738	2.344 8483	2.411 1416	2.479 9726	166
167	2.315 8617	2.348 2457	2.414 9813	2.484 2846	167
168	2.319 0345	2.351 6280	2.418 8062	2.488 5825	168
169	2.322 1920	2.354 9952	2.422 6166	2.492 8666	169
170	2.325 3345	2.358 3476	2.426 4127	2.497 1370	170
171	2.328 4623	2.361 6854	2.430 1945	2.501 3937	171
172	2.331 5753	2.365 0087	2.433 9624	2.505 6372	172
173	2.334 6740	2.368 3178	2.437 7166	2.509 8675	173
174	2.337 7584	2.371 6127	2.441 4570	2.514 0848	174
175	2.340 8286	2.374 8936	2.445 1840	2.518 2892	175
176	2.343 8848	2.378 1608	2.448 8976	2.522 4809	176
177	2.346 9274	2.381 4144	2.452 5982	2.526 6601	177
178	2.349 9562	2.384 6544	2.456 2857	2.530 8269	178
179	2.352 9716	2.387 8812	2.459 9603	2.534 9815	179
180	2.355 9736	2.391 0949	2.463 6224	2.539 1241	180
181	2.358 9624	2.394 2954	2.467 2718	2.543 2547	181
182	2.361 9383	2.397 4832	2.470 9089	2.547 3736	182
183	2.364 9012	2.400 6581	2.474 5337	2.551 4808	183
184	2.367 8514	2.403 8206	2.478 1464	2.555 5766	184
185	2.370 7890	2.406 9707	2.481 7471	2.559 6610	185
186	2.373 7141	2.410 1084	2.485 3361	2.563 7342	186
187	2.376 6269	2.413 2339	2.488 9133	2.567 7964	187
188	2.379 5275	2.416 3474	2.492 4789	2.571 8475	188
189	2.382 4159	2.419 4491	2.496 0331	2.575 8879	189
190	2.385 2925	2.422 5389	2.499 5761	2.579 9177	190
191	2.388 1573	2.425 6171	2.503 1078	2.583 9369	191
192	2.391 0104	2.428 6839	2.506 6286	2.587 9456	192
193	2.393 8520	2.431 7392	2.510 1384	2.591 9440	193
194	2.396 6820	2.434 7833	2.513 6374	2.595 9323	194
195	2.399 5009	2.437 8162	2.517 1257	2.599 9105	195
196	2.402 3085	2.440 8381	2.520 6035	2.603 8787	196
197	2.405 1049	2.443 8491	2.524 0708	2.607 8371	197
198	2.407 8905	2.446 8492	2.527 5278	2.611 7858	198
199	2.410 6652	2.449 8387	2.530 9746	2.615 7249	199
200	2.413 4291	2.452 8176	2.534 4112	2.619 6544	200

TABLE 6. $s_{\bar{m}i} = \frac{(1+i)^n - 1}{i}$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	1 $\frac{1}{4}\%$	n
1	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	1
2	2.0075 0000	2.0087 5000	2.0100 0000	2.0125 0000	2
3	3.0225 5625	3.0263 2656	3.0301 0000	3.0376 5625	3
4	4.0452 2542	4.0528 0692	4.0604 0100	4.0756 2695	4
5	5.0755 6461	5.0882 6898	5.1010 0501	5.1265 7229	5
6	6.1136 3135	6.1327 9133	6.1520 1506	6.1906 5444	6
7	7.1594 8358	7.1864 5326	7.2135 3521	7.2680 3762	7
8	8.2131 7971	8.2493 3472	8.2856 7056	8.3588 8809	8
9	9.2747 7856	9.3215 1640	9.3685 2727	9.4633 7420	9
10	10.3443 3940	10.4030 7967	10.4622 1254	10.5816 6637	10
11	11.4219 2194	11.4941 0662	11.5668 3467	11.7139 3720	11
12	12.5075 8636	12.5946 8005	12.6825 0301	12.8603 6142	12
13	13.6013 9325	13.7048 8350	13.8093 2804	14.0211 1594	13
14	14.7034 0370	14.8248 0123	14.9474 2132	15.1963 7988	14
15	15.8136 7923	15.9545 1824	16.0968 9554	16.3863 3463	15
16	16.9322 8183	17.0941 2028	17.2578 6449	17.5911 6382	16
17	18.0592 7394	18.2436 9383	18.4304 4314	18.8110 5336	17
18	19.1947 1849	19.4033 2615	19.6147 4757	20.0461 9153	18
19	20.3386 7888	20.5731 0526	20.8108 9504	21.2967 6893	19
20	21.4912 1897	21.7531 1993	22.0190 0399	22.5629 7854	20
21	22.6524 0312	22.9434 5973	23.2391 9403	23.8450 1577	21
22	23.8222 9614	24.1442 1500	24.4715 8598	25.1430 7847	22
23	25.0009 6336	25.3554 7688	25.7163 0183	26.4573 6695	23
24	26.1884 7059	26.5773 3730	26.9734 6485	27.7880 8403	24
25	27.3848 8412	27.8098 8900	28.2431 9950	29.1354 3508	25
26	28.5902 7075	29.0532 2553	29.5256 3150	30.4996 2802	26
27	29.8046 9778	30.3074 4126	30.8208 8781	31.8808 7337	27
28	31.0282 3301	31.5726 3137	32.1290 9669	33.2793 8429	28
29	32.2609 4476	32.8488 9189	33.4503 8766	34.6953 7659	29
30	33.5029 0184	34.1363 1970	34.7848 9153	36.1290 6880	30
31	34.7541 7361	35.4350 1249	36.1327 4045	37.5806 8216	31
32	36.0148 2991	36.7450 6885	37.4940 6785	39.0504 4069	32
33	37.2849 4113	38.0665 8820	38.8690 0853	40.5385 7120	33
34	38.5645 7819	39.3996 7085	40.2576 9862	42.0453 0334	34
35	39.8538 1253	40.7444 1797	41.6602 7560	43.5708 6963	35
36	41.1527 1612	42.1009 3163	43.0768 7836	45.1155 0550	36
37	42.4613 6149	43.4693 1478	44.5076 4714	46.6794 4932	37
38	43.7798 2170	44.8496 7128	45.9527 2361	48.2926 4243	38
39	45.1081 7037	46.2421 0591	47.4122 5085	49.8862 2921	39
40	46.4464 8164	47.6467 2433	48.8863 7336	51.4895 5708	40
41	47.7948 3026	49.0636 3317	50.3752 3709	53.1331 7654	41
42	49.1532 9148	50.4929 3996	51.8789 8946	54.7973 4125	42
43	50.5219 4117	51.9347 5319	53.3977 7936	56.4823 0801	43
44	51.9008 5573	53.3891 8228	54.9317 5715	58.1883 3687	44
45	53.2901 1215	54.8563 3762	56.4810 7472	59.9156 9108	45
46	54.6897 8799	56.3363 3058	58.0458 8547	61.6646 3721	46
47	56.0999 6140	57.8292 7347	59.6263 4432	63.4354 4518	47
48	57.5207 1111	59.3352 7961	61.2226 0777	65.2283 8824	48
49	58.9521 1644	60.8544 6331	62.8348 3385	67.0437 4310	49
50	60.3942 5732	62.3869 3986	64.4631 8218	68.8817 8989	50

TABLE 6. $\log s_{\bar{m}i}$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	1 $\frac{1}{4}\%$	n
1	0.000 0000	0.000 0000	0.000 0000	0.000 0000	1
2	0.302 6556	0.302 9258	0.303 1961	0.303 7359	2
3	0.480 3744	0.480 9158	0.481 4570	0.482 5386	3
4	0.606 9427	0.607 7559	0.608 5689	0.610 1944	4
5	0.705 4844	0.706 5701	0.707 6557	0.709 8270	5
6	0.786 2992	0.787 6582	0.789 0173	0.791 7365	6
7	0.854 8817	0.856 5146	0.858 1481	0.861 4171	7
8	0.914 5113	0.916 4189	0.918 3277	0.922 1484	8
9	0.967 3036	0.969 4866	0.971 6713	0.976 0460	9
10	1.014 7027	1.017 1620	1.019 6234	1.024 5542	10
11	1.057 7392	1.060 4752	1.063 2145	1.068 7029	11
12	1.097 1735	1.100 1872	1.103 2049	1.109 2531	12
13	1.133 5834	1.136 8753	1.140 1725	1.146 7827	13
14	1.167 4179	1.170 9889	1.174 5662	1.181 7401	14
15	1.199 0329	1.202 8837	1.206 7422	1.214 4819	15
16	1.228 7155	1.232 8467	1.236 9870	1.245 2944	16
17	1.256 7003	1.261 1128	1.265 5356	1.274 4130	17
18	1.283 1817	1.287 8762	1.292 5827	1.302 0318	18
19	1.308 3227	1.313 2998	1.318 2908	1.328 3137	19
20	1.332 2610	1.337 5216	1.342 7976	1.353 3964	20
21	1.355 1143	1.360 6589	1.366 2209	1.377 3976	21
22	1.376 9836	1.382 8131	1.388 6622	1.400 4184	22
23	1.397 9568	1.404 0718	1.410 2084	1.422 5466	23
24	1.418 1101	1.424 5115	1.430 9367	1.443 8585	24
25	1.437 5109	1.444 1993	1.450 9138	1.464 4214	25
26	1.456 2183	1.463 1944	1.470 1991	1.484 2945	26
27	1.474 2847	1.481 5492	1.488 8451	1.503 5301	27
28	1.491 7571	1.499 3107	1.506 8985	1.522 1752	28
29	1.508 6771	1.516 5208	1.524 4012	1.540 2716	29
30	1.525 0824	1.533 2167	1.541 3906	1.557 8567	30
31	1.541 0070	1.549 4325	1.557 9008	1.574 9646	31
32	1.556 4814	1.565 1991	1.573 9626	1.591 6259	32
33	1.571 5334	1.580 5440	1.589 6035	1.607 8683	33
34	1.586 1886	1.595 4926	1.604 8489	1.623 7174	34
35	1.600 4699	1.610 0681	1.619 7222	1.639 1962	35
36	1.614 3985	1.624 2917	1.634 2442	1.654 3258	36
37	1.627 9939	1.638 1827	1.648 4346	1.669 1256	37
38	1.641 2740	1.651 7592	1.662 3112	1.683 6137	38
39	1.654 2552	1.665 0376	1.675 8905	1.697 8065	39
40	1.666 9528	1.678 0331	1.689 1878	1.711 7192	40
41	1.679 3810	1.690 7597	1.702 2171	1.725 3657	41
42	1.691 5526	1.703 2307	1.714 9915	1.738 7594	42
43	1.703 4800	1.715 4580	1.727 5232	1.751 9124	43
44	1.715 1745	1.727 4533	1.739 8235	1.764 8359	44
45	1.726 6466	1.739 2268	1.751 9029	1.777 5405	45
46	1.737 9062	1.750 7885	1.763 7714	1.790 0361	46
47	1.748 9626	1.762 1477	1.775 4381	1.802 3320	47
48	1.759 8242	1.773 3130	1.786 9118	1.814 4366	48
49	1.770 4994	1.784 2924	1.798 2004	1.826 3582	49
50	1.780 9957	1.795 0937	1.809 3117	1.838 1043	50

TABLE 6. $s_{\overline{m}i} = \frac{(1+i)^n - 1}{i}$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	1 $\frac{1}{4}\%$	n
51	61.8472 1424	63.9328 2559	66.1078 1401	70.7428 1226	51
52	63.3110 6835	65.4922 3781	67.7688 9215	72.6270 9741	52
53	64.7859 0136	67.0652 9489	69.4465 8107	74.5349 3613	53
54	66.2717 9562	68.6521 1622	71.1410 4688	76.4666 2283	54
55	67.7688 3409	70.2528 2224	72.8524 5735	78.4224 5562	55
56	69.2771 0035	71.8675 3443	74.5809 8192	80.4027 3631	56
57	70.7966 7860	73.4963 7536	76.3267 9174	82.4077 7052	57
58	72.3276 5369	75.1394 0664	78.0900 5966	84.4378 6765	58
59	73.8701 1109	76.7969 3900	79.8709 6025	86.4933 4099	59
60	75.4241 3693	78.4689 1221	81.6696 6986	88.5745 0776	60
61	76.9898 1795	80.1558 1519	83.4863 6655	90.6816 8910	61
62	78.5672 4159	81.8568 7595	85.3212 3022	92.8152 1022	62
63	80.1564 9590	83.5731 2362	87.1744 4252	94.9754 0034	63
64	81.7576 6962	85.3043 8845	89.0461 8695	97.1625 9285	64
65	83.3708 5214	87.0508 0185	90.9366 4882	99.3771 2526	65
66	84.9961 3353	88.8124 9636	92.8460 1531	101.6193 3933	66
67	86.6336 0453	90.5896 0571	94.7744 7546	103.8895 8107	67
68	88.2833 5657	92.3822 6476	96.7222 2021	106.1882 0083	68
69	89.9454 8174	94.1906 0957	98.6894 4242	108.5155 5334	69
70	91.6200 7285	96.0147 7741	100.6763 3684	110.8719 9776	70
71	93.3072 2340	97.8549 0671	102.6831 0021	113.2578 9773	71
72	95.0070 2758	99.7111 3714	104.7099 3121	115.6736 2145	72
73	96.7195 8028	101.5836 0959	106.7570 3052	118.1195 4172	73
74	98.4449 7714	103.4724 6618	108.8246 0083	120.5960 3599	74
75	100.1833 1446	105.3778 5025	110.9128 4684	123.1034 8644	75
76	101.9346 8932	107.2999 0644	113.0219 7530	125.6422 8002	76
77	103.6991 9949	109.2387 8063	115.1521 9506	128.2128 0852	77
78	105.4769 4349	111.1946 1996	117.3037 1701	130.8154 6863	78
79	107.2680 2056	113.1675 7288	119.4767 5418	133.4506 6199	79
80	109.0725 3072	115.1577 8914	121.6715 2172	136.1187 9526	80
81	110.8905 7470	117.1654 1980	123.8882 3694	138.8202 8020	81
82	112.7222 3401	119.1906 1722	126.1271 1931	141.5555 3370	82
83	114.5676 7091	121.2335 3512	128.3893 9050	144.3249 7787	83
84	116.4269 2845	123.2943 2895	130.6722 7440	147.1290 4010	84
85	118.3001 3041	125.3731 5393	132.9789 9715	149.9681 5310	85
86	120.1873 8139	127.4701 6903	135.3087 8712	152.8427 5501	86
87	122.0887 8675	129.5855 3301	137.6618 7499	155.7532 8945	87
88	124.0044 5265	131.7194 0642	140.0384 9374	158.7002 0557	88
89	125.9344 8604	133.8719 5123	142.4388 7868	161.6839 5814	89
90	127.8789 9469	136.0433 3080	144.8632 6746	164.7050 0762	90
91	129.8380 8715	138.2337 0994	147.3119 0014	167.7638 2021	91
92	131.8118 7280	140.4432 5491	149.7850 1914	170.8608 6796	92
93	133.8004 6185	142.6721 3339	152.2828 6933	173.9966 2881	93
94	135.8039 6531	144.9205 1455	154.8056 9803	177.1715 8667	94
95	137.8224 9505	147.1883 6906	157.3537 5501	180.3862 3151	95
96	139.8561 6377	149.4764 6903	159.9272 9256	183.6410 5940	96
97	141.9050 8499	151.7843 8813	162.5265 6548	186.9365 7264	97
98	143.9693 7313	154.1125 0153	165.1518 3114	190.2732 7980	98
99	146.0491 4343	156.4609 8592	167.8033 4945	193.6516 9580	99
100	148.1445 1201	158.8300 1955	170.4813 8294	197.0723 4200	100

TABLE 6. $\log s_{\overline{m}i}$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	$1\frac{1}{4}\%$	n
51	1.791 3201	1.805 7239	1.820 2527	1.849 6823	51
52	1.801 4797	1.816 1899	1.831 0303	1.861 0987	52
53	1.811 4805	1.826 4978	1.841 6508	1.872 3599	53
54	1.821 3288	1.836 6539	1.852 1203	1.883 4719	54
55	1.831 0300	1.846 6638	1.862 4442	1.894 4404	55
56	1.840 5897	1.856 5328	1.872 6280	1.905 2708	56
57	1.850 0129	1.866 2659	1.882 6770	1.915 9681	57
58	1.859 3044	1.875 8681	1.892 5957	1.926 5372	58
59	1.868 4688	1.885 3440	1.902 3888	1.936 9827	59
60	1.877 5104	1.894 6976	1.912 0608	1.947 3087	60
61	1.886 4333	1.903 9334	1.921 6156	1.957 5196	61
62	1.895 2415	1.913 0551	1.931 0570	1.967 6191	62
63	1.903 9387	1.922 0666	1.940 3892	1.977 6111	63
64	1.912 5285	1.930 9714	1.949 6153	1.987 4990	64
65	1.921 0142	1.939 7728	1.958 7389	1.997 2864	65
66	1.929 3992	1.948 4741	1.967 7632	2.006 9762	66
67	1.937 6864	1.957 0784	1.976 6914	2.016 5720	67
68	1.945 8789	1.965 5886	1.985 5262	2.026 0762	68
69	1.953 9794	1.974 0076	1.994 2706	2.035 4921	69
70	1.961 9906	1.982 3381	2.002 9272	2.044 8218	70
71	1.969 9152	1.990 5826	2.011 4990	2.054 0685	71
72	1.977 7557	1.998 7437	2.019 9877	2.063 2342	72
73	1.985 5144	2.006 8236	2.028 3964	2.072 3216	73
74	1.993 1936	2.014 8248	2.036 7270	2.081 3329	74
75	2.000 7954	2.022 7493	2.044 9818	2.090 2704	75
76	2.008 3220	2.030 5993	2.053 1629	2.099 1358	76
77	2.015 7754	2.038 3768	2.061 2722	2.107 9313	77
78	2.023 1575	2.046 0838	2.069 3117	2.116 6591	78
79	2.030 4703	2.053 7220	2.077 2836	2.125 3208	79
80	2.037 7154	2.061 2933	2.085 1888	2.133 9180	80
81	2.044 8346	2.068 7995	2.093 0300	2.142 4529	81
82	2.052 0097	2.076 2421	2.100 8084	2.150 9267	82
83	2.059 0621	2.083 6228	2.108 5257	2.159 3415	83
84	2.066 0534	2.090 9431	2.116 1835	2.167 6983	84
85	2.072 9852	2.098 2045	2.123 7830	2.175 9991	85
86	2.079 8588	2.105 4086	2.131 3260	2.184 2449	86
87	2.086 6753	2.112 5565	2.138 8137	2.192 4372	87
88	2.093 4373	2.119 6498	2.146 2474	2.200 5774	88
89	2.100 1447	2.126 6896	2.153 6286	2.208 6670	89
90	2.106 7992	2.133 6772	2.160 9584	2.216 7068	90
91	2.113 4021	2.140 6139	2.168 2378	2.224 6982	91
92	2.119 9546	2.147 5008	2.175 4683	2.232 6426	92
93	2.126 4576	2.154 3392	2.182 6511	2.240 5407	93
94	2.132 9125	2.161 1298	2.189 7869	2.248 3941	94
95	2.139 3201	2.167 8741	2.196 8771	2.256 2033	95
96	2.145 6816	2.174 5728	2.203 9226	2.263 9698	96
97	2.151 9979	2.181 2271	2.210 9244	2.271 6943	97
98	2.158 2701	2.187 8378	2.217 8833	2.279 3778	98
99	2.164 4990	2.194 4061	2.224 8005	2.287 0213	99
100	2.170 6856	2.200 9326	2.231 6769	2.294 6255	100

TABLE 6. $s_{ni} = \frac{(1+i)^n - 1}{i}$

<i>n</i>	1½%	1¾%	2%	2½%	<i>n</i>
1	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	1
2	2.0150 0000	2.0175 0000	2.0200 0000	2.0250 0000	2
3	3.0452 2500	3.0528 0625	3.0604 0000	3.0756 2500	3
4	4.0909 0338	4.1062 3036	4.1216 0800	4.1523 1563	4
5	5.1522 6693	5.1780 8938	5.2040 4016	5.2563 2852	5
6	6.2295 5093	6.2687 0596	6.3081 2096	6.3877 3673	6
7	7.3229 9419	7.3784 0831	7.4342 8338	7.5474 3015	7
8	8.4328 3911	8.5075 3045	8.5829 6905	8.7361 1590	8
9	9.5593 3169	9.6564 1224	9.7546 2843	9.9545 1880	9
10	10.7027 2167	10.8253 9945	10.9497 2100	11.2033 8177	10
11	11.8632 6249	12.0148 4394	12.1687 1542	12.4834 6631	11
12	13.0412 1143	13.2251 0371	13.4120 8973	13.7955 5297	12
13	14.2368 2960	14.4565 4303	14.6803 3152	15.1404 4179	13
14	15.4503 8205	15.7095 3253	15.9739 3815	16.5189 5284	14
15	16.6821 3778	16.9844 4935	17.2934 1692	17.9319 2666	15
16	17.9323 6984	18.2816 7721	18.6392 8525	19.3802 2483	16
17	19.2013 5539	19.6016 0656	20.0120 7096	20.8647 3045	17
18	20.4893 7572	20.9446 3468	21.4123 1238	22.3863 4871	18
19	21.7967 1636	22.3111 6578	22.8405 5863	23.9460 0743	19
20	23.1236 6710	23.7016 1119	24.2973 6980	25.5446 5761	20
21	24.4705 2211	25.1163 8938	25.7833 1719	27.1832 7405	21
22	25.8375 7994	26.5559 2620	27.2989 8354	28.8628 5590	22
23	27.2251 4364	28.0206 5490	28.8449 6321	30.5844 2730	23
24	28.6335 2080	29.5110 1637	30.4218 6247	32.3490 3798	24
25	30.0630 2361	31.0274 5915	32.0302 9972	34.1577 6393	25
26	31.5139 6896	32.5704 3969	33.6709 0572	36.0117 0803	26
27	32.9866 7850	34.1404 2238	35.3443 2383	37.9120 0073	27
28	34.4814 7867	35.7378 7977	37.0512 1031	39.8598 0075	28
29	35.9987 0085	37.3632 9267	38.7922 3451	41.8562 9577	29
30	37.5386 8137	39.0171 5029	40.5680 7921	43.9027 0316	30
31	39.1017 6159	40.6999 5042	42.3794 4079	46.0002 7074	31
32	40.6882 8801	42.4121 9955	44.2270 2961	48.1502 7751	32
33	42.2986 1233	44.1544 1305	46.1115 7020	50.3540 3445	33
34	43.9330 9152	45.9271 1527	48.0338 0160	52.6128 8531	34
35	45.5920 8789	47.7308 3979	49.9944 7763	54.9282 0744	35
36	47.2759 6921	49.5661 2949	51.9943 6719	57.3014 1263	36
37	48.9851 0874	51.4335 3675	54.0342 5453	59.7339 4794	37
38	50.7198 8538	53.3336 2365	56.1149 3962	62.2272 9664	38
39	52.4806 8366	55.2669 6206	58.2372 3841	64.7829 7906	39
40	54.2678 9391	57.2341 3390	60.4019 8318	67.4025 5354	40
41	56.0819 1232	59.2357 3124	62.6100 2284	70.0876 1737	41
42	57.9231 4100	61.2723 5654	64.8622 2330	72.8398 0781	42
43	59.7919 8812	63.3446 2278	67.1594 6777	75.6608 0300	43
44	61.6888 6794	65.4531 5367	69.5026 5712	78.5523 2308	44
45	63.6142 0096	67.5985 8386	71.8927 1027	81.5161 3116	45
46	65.5684 1398	69.7815 5908	74.3305 6447	84.5540 3443	46
47	67.5519 4018	72.0027 3637	76.8171 7576	87.6678 8530	47
48	69.5652 1929	74.2627 8425	79.3535 1927	90.8595 8243	48
49	71.6086 9758	76.5623 8298	81.9405 8966	94.1310 7199	49
50	73.6828 2804	78.9022 2468	84.5794 0145	97.4843 4879	50

TABLE 6. $\log s_{\bar{m}i}$

n	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	2%	$2\frac{1}{2}\%$	n
1	0.000 0000	0.000 0000	0.000 0000	0.000 0000	1
2	0.304 2751	0.304 8135	0.305 3514	0.306 4250	2
3	0.483 6194	0.484 6993	0.485 7782	0.487 9334	3
4	0.611 8193	0.613 4433	0.615 0666	0.618 3112	4
5	0.711 9983	0.714 1695	0.716 3407	0.720 6825	5
6	0.794 4568	0.797 1779	0.799 9000	0.805 3470	6
7	0.864 6887	0.867 9627	0.871 2391	0.877 7991	7
8	0.925 9738	0.929 8036	0.933 6376	0.941 3184	8
9	0.980 4275	0.984 8157	0.989 2107	0.998 0203	9
10	1.029 4943	1.034 4439	1.039 4030	1.049 3491	10
11	1.074 2041	1.079 7182	1.085 2447	1.096 3352	11
12	1.115 3179	1.121 3991	1.127 4964	1.139 7391	12
13	1.153 4133	1.160 0644	1.166 7359	1.180 1386	13
14	1.188 9392	1.196 1633	1.203 4120	1.217 9825	14
15	1.222 2517	1.230 0515	1.237 8808	1.253 6269	15
16	1.253 6377	1.262 0160	1.270 4292	1.287 3588	16
17	1.283 3319	1.292 2917	1.301 2920	1.319 4128	17
18	1.311 5288	1.321 0728	1.330 6635	1.349 9833	18
19	1.338 3910	1.348 5222	1.358 7067	1.379 2331	19
20	1.364 0567	1.374 7779	1.385 5593	1.407 3001	20
21	1.388 6432	1.399 9572	1.411 3388	1.434 3018	21
22	1.412 2518	1.424 1615	1.436 1465	1.460 3393	22
23	1.434 9702	1.447 4783	1.460 0699	1.485 5004	23
24	1.456 8748	1.469 9841	1.483 1858	1.509 8614	24
25	1.478 0326	1.491 7462	1.505 5610	1.533 4894	25
26	1.498 5031	1.512 8237	1.527 2548	1.556 4437	26
27	1.518 3387	1.533 2689	1.548 3197	1.578 7767	27
28	1.537 5859	1.553 1287	1.568 8024	1.600 5351	28
29	1.556 2868	1.572 4451	1.588 7448	1.621 7608	29
30	1.574 4790	1.591 2556	1.60 1844	1.642 4913	30
31	1.592 1964	1.609 5939	1.627 1552	1.662 7604	31
32	1.609 4694	1.627 4908	1.645 6878	1.682 5988	32
33	1.626 3261	1.644 9741	1.663 8099	1.702 0343	33
34	1.642 7918	1.662 0692	1.681 5470	1.721 0921	34
35	1.658 8895	1.678 7990	1.698 9221	1.739 7954	35
36	1.674 6404	1.695 1850	1.715 9563	1.758 1653	36
37	1.690 0641	1.711 2464	1.732 6692	1.776 2212	37
38	1.705 1782	1.727 0011	1.749 0785	1.793 9810	38
39	1.719 9995	1.742 4656	1.765 2008	1.811 4610	39
40	1.734 5430	1.757 6552	1.781 0512	1.828 6763	40
41	1.748 8228	1.772 5837	1.796 6439	1.845 6413	41
42	1.762 8521	1.787 2646	1.811 9918	1.862 3688	42
43	1.776 6430	1.801 7097	1.827 1072	1.878 8710	43
44	1.790 2068	1.815 9306	1.842 0014	1.895 1590	44
45	1.803 5541	1.829 9376	1.856 6849	1.911 2436	45
46	1.816 6947	1.843 7407	1.871 1674	1.927 1344	46
47	1.829 6378	1.857 3490	1.885 4583	1.942 8405	47
48	1.842 3922	1.870 7712	1.899 5662	1.958 3707	48
49	1.854 9658	1.884 0154	1.913 4991	1.973 7330	49
50	1.867 3663	1.897 0893	1.927 2646	1.988 9349	50

TABLE 6. $s_{\overline{ni}} = \frac{(1+i)^n - 1}{i}$

n	1½%	1¾%	2%	2½%	n
51	75.7880 7046	81.2830 1361	87.2709 8948	100.9214 5751	51
52	77.9248 9152	83.7054 6635	90.0164 0927	104.4444 9395	52
53	80.0937 6489	86.1703 1201	92.8167 3746	108.0556 0629	53
54	82.2951 7136	88.6782 9247	95.6730 7221	111.7569 9645	54
55	84.5295 9893	91.2301 6259	98.5865 3363	115.5509 2136	55
56	86.7975 4292	93.8266 9043	101.5582 6432	119.4396 9440	56
57	89.0995 0606	96.4686 5752	104.5894 2961	123.4256 8676	57
58	91.4359 9865	99.1568 5902	107.6812 1820	127.5113 2893	58
59	93.8075 3863	101.8921 0405	110.8348 4257	131.6991 1215	59
60	96.2146 5171	104.6752 1588	114.0515 3942	135.9915 8995	60
61	98.6578 7149	107.5070 3215	117.3325 7021	140.3913 7970	61
62	101.1377 3956	110.3884 0522	120.6792 2161	144.9011 6419	62
63	103.6548 0565	113.3202 0231	124.0928 0604	149.5236 9330	63
64	106.2096 2774	116.3033 0585	127.5746 6216	154.2617 8563	64
65	108.8027 7215	119.3386 1370	131.1261 5541	159.1183 3027	65
66	111.4348 1374	122.4270 3944	134.7486 7852	164.0962 8853	66
67	114.1063 3594	125.5695 1263	138.4436 5209	169.1986 9574	67
68	116.8179 3098	128.7669 7910	142.2125 2513	174.4286 6314	68
69	119.5701 9995	132.0204 0124	146.0567 7563	179.7893 7971	69
70	122.3637 5295	135.3307 5826	149.9779 1114	185.2841 1421	70
71	125.1992 0924	138.6990 4653	153.9774 6937	190.9162 1706	71
72	128.0771 9738	142.1262 7984	158.0570 1875	196.6891 2249	72
73	130.9983 5534	145.6134 8974	162.2181 5913	202.6063 5055	73
74	133.9633 3067	149.1617 2581	166.4625 2231	208.6715 0931	74
75	136.9727 8063	152.7720 5601	170.7917 7276	214.8882 9705	75
76	140.0273 7234	156.4455 6699	175.2076 0821	221.2605 0447	76
77	143.1277 8292	160.1833 6441	179.7117 6038	227.7920 1709	77
78	146.2746 9967	163.9865 7329	184.3059 9558	234.4868 1751	78
79	149.4688 2016	167.8563 3832	188.9921 1549	241.3489 8795	79
80	152.7108 5247	171.7938 2424	193.7719 5780	248.3827 1265	80
81	156.0015 1525	175.8002 1617	198.6473 9696	255.5922 8047	81
82	159.3415 3798	179.8767 1995	203.6203 4490	262.9820 8748	82
83	162.7316 6105	184.0245 6255	208.6927 5180	270.5566 3966	83
84	166.1726 3597	188.2449 9239	213.8666 0683	278.2205 5566	84
85	169.6652 2551	192.5392 7976	219.1439 3897	286.2785 6955	85
86	173.2102 0389	196.9087 1716	224.5268 1775	294.4355 3379	86
87	176.8083 5695	201.3546 1971	230.0173 5411	302.7964 2213	87
88	180.4604 8230	205.8783 2555	235.6177 0119	311.3663 3268	88
89	184.1673 8954	210.4811 9625	241.3300 5521	320.1504 9100	89
90	187.9299 0038	215.1646 1718	247.1566 5632	329.1542 5328	90
91	191.7488 4889	219.9299 9798	253.0997 8944	338.3831 0961	91
92	195.6250 8162	224.7787 7295	259.1617 8523	347.8426 8735	92
93	199.5594 5784	229.7124 0148	265.3450 2094	357.5387 5453	93
94	203.5528 4971	234.7323 6850	271.6519 2135	367.4772 2339	94
95	207.6061 4246	239.8401 8495	278.0849 5978	377.6641 5398	95
96	211.7202 3459	245.0373 8819	284.6466 5898	388.1057 5783	96
97	215.8960 3811	250.3255 4248	291.3395 9216	398.8084 0177	97
98	220.1344 7868	255.7062 3947	298.1663 8400	409.7786 1182	98
99	224.4364 9586	261.1810 9866	305.1297 1168	421.0230 7711	99
100	228.8030 4330	266.7517 6789	312.2323 0591	432.5486 5404	100

TABLE 6. $\log s_{\bar{n}i}$

n	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	2%	$2\frac{1}{2}\%$	n
51	1.879 6008	1.909 9997	1.940 8699	2.003 9836	51
52	1.891 6761	1.922 7538	1.954 3216	2.018 8855	52
53	1.903 5987	1.935 3576	1.967 6262	2.033 6473	53
54	1.915 3743	1.947 8173	1.980 7897	2.048 2747	54
55	1.927 0088	1.960 1384	1.993 8175	2.062 7734	55
56	1.938 5074	1.972 3263	2.006 7153	2.077 1487	56
57	1.949 8752	1.984 3862	2.019 4877	2.091 4055	57
58	1.961 1172	1.996 3227	2.032 1399	2.105 5487	58
59	1.972 2377	2.008 1405	2.044 6761	2.119 5828	59
60	1.983 2412	2.019 8438	2.057 1012	2.133 5121	60
61	1.994 1317	2.031 4367	2.069 4186	2.147 3405	61
62	2.004 9130	2.042 9234	2.081 6324	2.161 0720	62
63	2.015 5894	2.054 3073	2.093 7466	2.174 7100	63
64	2.026 1637	2.065 5920	2.105 7645	2.188 2583	64
65	2.036 6400	2.076 7809	2.117 6894	2.201 7201	65
66	2.047 0208	2.087 8773	2.129 5246	2.215 0987	66
67	2.057 3096	2.098 8841	2.141 2731	2.228 3970	67
68	2.067 5094	2.109 8045	2.152 9377	2.241 6177	68
69	2.077 6229	2.120 6410	2.164 5217	2.254 7640	69
70	2.087 6529	2.131 3966	2.176 0272	2.267 8381	70
71	2.097 6015	2.142 7033	2.187 4572	2.280 8427	71
72	2.107 4718	2.152 6744	2.198 8137	2.293 7803	72
73	2.117 2659	2.163 2016	2.210 0995	2.306 6531	73
74	2.126 9858	2.173 6573	2.221 3164	2.319 4631	74
75	2.136 6343	2.184 0440	2.232 4669	2.332 2127	75
76	2.146 2130	2.194 3633	2.243 5529	2.344 9038	76
77	2.155 7240	2.204 6174	2.254 5765	2.357 5385	77
78	2.165 1692	2.214 8083	2.265 5394	2.370 1183	78
79	2.174 5505	2.224 9376	2.276 4436	2.382 6455	79
80	2.183 8700	2.235 0075	2.287 2909	2.395 1213	80
81	2.193 1287	2.245 0193	2.298 0829	2.407 5477	81
82	2.202 3289	2.254 9749	2.308 8212	2.419 9261	82
83	2.211 4721	2.264 8758	2.319 5074	2.432 2581	83
84	2.220 5594	2.274 7234	2.330 1429	2.444 5453	84
85	2.229 5928	2.284 5194	2.340 7294	2.456 7888	85
86	2.238 5735	2.294 2649	2.351 2681	2.468 9901	86
87	2.247 5028	2.303 9616	2.361 7606	2.481 1507	87
88	2.256 3822	2.313 6106	2.372 2079	2.493 2716	88
89	2.265 2127	2.323 2132	2.382 6114	2.505 3541	89
90	2.273 9958	2.332 7707	2.392 9723	2.517 3995	90
91	2.282 7326	2.342 2845	2.403 2917	2.529 4086	91
92	2.291 4246	2.351 7553	2.413 5710	2.541 3828	92
93	2.300 0724	2.361 1844	2.423 8109	2.553 3231	93
94	2.308 6772	2.370 5730	2.434 0127	2.565 2303	94
95	2.317 2401	2.379 9219	2.444 1776	2.577 1058	95
96	2.325 7623	2.389 2323	2.454 3061	2.588 9500	96
97	2.334 2445	2.398 5051	2.464 3995	2.600 7642	97
98	2.342 6881	2.407 7413	2.474 4586	2.612 5493	98
99	2.351 0935	2.416 9417	2.484 4844	2.624 3059	99
100	2.359 4617	2.426 1073	2.494 4777	2.636 0350	100

TABLE 6. $s_{ni} = \frac{(1+i)^n - 1}{i}$

<i>n</i>	3%	3½%	4%	4½%	<i>n</i>
1	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	1
2	2.0300 0000	2.0350 0000	2.0400 0000	2.0450 0000	2
3	3.0909 0000	3.1062 2500	3.1216 0000	3.1370 2500	3
4	4.1836 2700	4.2149 4288	4.2464 6400	4.2781 9113	4
5	5.3091 3581	5.3624 6588	5.4163 2256	5.4707 0973	5
6	6.4684 0988	6.5501 5218	6.6329 7546	6.7168 9166	6
7	7.6624 6218	7.7794 0751	7.8982 9448	8.0191 5179	7
8	8.8923 3605	9.0516 8677	9.2142 2626	9.3800 1362	8
9	10.1591 0613	10.3684 9581	10.5827 9531	10.8021 1423	9
10	11.4638 7931	11.7313 9316	12.0061 0712	12.2882 0937	10
11	12.8077 9569	13.1419 9192	13.4863 5141	13.8411 7879	11
12	14.1920 2956	14.6019 6164	15.0258 0546	15.4640 3184	12
13	15.6177 9045	16.1130 3030	16.6268 3768	17.1599 1327	13
14	17.0863 2416	17.6769 8636	18.2919 1119	18.9321 0937	14
15	18.5989 1389	19.2956 8088	20.0235 8764	20.7840 5429	15
16	20.1568 8130	20.9710 2971	21.8245 3114	22.7193 3673	16
17	21.7615 8774	22.7050 1575	23.6975 1239	24.7417 0689	17
18	23.4144 3537	24.4996 9130	25.6454 1288	26.8550 8370	18
19	25.1168 6844	26.3571 8050	27.6712 2940	29.0635 6246	19
20	26.8703 7449	28.2796 8181	29.7780 7858	31.3714 2277	20
21	28.6764 8572	30.2694 7068	31.9692 0172	33.7831 3680	21
22	30.5367 8030	32.3289 0215	34.2479 6979	36.3033 7795	22
23	32.4528 8370	34.4604 1373	36.6178 8858	38.9370 2996	23
24	34.4264 7022	36.6665 2821	39.0826 0412	41.6891 9631	24
25	36.4592 6432	38.9498 5669	41.6459 0829	44.5652 1015	25
26	38.5530 4225	41.3131 0168	44.3117 4462	47.5706 4460	26
27	40.7096 3352	43.7590 6024	47.0842 1440	50.7113 2361	27
28	42.9309 2252	46.2906 2734	49.9675 8298	53.9933 3317	28
29	45.2188 5020	48.9107 9930	52.9662 8630	57.4230 3316	29
30	47.5754 1571	51.6226 7728	56.0849 3775	61.0070 6966	30
31	50.0026 7818	54.4294 7098	59.3283 3526	64.7523 8779	31
32	52.5027 5852	57.3345 0247	62.7014 6867	68.6662 4524	32
33	55.0778 4128	60.3412 1005	66.2095 2742	72.7562 2628	33
34	57.7301 7652	63.4531 5240	69.8579 0851	77.0302 5646	34
35	60.4620 8181	66.6740 1274	73.6522 2486	81.4966 1800	35
36	63.2759 4427	70.0076 0318	77.5983 1385	86.1639 6581	36
37	66.1742 2259	73.4578 6930	81.7022 4640	91.0413 4427	37
38	69.1594 4927	77.0288 9472	85.9703 3626	96.1382 0476	38
39	72.2342 3275	80.7249 0604	90.4091 4971	101.4644 2398	39
40	75.4012 5973	84.5502 7775	95.0255 1570	107.0303 2306	40
41	78.6632 9753	88.5095 3747	99.8265 3633	112.8466 8760	41
42	82.0231 9645	92.6073 7128	104.8195 9778	118.9247 8854	42
43	85.4838 9234	96.8486 2928	110.0123 8169	125.2764 0402	43
44	89.0484 0911	101.2383 3130	115.4128 7696	131.9138 4220	44
45	92.7198 6139	105.7816 7290	121.0293 9204	138.8499 6510	45
46	96.5014 5723	110.4840 3145	126.8705 6772	146.0982 1353	46
47	100.3965 0095	115.3509 7255	132.9453 9043	153.6726 3314	47
48	104.4083 9598	120.3882 5659	139.2632 0604	161.5879 0163	48
49	108.5406 4785	125.6018 4557	145.8337 3429	169.8593 5720	49
50	112.7968 6729	130.9979 1016	152.6670 8366	178.5030 2828	50

TABLE 6. $\log s_{\bar{m}i}$

n	3%	3½%	4%	4½%	n
1	0.000 0000	0.000 0000	0.000 0000	0.000 0000	1
2	0.307 4960	0.308 5644	0.309 6302	0.310 6933	2
3	0.490 0850	0.492 2329	0.494 3773	0.496 5180	3
4	0.621 5530	0.624 7917	0.628 0274	0.631 2602	4
5	0.725 0238	0.729 3645	0.733 7045	0.738 0437	5
6	0.810 7976	0.816 2513	0.821 7084	0.827 1684	6
7	0.884 3683	0.890 9465	0.897 5333	0.904 1284	7
8	0.949 0159	0.956 7296	0.964 4588	0.972 2034	8
9	1.006 8555	1.015 7158	1.024 6004	1.033 5088	9
10	1.059 3316	1.069 3496	1.079 4022	1.089 4886	10
11	1.107 4744	1.118 6612	1.129 8945	1.141 1731	11
12	1.152 0445	1.164 4112	1.176 8378	1.189 3227	12
13	1.193 6196	1.207 1772	1.220 8097	1.234 5151	13
14	1.232 6486	1.247 4083	1.262 2591	1.277 1990	14
15	1.269 4876	1.285 4601	1.301 5419	1.317 7302	15
16	1.304 4233	1.321 6197	1.338 9450	1.356 3956	16
17	1.337 6906	1.356 1218	1.374 7028	1.393 4296	17
18	1.369 4837	1.389 1606	1.409 0097	1.429 0265	18
19	1.399 9655	1.420 8990	1.442 0285	1.463 3488	19
20	1.429 2737	1.451 4745	1.473 8966	1.496 5342	20
21	1.457 5259	1.481 0048	1.504 7318	1.528 7000	21
22	1.484 8233	1.509 5910	1.534 6348	1.559 9470	22
23	1.511 2533	1.537 3205	1.563 6933	1.590 3628	23
24	1.536 8925	1.564 2698	1.591 9835	1.620 0235	24
25	1.561 8079	1.590 5059	1.619 5723	1.648 9960	25
26	1.586 0587	1.616 0878	1.646 5189	1.677 3391	26
27	1.609 6972	1.641 0680	1.672 8754	1.705 1050	27
28	1.632 7702	1.665 4931	1.698 6883	1.732 3402	28
29	1.655 3195	1.689 4047	1.723 9995	1.759 0861	29
30	1.677 3826	1.712 8405	1.748 8462	1.785 3802	30
31	1.698 9934	1.735 8342	1.773 2621	1.811 2558	31
32	1.720 1821	1.758 4160	1.797 2777	1.836 7433	32
33	1.740 9769	1.780 6141	1.820 9205	1.861 8702	33
34	1.761 4029	1.802 4532	1.844 2155	1.886 6613	34
35	1.781 4831	1.823 9566	1.867 1858	1.911 1396	35
36	1.801 2386	1.845 1452	1.889 8523	1.935 3257	36
37	1.820 6888	1.866 0383	1.912 2340	1.959 2387	37
38	1.839 8515	1.886 6537	1.934 3486	1.982 8960	38
39	1.858 7431	1.907 0076	1.956 2124	2.006 3138	39
40	1.877 3786	1.927 1150	1.977 8402	2.029 5068	40
41	1.895 7721	1.946 9900	1.999 2460	2.052 4889	41
42	1.913 9366	1.966 6455	2.020 4425	2.075 2724	42
43	1.931 8843	1.986 0935	2.041 4416	2.097 8692	43
44	1.949 6262	2.005 3449	2.062 2543	2.120 2903	44
45	1.967 1727	2.024 4104	2.082 8908	2.142 5458	45
46	1.984 5339	2.043 2994	2.103 3608	2.164 6449	46
47	2.001 7186	2.062 0213	2.123 6732	2.186 5965	47
48	2.018 7354	2.080 5841	2.143 8364	2.208 4089	48
49	2.035 5924	2.098 9960	2.163 8580	2.230 0895	49
50	2.052 2970	2.117 2644	2.183 7454	2.251 6456	50

TABLE 6. $s_{ni} = \frac{(1+i)^n - 1}{i}$

<i>n</i>	3%	3½%	4%	4½%	<i>n</i>
51	117.1807 7331	136.5828 3702	159.7737 6700	187.5356 6455	51
52	121.6961 9651	142.3632 3631	167.1647 1768	196.9747 6946	52
53	126.3470 8240	148.3459 4958	174.8513 0639	206.8386 3408	53
54	131.1374 9488	154.5380 5782	182.8453 5865	217.1463 7262	54
55	136.0716 1972	160.9468 8984	191.1591 7299	227.9179 5938	55
56	141.1537 6831	167.5800 3099	199.8055 3991	239.1742 6756	56
57	146.3883 8136	174.4453 3207	208.7977 6151	250.9371 0960	57
58	151.7800 3280	181.5509 1869	218.1496 7197	263.2292 7953	58
59	157.3334 3379	188.9052 0085	227.8756 5885	276.0745 9711	59
60	163.0534 3680	196.5168 8288	237.9906 8520	289.4979 5398	60
61	168.9450 3991	204.3949 7378	248.5103 1261	303.5253 6190	61
62	175.0133 9110	212.5487 9786	259.4507 2511	318.1840 0319	62
63	181.2637 9284	220.9880 0579	270.8287 5412	333.5022 8333	63
64	187.7017 0662	229.7225 8599	282.6619 0428	349.5098 8608	64
65	194.3327 5782	238.7628 7650	294.9683 8045	366.2378 3096	65
66	201.1627 4055	248.1195 7718	307.7671 1567	383.7185 3335	66
67	208.1976 2277	257.8037 6238	321.0778 0030	401.9858 6735	67
68	215.4435 5145	267.8268 9406	334.9209 1231	421.0752 3138	68
69	222.9068 5800	278.2008 3535	349.3177 4880	441.0236 1679	69
70	230.5940 6374	288.9378 6459	364.2904 5876	461.8696 7955	70
71	238.5118 8565	300.0506 8985	379.8620 7711	483.6538 1513	71
72	246.6672 4222	311.5524 6400	396.0565 6019	506.4182 3681	72
73	255.0672 5949	323.4568 0024	412.8988 2260	530.2070 5747	73
74	263.7192 7727	335.7777 8824	430.4147 7550	555.0663 7505	74
75	272.6308 5559	348.5300 1083	448.6313 6652	581.0443 6193	75
76	281.8097 8126	361.7285 6121	467.5766 2118	608.1913 5822	76
77	291.2640 7469	375.3890 6085	487.2796 8603	636.5599 6934	77
78	301.0019 9693	389.5276 7798	507.7708 7347	666.2051 6796	78
79	311.0320 5684	404.1611 4671	529.0817 0841	697.1844 0052	79
80	321.3630 1855	419.3067 8685	551.2449 7675	729.5576 9854	80
81	332.0039 0910	434.9825 2439	574.2947 7582	763.3877 9497	81
82	342.9640 2638	451.2069 1274	598.2665 6685	798.7402 4575	82
83	354.2529 4717	467.9991 5469	623.1972 2952	835.6835 5680	83
84	365.8805 3558	485.3791 2510	649.1251 1870	874.2893 1686	84
85	377.8569 5165	503.3673 9448	676.0901 2345	914.6323 3612	85
86	390.1926 6020	521.9852 5329	704.1337 2839	956.7907 9125	86
87	402.8984 4001	541.2547 3715	733.2990 7753	1000.8463 7685	87
88	415.9853 9321	561.1986 5295	763.6310 4063	1046.8844 6381	88
89	429.4649 5500	581.8406 0581	795.1762 8225	1094.9942 6468	89
90	443.3489 0365	603.2050 2701	827.9833 3354	1145.2690 0659	90
91	457.6493 7076	625.3172 0295	862.1026 6688	1197.8061 1189	91
92	472.3788 5189	648.2033 0506	897.5867 7356	1252.7073 8692	92
93	487.5502 1744	671.8904 2073	934.4902 4450	1310.0792 1933	93
94	503.1767 2397	696.4065 8546	972.8698 5428	1370.0327 8420	94
95	519.2720 2569	721.7808 1595	1012.7846 4845	1432.6842 5949	95
96	535.8501 8645	748.0431 4451	1054.2960 3439	1498.1550 5117	96
97	552.9256 9205	775.2246 5457	1097.4678 7577	1566.5720 2847	97
98	570.5134 6281	803.3575 1748	1142.3665 9080	1638.0677 6976	98
99	588.6288 6669	832.4750 3059	1189.0612 5443	1712.7808 1939	99
100	607.2877 3270	862.6116 5666	1237.6237 0461	1790.8559 5627	100

TABLE 6. $\log s_{\bar{m}i}$

n	3%	3½%	4%	4½%	n
51	2.068 8564	2.135 3960	2.203 5055	2.273 0839	51
52	2.085 2770	2.153 3977	2.223 1445	2.294 4106	52
53	2.101 5652	2.171 2755	2.242 6688	2.315 6315	53
54	2.117 7269	2.189 0355	2.262 0840	2.336 7526	54
55	2.133 7676	2.206 6826	2.281 3951	2.357 7786	55
56	2.149 6925	2.224 2222	2.300 6074	2.378 7145	56
57	2.165 5066	2.241 6592	2.319 7259	2.399 5648	57
58	2.181 2145	2.258 9984	2.338 7546	2.420 3342	58
59	2.196 8210	2.276 2439	2.357 6980	2.441 0264	59
60	2.212 3298	2.293 3999	2.376 5599	2.461 6456	60
61	2.227 7453	2.310 4702	2.395 3444	2.482 1950	61
62	2.243 0713	2.327 4586	2.414 0548	2.502 6783	62
63	2.258 3110	2.344 3686	2.432 6948	2.523 0988	63
64	2.273 4682	2.361 2037	2.451 2673	2.543 4594	64
65	2.288 5461	2.377 9668	2.469 7754	2.563 7631	65
66	2.303 5474	2.394 6611	2.488 2222	2.584 0127	66
67	2.318 4757	2.411 2893	2.506 6103	2.604 2107	67
68	2.333 3335	2.427 8541	2.524 9422	2.624 3596	68
69	2.348 1235	2.444 3583	2.543 2206	2.644 4618	69
70	2.362 8482	2.460 8045	2.561 4478	2.664 5195	70
71	2.377 5100	2.477 1946	2.579 6260	2.684 5346	71
72	2.392 1114	2.493 5312	2.597 7572	2.704 5093	72
73	2.406 6548	2.509 8163	2.615 8436	2.724 4455	73
74	2.421 1419	2.526 0519	2.633 8872	2.744 3449	74
75	2.435 5750	2.542 2402	2.651 8896	2.764 2093	75
76	2.449 9560	2.558 3828	2.669 8527	2.784 0402	76
77	2.464 2869	2.574 4816	2.687 7783	2.803 8393	77
78	2.478 5693	2.590 5383	2.705 6677	2.823 6079	78
79	2.492 8052	2.606 5544	2.723 5227	2.843 3476	79
80	2.506 9958	2.622 5319	2.741 3446	2.863 0596	80
81	2.521 1432	2.638 4718	2.759 1349	2.882 7452	81
82	2.535 2485	2.654 3757	2.776 8947	2.902 4055	82
83	2.549 3134	2.670 2451	2.794 6254	2.922 0418	83
84	2.563 3392	2.686 0810	2.812 3284	2.941 6551	84
85	2.577 3274	2.701 8850	2.830 0045	2.961 2465	85
86	2.591 2791	2.717 6582	2.847 6550	2.980 8169	86
87	2.605 1955	2.733 4016	2.865 2811	3.000 3674	87
88	2.619 0781	2.749 1166	2.882 8835	3.019 8987	88
89	2.632 9277	2.764 8040	2.900 4634	3.039 4118	89
90	2.646 7456	2.780 4649	2.918 0215	3.058 9075	90
91	2.660 5329	2.796 1003	2.935 5589	3.078 3865	91
92	2.674 2904	2.811 7112	2.953 0764	3.097 8497	92
93	2.688 0193	2.827 2984	2.970 5747	3.117 2975	93
94	2.701 7205	2.842 8628	2.988 0547	3.136 7310	94
95	2.715 3948	2.858 4052	3.005 5171	3.156 1505	95
96	2.729 0433	2.873 9265	3.022 9625	3.175 5567	96
97	2.742 6668	2.889 4276	3.040 3918	3.194 9503	97
98	2.756 2659	2.904 9088	3.057 8055	3.214 3319	98
99	2.769 8415	2.920 3712	3.075 2042	3.233 7018	99
100	2.783 3944	2.935 8153	3.092 5886	3.253 0606	100

TABLE 6. $s_{\overline{m}|i} = \frac{(1+i)^n - 1}{i}$

<i>n</i>	5%	5½%	6%	7%	<i>n</i>
1	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	1
2	2.0500 0000	2.0550 0000	2.0600 0000	2.0700 0000	2
3	3.1525 0000	3.1680 2500	3.1836 0000	3.2149 0000	3
4	4.3101 2500	4.3422 6638	4.3746 1600	4.4399 4300	4
5	5.5256 3125	5.5810 9103	5.6370 9296	5.7507 3901	5
6	6.8019 1281	6.8880 5103	6.9753 1854	7.1532 9074	6
7	8.1420 0845	8.2668 9384	8.3938 3765	8.6540 2109	7
8	9.5491 0888	9.7215 7300	9.8974 6791	10.2598 0257	8
9	11.0265 6432	11.2562 5951	11.4913 1598	11.9779 8875	9
10	12.5778 9254	12.8753 5379	13.1807 9494	13.8164 4796	10
11	14.2067 8716	14.5834 9825	14.9716 4264	15.7835 9932	11
12	15.9171 2652	16.3855 9065	16.8699 4120	17.8884 5127	12
13	17.7129 8285	18.2867 9814	18.8821 3767	20.1406 4286	13
14	19.5986 3199	20.2925 7203	21.0150 6593	22.5504 8786	14
15	21.5785 6359	22.4086 6350	23.2759 6988	25.1290 2201	15
16	23.6574 9177	24.6411 3999	25.6725 2808	27.8880 5355	16
17	25.8403 6636	26.9964 0269	28.2128 7976	30.8402 1730	17
18	28.1323 8467	29.4812 0483	30.9056 5255	33.9990 3251	18
19	30.5390 0391	32.1026 7110	33.7599 9170	37.3789 6479	19
20	33.0659 5410	34.8683 1801	36.7855 9120	40.9954 9232	20
21	35.7192 5181	37.7860 7550	39.9927 2668	44.8651 7678	21
22	38.5052 1440	40.8643 0965	43.3922 9028	49.0057 3916	22
23	41.4304 7512	44.1118 4669	46.9958 2769	53.4361 4090	23
24	44.5019 9887	47.5379 9825	50.8155 7735	58.1766 7076	24
25	47.7270 9882	51.1525 8816	54.8645 1200	63.2490 3772	25
26	51.1134 5376	54.9659 8051	59.1563 8272	68.6764 7036	26
27	54.6691 2645	58.9891 0943	63.7057 6568	74.4838 2328	27
28	58.4025 8277	63.2335 1045	68.5281 1162	80.6976 9091	28
29	62.3227 1191	67.7113 5353	73.6397 9832	87.3465 2927	29
30	66.4388 4750	72.4354 7797	79.0581 8622	94.4607 8632	30
31	70.7607 8988	77.4194 2926	84.8016 7739	102.0730 4137	31
32	75.2988 2937	82.6774 9787	90.8897 7803	110.2181 5426	32
33	80.0637 7084	88.2247 6025	97.3431 6471	118.9334 2506	33
34	85.0669 5938	94.0771 2207	104.1837 5460	128.2587 6481	34
35	90.3203 0735	100.2513 6378	111.4347 7987	138.2368 7835	35
36	95.8363 2272	106.7651 8879	119.1208 6666	148.9134 5984	36
37	101.6281 3886	113.6372 7417	127.2681 1866	160.3374 0202	37
38	107.7095 4580	120.8873 2425	135.9042 0578	172.5610 2017	38
39	114.0950 2309	128.5361 2708	145.0584 5813	185.6402 9158	39
40	120.7997 7424	136.6056 1407	154.7619 6562	199.6351 1199	40
41	127.8397 6295	145.1189 2285	165.0476 8356	214.6095 6983	41
42	135.2317 5110	154.1004 6360	175.9505 4457	230.6322 3972	42
43	142.9933 3866	163.5759 8910	187.5075 7724	247.7764 9650	43
44	151.1430 0559	173.5726 6850	199.7580 3188	266.1208 5125	44
45	159.7001 5587	184.1191 6527	212.7435 1379	285.7493 1084	45
46	168.6851 6366	195.2457 1936	226.5081 2462	306.7517 6260	46
47	178.1194 2185	206.9842 3392	241.0986 1210	329.2243 8598	47
48	188.0253 9294	219.3683 6679	256.5645 2882	353.2700 9300	48
49	198.4266 6259	232.4336 2696	272.9584 0055	378.9989 9951	49
50	209.3479 9572	246.2174 7645	290.3359 0458	406.5289 2947	50

TABLE 6. $\log s_{\bar{m}i}$

n	5%	5½%	6%	7%	n
1	0.000 0000	0.000 0000	0.000 0000	0.000 0000	1
2	0.311 7539	0.312 8118	0.313 8672	0.315 9703	2
3	0.498 6551	0.500 7886	0.502 9185	0.507 1675	3
4	0.634 4898	0.637 7164	0.640 9399	0.647 3774	4
5	0.742 3819	0.746 7191	0.751 0552	0.759 7236	5
6	0.832 6310	0.838 0963	0.843 5640	0.854 5058	6
7	0.910 7316	0.917 3424	0.923 9606	0.937 2180	7
8	0.979 9628	0.987 7365	0.995 5241	1.011 1390	8
9	1.042 4402	1.051 3941	1.060 3697	1.078 3839	9
10	1.099 6079	1.109 7592	1.119 9416	1.140 3964	10
11	1.152 4959	1.163 8617	1.175 2694	1.198 2061	11
12	1.201 8647	1.214 4621	1.227 1136	1.252 5728	12
13	1.248 2917	1.262 1377	1.276 0512	1.304 0733	13
14	1.292 2258	1.307 3371	1.322 5308	1.353 1559	14
15	1.334 0226	1.350 4160	1.366 9077	1.400 1756	15
16	1.373 9687	1.391 6608	1.409 4686	1.445 4182	16
17	1.412 2959	1.431 3059	1.450 4475	1.489 1175	17
18	1.449 2065	1.469 5452	1.490 0379	1.531 4665	18
19	1.484 8549	1.506 5412	1.528 4023	1.572 6273	19
20	1.519 3811	1.542 4310	1.565 6778	1.612 7361	20
21	1.552 9024	1.577 3318	1.601 9810	1.651 9094	21
22	1.585 5195	1.611 3442	1.637 4126	1.690 2470	22
23	1.617 3199	1.644 5552	1.672 0593	1.727 8351	23
24	1.648 3795	1.677 0409	1.705 9969	1.764 7489	24
25	1.678 7650	1.708 8676	1.739 2915	1.801 0539	25
26	1.708 5353	1.740 0939	1.772 0016	1.836 8080	26
27	1.737 7421	1.770 7718	1.804 1787	1.872 0620	27
28	1.766 4321	1.800 9473	1.835 8688	1.906 8611	28
29	1.794 6464	1.830 6615	1.867 1126	1.941 2456	29
30	1.822 4221	1.859 9514	1.897 9468	1.975 2515	30
31	1.849 7927	1.888 8500	1.928 4045	2.008 9111	31
32	1.876 7882	1.917 3873	1.958 5150	2.042 2531	32
33	1.903 4361	1.945 5905	1.988 3054	2.075 3039	33
34	1.929 7609	1.973 4841	2.017 8000	2.108 0870	34
35	1.955 7854	2.001 0903	2.047 0207	2.140 6239	35
36	1.981 5302	2.028 4297	2.075 9879	2.172 9340	36
37	2.007 0140	2.055 5207	2.104 7196	2.205 0349	37
38	2.032 2544	2.082 3808	2.133 2329	2.236 9427	38
39	2.057 2667	2.109 0252	2.161 5431	2.268 6723	39
40	2.082 0661	2.135 4685	2.189 6642	2.300 2369	40
41	2.106 6660	2.161 7241	2.217 6094	2.331 6491	41
42	2.131 0786	2.187 8039	2.245 3906	2.362 9200	42
43	2.155 3158	2.213 7196	2.273 0188	2.394 0600	43
44	2.179 3880	2.239 4814	2.300 5042	2.425 0789	44
45	2.203 3053	2.265 0990	2.327 8563	2.455 9852	45
46	2.227 0769	2.290 5815	2.355 0838	2.486 7871	46
47	2.250 7113	2.315 9373	2.382 1947	2.517 4920	47
48	2.274 2165	2.341 1740	2.409 1966	2.548 1069	48
49	2.297 6000	2.366 2990	2.436 0965	2.578 6281	49
50	2.320 8688	2.391 3189	2.462 9008	2.609 0915	50

TABLE 6. $s_{\bar{n}i} = \frac{(1+i)^n - 1}{i}$

<i>n</i>	5%	5½%	6%	7%	<i>n</i>
51	220.8153 9550	260.7594 3765	308.7560 5886	435.9859 5454	51
52	232.8561 6528	276.1012 0672	328.2814 2239	457.5049 7135	52
53	245.4989 7354	292.2867 7309	348.9783 0773	501.2303 1935	53
54	258.7739 2222	309.3625 4561	370.9170 0620	537.3164 4170	54
55	272.7126 1833	327.3774 8562	394.1720 2657	575.9285 9262	55
56	287.3482 4924	346.3832 4733	418.8223 4816	617.2435 9410	56
57	302.7156 6171	366.4343 2593	444.9516 8905	661.4506 4569	57
58	318.8514 4479	387.5882 1386	472.6487 9040	708.7521 9089	58
59	335.7940 1703	409.9055 6562	502.0077 1782	759.3648 4425	59
60	353.5837 1788	433.4503 7173	533.1281 8089	813.5203 8335	60
61	372.2629 0378	458.2901 4217	566.1158 7174	871.4668 1019	61
62	391.8760 4897	484.4960 9999	601.0828 2405	933.4694 8690	62
63	412.4698 5141	512.1433 8549	638.1477 9349	999.8123 5098	63
64	434.0933 4398	541.3112 7170	677.4366, 6110	1070.7992 1555	64
65	456.7980 1118	572.0833 9164	719.0828 6076	1146.7551 6064	65
66	480.6379 1174	604.5479 7818	763.2278 3241	1228.0280 2188	66
67	505.6698 0733	638.7981 1698	810.0215 0236	1314.9899 8341	67
68	531.9532 9770	674.9320 1341	859.6227 9250	1408.0392 8225	68
69	559.5509 6258	713.0532 7415	912.2001 6005	1507.6020 3201	69
70	588.5285 1071	753.2712 0423	967.9321 6965	1614.1341 7425	70
71	618.9549 3625	795.7011 2046	1027.0080 9983	1728.1235 6645	71
72	650.9026 8306	840.4646 8209	1089.6285 8582	1850.0922 1610	72
73	684.4478 1721	887.6902 3960	1156.0063 0097	1980.5986 7123	73
74	719.6702 0807	937.5132 0278	1226.3666 7903	2120.2405 7821	74
75	756.6537 1848	990.0764 2893	1300.9486 7977	2269.6574 1869	75
76	795.4864 0440	1045.5306 3252	1380.0056 0055	2429.5334 3800	76
77	836.2607 2462	1104.0348 1731	1463.8059 3659	2600.6007 7866	77
78	879.0737 6085	1165.7567 3226	1552.6342 9278	2783.6428 3316	78
79	924.0274 4889	1230.8733 5254	1646.7923 5035	2979.4978 3148	79
80	971.2288 2134	1299.5713 8693	1746.5998 9137	3189.0626 7969	80
81	1020.7902 6240	1372.0478 1321	1852.3958 8485	3413.2970 6727	81
82	1072.8297 7552	1448.5104 4294	1964.5396 3794	3653.2278 6198	82
83	1127.4712 6430	1529.1785 1730	2083.4120 1622	3909.9538 1231	83
84	1184.8448 2752	1614.2833 3575	2209.4167 3719	4184.6505 7918	84
85	1245.0870 6889	1704.0689 1921	2342.9817 4142	4478.5761 1972	85
86	1308.3414 2234	1798.7927 0977	2484.5606 4391	4793.0764 4810	86
87	1374.7584 9345	1898.7263 0881	2634.6342 8466	5129.5917 9946	87
88	1444.4964 1812	2004.1562 5579	2793.7123 4174	5489.6632 2543	88
89	1517.7212 3903	2115.3848 4986	2962.3350 8225	5874.9396 5121	89
90	1594.6073 0098	2232.7310 1660	3141.0751 8718	6287.1854 2679	90
91	1675.3376 6603	2356.5312 2252	3330.5396 9841	6728.2884 0667	91
92	1760.1045 4933	2487.1404 3976	3531.3720 8032	7200.2685 9513	92
93	1849.1097 7680	2624.9331 6394	3744.2544 0514	7705.2873 9679	93
94	1942.5652 6564	2770.3044 8796	3969.9096 6944	8245.6575 1457	94
95	2040.6935 2892	2923.6712 3480	4209.1042 4961	8823.8535 4059	95
96	2143.7282 0537	3085.4731 5271	4462.6505 0459	9442.5232 8843	96
97	2251.9146 1564	3256.1741 7611	4731.4095 3486	10104.4999 1862	97
98	2365.5103 4642	3436.2637 5580	5016.2941 0696	10812.8149 1292	98
99	2484.7858 6374	3626.2582 6237	5318.2717 5337	11570.7119 5683	99
100	2610.0251 5693	3826.7024 6680	5638.3680 5857	12381.6617 9381	100

TABLE 6. $\log s_{\overline{m}i}$

n	5%	5½%	6%	7%	n
51	2.344 0293	2.416 2399	2.489 6155	2.639 4725	51
52	2.367 0877	2.441 0682	2.516 2462	2.669 7862	52
53	2.390 0497	2.465 8092	2.542 7983	2.700 0373	53
54	2.412 9205	2.490 4677	2.569 2767	2.730 2300	54
55	2.435 7052	2.515 0488	2.595 6858	2.760 3686	55
56	2.458 4084	2.539 5568	2.622 0297	2.790 4566	56
57	2.481 0349	2.563 9961	2.648 3128	2.820 4973	57
58	2.503 5883	2.588 3705	2.674 5385	2.850 4944	58
59	2.526 0729	2.612 6838	2.700 7103	2.880 4504	59
60	2.548 4922	2.636 9394	2.726 8316	2.910 3684	60
61	2.570 8497	2.661 1405	2.752 9053	2.940 2508	61
62	2.593 1486	2.685 2902	2.778 9343	2.970 1001	62
63	2.615 3922	2.709 3915	2.804 9213	2.999 9185	63
64	2.637 5831	2.733 4471	2.830 8686	3.029 7081	64
65	2.659 7242	2.757 4593	2.856 7789	3.059 4707	65
66	2.681 8180	2.781 4307	2.882 6541	3.089 2082	66
67	2.703 8670	2.805 3636	2.908 4965	3.118 9224	67
68	2.725 8735	2.829 2600	2.934 3079	3.148 6148	68
69	2.747 8396	2.853 1220	2.960 0901	3.178 2866	69
70	2.769 7674	2.876 9513	2.985 8449	3.207 9396	70
71	2.791 6590	2.900 7499	3.011 5737	2.237 5748	71
72	2.813 5160	2.924 5194	3.037 2785	3.267 1934	72
73	2.835 3403	2.948 2614	3.062 9602	3.296 7965	73
74	2.857 1335	2.971 9773	3.088 6203	3.326 3851	74
75	2.878 8971	2.995 6687	3.114 2602	3.355 9603	75
76	2.900 6328	3.019 3367	3.139 8809	3.385 5229	76
77	2.922 3416	3.042 9828	3.165 4835	3.415 0736	77
78	2.944 0253	3.066 6079	3.191 0692	3.444 6135	78
79	2.965 6848	3.090 2134	3.216 6389	3.474 1431	79
80	2.987 3215	3.113 8001	3.242 1935	3.503 6630	80
81	3.008 9366	3.137 3693	3.267 7338	3.533 1741	81
82	3.030 5308	3.160 9216	3.293 2608	3.562 6767	82
83	3.052 1055	3.184 4582	3.318 7751	3.592 1716	83
84	3.073 6614	3.207 9797	3.344 2776	3.621 6592	84
85	3.095 1997	3.231 4872	3.369 7689	3.651 1400	85
86	3.116 7210	3.254 9811	3.395 2496	3.680 6144	86
87	3.138 2264	3.278 4624	3.420 7204	3.710 0828	87
88	3.159 7164	3.301 9316	3.446 1817	3.739 5478	88
89	3.181 1920	3.325 3894	3.471 6342	3.769 0034	89
90	3.202 6538	3.348 8364	3.497 0784	3.798 4563	90
91	3.224 1024	3.372 2731	3.522 5146	3.827 9046	91
92	3.245 5384	3.395 7003	3.547 9435	3.857 3487	92
93	3.266 9627	3.419 1183	3.573 3653	3.886 7889	93
94	3.288 3756	3.442 5275	3.598 7807	3.916 2253	94
95	3.309 7778	3.465 9285	3.624 1896	3.945 6583	95
96	3.331 1697	3.489 3218	3.649 5929	3.975 0881	96
97	3.352 5519	3.512 7076	3.674 9906	4.004 5148	97
98	3.373 9249	3.536 0865	3.700 3830	4.033 9388	98
99	3.395 2889	3.559 4587	3.725 7705	4.063 3601	99
100	3.416 6447	3.582 8247	3.751 1534	4.092 7789	100

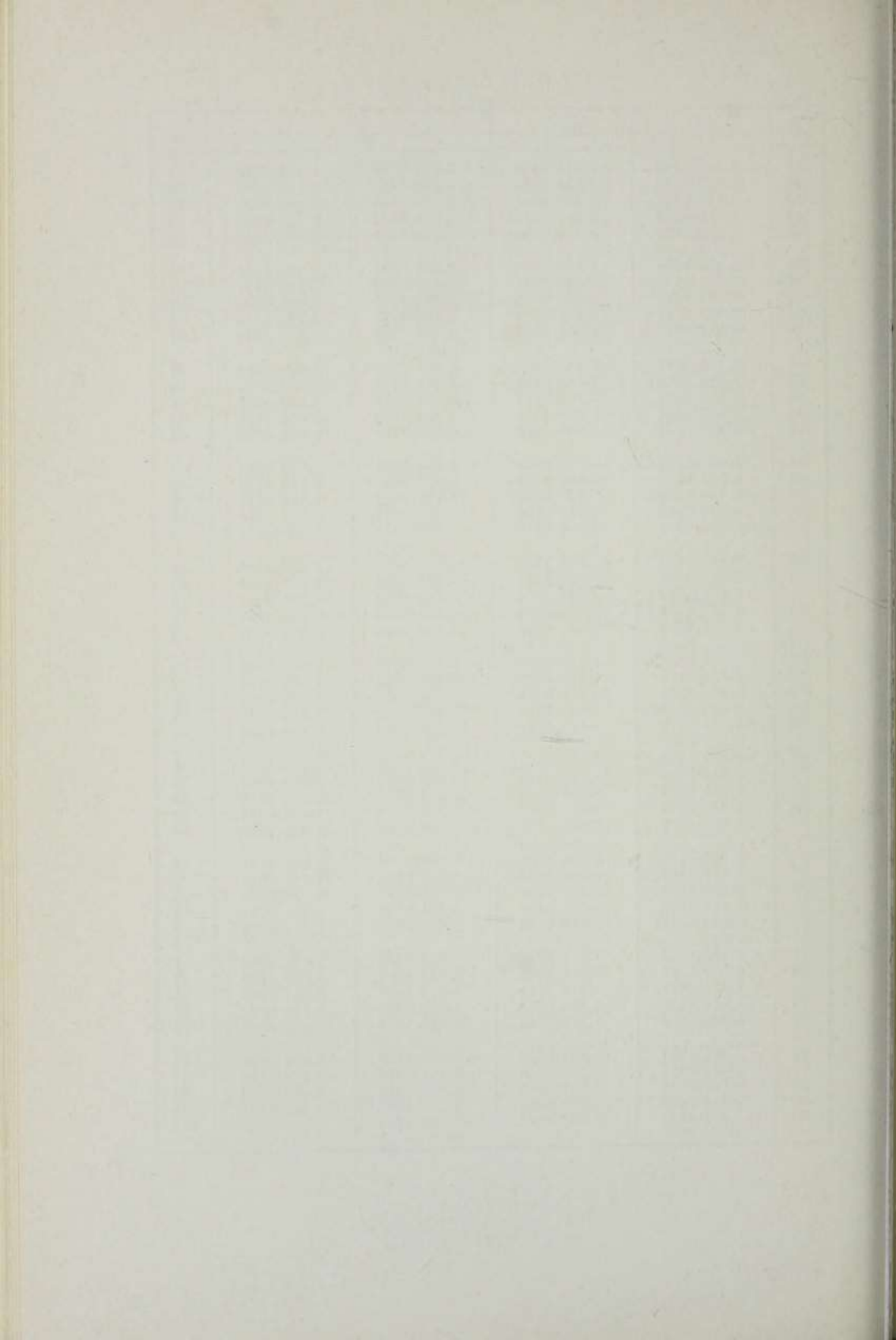


TABLE 7

$a_{\overline{m}i}$ and $\log a_{\overline{m}i}$

TABLE 7. $a_{ni} = \frac{1 - (1 + i)^{-n}}{i}$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
1	0.9975 0623	0.9966 7774	0.9950 2488	0.9933 7748	1
2	1.9925 2492	1.9900 4426	1.9850 9938	1.9801 7631	2
3	2.9850 6227	2.9801 1056	2.9702 4814	2.9604 4004	3
4	3.9751 2446	3.9668 8760	3.9504 9566	3.9342 1196	4
5	4.9627 1766	4.9503 8631	4.9258 6633	4.9015 3506	5
6	5.9478 4804	5.9306 1759	5.8963 8441	5.8624 5205	6
7	6.9305 2174	6.9075 9228	6.8620 7404	6.8170 0535	7
8	7.9107 4487	7.8813 2121	7.8229 5924	7.7652 3710	8
9	8.8885 2357	8.8518 1516	8.7790 6392	8.7071 8917	9
10	9.8638 6391	9.8190 8487	9.7304 1186	9.6429 0315	10
11	10.8367 7198	10.7831 4107	10.6770 2673	10.5724 2035	11
12	11.8072 5384	11.7439 9442	11.6189 3207	11.4957 8180	12
13	12.7753 1555	12.7016 5557	12.5561 5131	12.4130 2828	13
14	13.7409 6314	13.6561 3512	13.4887 0777	13.3242 0028	14
15	14.7042 0264	14.6074 4364	14.4166 2465	14.2293 3802	15
16	15.6650 4004	15.5555 9167	15.3399 2502	15.1284 8148	16
17	16.6234 8133	16.5005 8970	16.2586 3186	16.0216 7035	17
18	17.5795 3250	17.4424 4821	17.1727 6802	16.9089 4405	18
19	18.5331 9950	18.3811 7762	18.0823 5624	17.7903 4177	19
20	19.4844 8828	19.3167 8832	18.9874 1915	18.6659 0242	20
21	20.4334 0477	20.2492 9069	19.8879 7925	19.5356 6466	21
22	21.3799 5488	21.1786 9504	20.7840 5896	20.3996 6688	22
23	22.3241 4452	22.1050 1167	21.6756 8055	21.2579 4723	23
24	23.2659 7957	23.0282 5083	22.5628 6622	22.1105 4361	24
25	24.2054 6591	23.9484 2275	23.4456 5803	22.9574 9365	25
26	25.1426 0939	24.8655 3763	24.3240 1794	23.7988 3475	26
27	26.0774 1585	25.7796 0561	25.1980 2780	24.6346 0406	27
28	27.0098 9112	26.6906 3682	26.0676 8936	25.4648 3847	28
29	27.9400 4102	27.5986 4135	26.9330 2423	26.2895 7464	29
30	28.8678 7134	28.5036 2925	27.7940 5397	27.1088 4898	30
31	29.7933 8787	29.4056 1055	28.6507 9997	27.9226 9766	31
32	30.7165 9638	30.3045 9523	29.5032 8355	28.7311 5662	32
33	31.6375 0262	31.2005 9325	30.3515 2592	29.5342 6154	33
34	32.5561 1234	32.0936 1454	31.1955 4818	30.3320 4789	34
35	33.4724 3126	32.9836 6898	32.0353 7132	31.1245 5088	35
36	34.3864 6510	33.8707 6642	32.8710 1624	31.9118 0551	36
37	35.2982 1955	34.7549 1670	33.7025 0372	32.6938 4653	37
38	36.2077 0030	35.6361 2960	34.5298 5445	33.4707 0848	38
39	37.1149 1302	36.5144 1488	35.3530 8900	34.2424 2564	39
40	38.0198 6336	37.3897 8228	36.1722 2786	35.0090 3209	40
41	38.9225 5697	38.2622 4147	36.9872 9141	35.7705 6168	41
42	39.8229 9947	39.1318 0213	37.7982 9991	36.5270 4803	42
43	40.7211 9648	39.9984 7388	38.6052 7354	37.2785 2453	43
44	41.6171 5359	40.8622 6633	39.4082 3238	38.0250 2437	44
45	42.5108 7640	41.7231 8903	40.2071 9640	38.7665 8050	45
46	43.4023 7047	42.5812 5153	41.0021 8547	39.5032 2566	46
47	44.2916 4137	43.4364 6332	41.7932 1937	40.2349 9238	47
48	45.1786 9463	44.2888 3387	42.5803 1778	40.9619 1296	48
49	46.0635 3580	45.1383 7263	43.3635 0028	41.6840 1949	49
50	46.9461 7037	45.9850 8900	44.1427 8635	42.4013 4387	50

TABLE 7. $\log a_{\bar{n}i}$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
1	9.998 9156	9.998 5547	9.997 8339	9.997 1143	1
2	0.299 4037	0.298 8627	0.297 7823	0.296 7039	2
3	0.474 9534	0.474 2323	0.472 7927	0.471 3563	3
4	0.599 3508	0.598 4499	0.596 6517	0.594 8577	4
5	0.695 7196	0.694 6391	0.692 4826	0.690 3321	5
6	0.774 3599	0.773 0999	0.770 5858	0.768 0793	6
7	0.840 7659	0.839 3267	0.836 4554	0.833 5936	7
8	0.898 2174	0.896 5991	0.893 3711	0.890 1547	8
9	0.948 8297	0.947 0323	0.943 4482	0.939 8780	9
10	0.994 0471	0.992 0710	0.988 1312	0.984 2078	10
11	1.034 9000	1.032 7453	1.028 4503	1.024 1744	11
12	1.072 1489	1.069 8158	1.065 1663	1.060 5386	12
13	1.106 3716	1.103 8603	1.098 8565	1.093 8778	13
14	1.138 0171	1.135 3278	1.129 9704	1.124 6411	14
15	1.167 4415	1.164 5742	1.158 8636	1.153 1847	15
16	1.194 9315	1.191 8865	1.185 8233	1.179 7953	16
17	1.220 7220	1.217 4995	1.211 0840	1.204 7078	17
18	1.245 0074	1.241 6075	1.234 8403	1.228 1165	18
19	1.267 9504	1.264 3733	1.257 2550	1.250 1843	19
20	1.289 6890	1.285 9349	1.278 4660	1.271 0490	20
21	1.310 3407	1.306 4098	1.298 5906	1.290 8282	21
22	1.330 0068	1.325 8992	1.317 7303	1.309 6231	22
23	1.348 7748	1.344 4907	1.335 9727	1.327 5213	23
24	1.366 7213	1.362 2609	1.353 3942	1.344 5994	24
25	1.383 9134	1.379 2769	1.370 0621	1.360 9245	25
26	1.400 4103	1.395 5979	1.386 0353	1.376 5557	26
27	1.416 2645	1.411 2763	1.401 3666	1.391 5456	27
28	1.431 5228	1.426 3589	1.416 1025	1.405 9409	28
29	1.446 2270	1.440 8877	1.430 2851	1.419 7836	29
30	1.460 4148	1.454 9002	1.443 9519	1.433 1111	30
31	1.474 1199	1.468 4302	1.457 1368	1.445 9574	31
32	1.487 3731	1.481 5085	1.469 8704	1.458 3531	32
33	1.500 2022	1.494 1628	1.482 1805	1.470 3262	33
34	1.512 6325	1.506 4186	1.494 0926	1.481 9017	34
35	1.524 6873	1.518 2990	1.505 6297	1.493 1031	35
36	1.536 3876	1.529 8250	1.516 8131	1.503 9514	36
37	1.547 7528	1.541 0163	1.527 6622	1.514 4661	37
38	1.558 8009	1.551 8906	1.538 1948	1.524 6649	38
39	1.569 5485	1.562 4643	1.548 4274	1.534 5645	39
40	1.580 0105	1.572 7529	1.558 3752	1.544 1801	40
41	1.590 2013	1.582 7704	1.568 0525	1.553 5258	41
42	1.600 1340	1.592 5298	1.577 4723	1.562 6146	42
43	1.609 8205	1.602 0434	1.586 6467	1.571 4587	43
44	1.619 2724	1.611 3224	1.595 5870	1.580 0695	44
45	1.628 5000	1.620 3775	1.604 3038	1.588 4575	45
46	1.637 5134	1.629 2185	1.612 8070	1.596 6326	46
47	1.646 3218	1.637 8544	1.621 1058	1.604 6039	47
48	1.654 9337	1.646 2943	1.629 2089	1.612 3803	48
49	1.663 3572	1.654 5459	1.637 1243	1.619 9696	49
50	1.671 6002	1.662 6170	1.644 8597	1.627 3796	50

TABLE 7. $a_{ni} = \frac{1 - (1 + i)^{-n}}{i}$

n	$\frac{1}{4}\%$	$\frac{3}{8}\%$	$\frac{1}{2}\%$	$\frac{3}{4}\%$	n
51	47.8266 0386	46.8289 9236	44.9181 9537	43.1139 1775	51
52	48.7048 4176	47.6700 9205	45.6897 4664	43.8217 7260	52
53	49.5808 8953	48.5083 9739	46.4574 5934	44.5249 3967	53
54	50.4547 5265	49.3439 1767	47.2213 5258	45.2234 5000	54
55	51.3264 3656	50.1766 6213	47.9814 4535	45.9173 3444	55
56	52.1959 4669	51.0066 3999	48.7377 5657	46.6066 2362	56
57	53.0632 8847	51.8338 6046	49.4903 0505	47.2913 4796	57
58	53.9284 6730	52.6583 3268	50.2391 0950	47.9715 3771	58
59	54.7914 8858	53.4800 6580	50.9841 8855	48.6472 2289	59
60	55.6523 5769	54.2990 6890	51.7255 6075	49.3184 3334	60
61	56.5110 7999	55.1153 5106	52.4632 4453	49.9851 9868	61
62	57.3676 6083	55.9289 2133	53.1972 5824	50.6475 4835	62
63	58.2221 0557	56.7397 8870	53.9276 2014	51.3055 1161	63
64	59.0744 1952	57.5479 6216	54.6543 4839	51.9591 1749	64
65	59.9246 0800	58.3534 5065	55.3774 6109	52.6083 9486	65
66	60.7726 7631	59.1562 6311	56.0969 7621	53.2533 7238	66
67	61.6186 2974	59.9564 0842	56.8129 1165	53.8940 7852	67
68	62.4624 7355	60.7538 9543	57.5252 8522	54.5305 4158	68
69	63.3042 1302	61.5487 3299	58.2341 1465	55.1627 8965	69
70	64.1438 5339	62.3409 2989	58.9394 1756	55.7908 5064	70
71	64.9813 9989	63.1304 9490	59.6412 1151	56.4147 5229	71
72	65.8168 5774	63.9174 3678	60.3395 1394	57.0345 2215	72
73	66.6502 3216	64.7017 6423	61.0343 4222	57.6501 8756	73
74	67.4815 2834	65.4834 8595	61.7257 1366	58.2617 7572	74
75	68.3107 5146	66.2626 1058	62.4136 4543	58.8693 1363	75
76	69.1379 0670	67.0391 4676	63.0981 5466	59.4728 2811	76
77	69.9629 9920	67.8131 0308	63.7792 5836	60.0723 4581	77
78	70.7860 3411	68.5844 8812	64.4569 7350	60.6678 9319	78
79	71.6070 1657	69.3533 1042	65.1313 1691	61.2594 9654	79
80	72.4259 5169	70.1195 7849	65.8023 0538	61.8471 8200	80
81	73.2428 4458	70.8833 0082	66.4699 5561	62.4309 7549	81
82	74.0577 0033	71.6444 8587	67.1342 8419	63.0109 0281	82
83	74.8705 2402	72.4031 4206	67.7953 0765	63.5869 8954	83
84	75.6813 2072	73.1592 7780	68.4530 4244	64.1592 6114	84
85	76.4900 9548	73.9129 0146	69.1075 0491	64.7277 4285	85
86	77.2968 5335	74.6640 2139	69.7587 1135	65.2924 5979	86
87	78.1015 9935	75.4126 4591	70.4066 7796	65.8534 3687	87
88	78.9043 3850	76.1587 8329	71.0514 2086	66.4106 9888	88
89	79.7050 7581	76.9024 4182	71.6929 5608	66.9642 7041	89
90	80.5038 1627	77.6436 2972	72.3312 9958	67.5141 7590	90
91	81.3005 6486	78.3823 5520	72.9664 6725	68.0604 3964	91
92	82.0953 2654	79.1186 2645	73.5984 7487	68.6030 8574	92
93	82.8881 0628	79.8524 5161	74.2273 3818	69.1421 3815	93
94	83.6789 0900	80.5838 3882	74.8530 7282	69.6776 2068	94
95	84.4677 3966	81.3127 9616	75.4756 9434	70.2095 5696	95
96	85.2546 0315	82.0393 3172	76.0952 1825	70.7379 7049	96
97	86.0395 0439	82.7634 5354	76.7116 5995	71.2628 8460	97
98	86.8224 4827	83.4851 6964	77.3250 3478	71.7843 2245	98
99	87.6034 3967	84.2044 8802	77.9353 5799	72.3023 0707	99
100	88.3824 8346	84.9214 1663	78.5426 4477	72.8168 6132	100

TABLE 7. $\log a_{\bar{m}i}$

n	$\frac{1}{4}\%$	$\frac{1}{8}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
51	1.679 6695	1.670 5148	1.652 4223	1.634 6175	51
52	1.687 5721	1.678 2460	1.659 8188	1.641 6900	52
53	1.695 3143	1.685 8169	1.667 0555	1.648 6033	53
54	1.702 9021	1.693 2337	1.674 1384	1.655 3637	54
55	1.710 3411	1.700 5018	1.681 0733	1.661 9767	55
56	1.717 6368	1.707 6267	1.687 8655	1.668 4476	56
57	1.724 7942	1.714 6135	1.694 5202	1.674 7817	57
58	1.731 8181	1.721 4671	1.701 0419	1.680 9836	58
59	1.738 7131	1.728 1919	1.707 4355	1.687 0581	59
60	1.745 4836	1.734 7924	1.713 7052	1.693 0093	60
61	1.752 1336	1.741 2726	1.719 8551	1.698 8414	61
62	1.758 6671	1.747 6365	1.725 8892	1.704 5584	62
63	1.765 0879	1.753 8877	1.731 8113	1.710 1641	63
64	1.771 3995	1.760 0299	1.737 6247	1.715 6618	64
65	1.777 6052	1.766 0665	1.743 3330	1.721 0550	65
66	1.783 7084	1.772 0007	1.748 9394	1.726 3471	66
67	1.789 7120	1.777 8356	1.754 4470	1.731 5410	67
68	1.795 6192	1.783 5741	1.759 8588	1.736 6398	68
69	1.801 4326	1.789 2191	1.765 1775	1.741 6462	69
70	1.807 1550	1.794 7733	1.770 4059	1.746 5630	70
71	1.812 7891	1.800 2392	1.775 5464	1.751 3927	71
72	1.818 3372	1.805 6194	1.780 6018	1.756 1378	72
73	1.823 8017	1.810 9161	1.785 5742	1.760 8007	73
74	1.829 1849	1.816 1318	1.790 4661	1.765 3837	74
75	1.834 4890	1.821 2685	1.795 2796	1.769 8890	75
76	1.839 7162	1.826 3285	1.800 0167	1.774 3185	76
77	1.844 8684	1.831 3137	1.804 6795	1.778 6746	77
78	1.849 9476	1.836 2259	1.809 2699	1.782 9589	78
79	1.854 9556	1.841 0672	1.813 7898	1.787 1734	79
80	1.859 8942	1.845 8393	1.818 2411	1.791 3199	80
81	1.864 7652	1.850 5439	1.822 6254	1.795 4001	81
82	1.869 5702	1.855 1828	1.826 9443	1.799 4157	82
83	1.874 3108	1.859 7575	1.831 1997	1.803 3682	83
84	1.878 9887	1.864 2694	1.835 3928	1.807 2594	84
85	1.883 6052	1.868 7202	1.839 5252	1.811 0905	85
86	1.888 1618	1.873 1114	1.843 5984	1.814 8630	86
87	1.892 6600	1.877 4442	1.847 6138	1.818 5785	87
88	1.897 1009	1.881 7200	1.851 5728	1.822 2380	88
89	1.901 4860	1.885 9401	1.855 4765	1.825 8432	89
90	1.905 8165	1.890 1058	1.859 3263	1.829 3949	90
91	1.910 0936	1.894 2183	1.863 1233	1.832 8947	91
92	1.914 3184	1.898 2787	1.866 8688	1.836 3436	92
93	1.918 4923	1.902 2882	1.870 5639	1.839 7428	93
94	1.922 6160	1.906 2479	1.874 2096	1.843 0933	94
95	1.926 6909	1.910 1589	1.877 8071	1.846 3963	95
96	1.930 7179	1.914 0222	1.881 3573	1.849 6526	96
97	1.934 6979	1.917 8386	1.884 8614	1.852 8634	97
98	1.938 6320	1.921 6094	1.888 3201	1.856 0296	98
99	1.942 5212	1.925 3352	1.891 7345	1.859 1521	99
100	1.946 3662	1.929 0172	1.895 1055	1.862 2320	100

TABLE 7. $a_{\overline{m}i} = \frac{1 - (1 + i)^{-n}}{i}$

n	$\frac{1}{2}\%$	$\frac{3}{4}\%$	1%	2%	n
101	89.1595 8450	85.6359 6344	79.1469 1021	73.3280 0792	101
102	89.9347 4763	86.3481 3635	79.7481 6937	73.8357 6944	102
103	90.7079 7768	87.0579 4323	80.3464 3718	74.3401 6830	103
104	91.4792 7948	87.7653 9195	80.9417 2854	74.8412 2677	104
105	92.2486 5784	88.4704 9034	81.5340 5825	75.3389 6697	105
106	93.0161 1755	89.1732 4621	82.1234 4104	75.8334 1088	106
107	93.7816 6339	89.8736 6735	82.7098 9158	76.3245 8032	107
108	94.5453 0014	90.5717 6150	83.2934 2446	76.8124 9699	108
109	95.3070 3256	91.2675 3641	83.8740 5419	77.2971 8242	109
110	96.0668 6539	91.9609 9977	84.4517 9522	77.7786 5801	110
111	96.8248 0338	92.6521 5927	85.0266 6191	78.2569 4503	111
112	97.5808 5126	93.3410 2255	85.5986 6856	78.7320 6458	112
113	98.3350 1372	94.0275 9726	86.1678 2942	79.2040 3764	113
114	99.0872 9548	94.7118 9098	86.7341 5862	79.6728 8505	114
115	99.8377 0123	95.3939 1131	87.2976 7027	80.1386 2751	115
116	100.5862 3564	96.0736 6578	87.8583 7838	80.6012 8559	116
117	101.3329 0338	96.7511 6194	88.4162 9690	81.0608 7970	117
118	102.0777 0911	97.4264 0727	88.9714 3970	81.5174 3015	118
119	102.8206 5747	98.0994 0927	89.5238 2059	81.9709 5708	119
120	103.5617 5308	98.7701 7538	90.0734 5333	82.4214 8052	120
121	104.3010 0058	99.4387 1304	90.6203 5157	82.8690 2036	121
122	105.0384 0457	100.1050 2964	91.1645 2892	83.3135 9636	122
123	105.7739 6965	100.7691 3256	91.7059 9893	83.7552 2815	123
124	106.5077 0040	101.4310 2916	92.2447 7505	84.1939 3523	124
125	107.2396 0139	102.0907 2677	92.7808 7070	84.6297 3696	125
126	107.9696 7720	102.7482 3269	93.3142 9920	85.0626 5259	126
127	108.6979 3237	103.4035 5420	93.8450 7384	85.4927 0122	127
128	109.4243 7144	104.0566 9857	94.3732 0780	85.9199 0185	128
129	110.1489 9894	104.7076 7303	94.8987 1422	86.3442 7334	129
130	110.8718 1939	105.3564 8478	95.4216 0619	86.7658 3442	130
131	111.5928 3730	106.0031 4101	95.9418 9671	87.1846 0371	131
132	112.3120 5716	106.6476 4888	96.4595 9872	87.6005 9969	132
133	113.0294 8345	107.2900 1552	96.9747 2509	88.0138 4072	133
134	113.7451 2065	107.9302 4806	97.4872 8865	88.4243 4507	134
135	114.4589 7321	108.5683 5358	97.9973 0214	88.8321 3084	135
136	115.1710 4560	109.2043 3915	98.5047 7825	89.2372 1604	136
137	115.8813 4224	109.8382 1181	99.0097 2960	89.6396 1856	137
138	116.5898 6758	110.4699 7859	99.5121 6875	90.0393 5616	138
139	117.2966 2601	111.0996 4646	100.0121 0821	90.4364 4649	139
140	118.0016 2196	111.7272 2242	100.5095 6041	90.8309 0709	140
141	118.7048 5981	112.3527 1341	101.0045 3772	91.2227 5536	141
142	119.4083 4395	112.9781 2636	101.4970 5246	91.6120 0861	142
143	120.1060 7875	113.5974 6817	101.9871 1688	91.9986 8402	143
144	120.8040 6858	114.2167 4572	102.4747 4316	92.3827 9867	144
145	121.5003 1778	114.8339 6586	102.9599 4344	92.7643 6952	145
146	122.1948 3071	115.4491 3545	103.4427 2979	93.1434 1340	146
147	122.8876 1168	116.0622 6128	103.9231 1422	93.5199 4706	147
148	123.5786 6502	116.6733 5015	104.4011 0868	93.8939 8712	148
149	124.2679 9503	117.2824 0882	104.8767 2505	94.2655 5010	149
150	124.9556 0601	117.8894 4404	105.3499 7518	94.6346 5239	150

TABLE 7. $\log a_{\bar{n}i}$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
101	1.950 1681	1.932 6562	1.898 4340	1.865 2698	101
102	1.953 9275	1.936 2530	1.901 7207	1.868 2668	102
103	1.957 6455	1.939 8084	1.904 9667	1.871 2236	103
104	1.961 3227	1.943 3233	1.908 1724	1.874 1409	104
105	1.964 9601	1.946 7984	1.911 3390	1.877 0197	105
106	1.968 5582	1.950 2346	1.914 4671	1.879 8606	106
107	1.972 1179	1.953 6325	1.917 5574	1.882 6644	107
108	1.975 6400	1.956 9929	1.920 6107	1.885 4319	108
109	1.979 1249	1.960 3163	1.923 6277	1.888 1636	109
110	1.982 5736	1.963 6037	1.926 6089	1.890 8605	110
111	1.985 9866	1.966 8555	1.929 5551	1.893 5229	111
112	1.989 3646	1.970 0725	1.932 4670	1.896 1517	112
113	1.992 7082	1.973 2553	1.935 3451	1.898 7473	113
114	1.996 0180	1.976 4045	1.938 1902	1.901 3106	114
115	1.999 2946	1.979 5207	1.941 0027	1.903 8419	115
116	2.002 5386	1.982 6044	1.943 7832	1.906 3419	116
117	2.005 7504	1.985 6562	1.946 5324	1.908 8114	117
118	2.008 9309	1.988 6767	1.949 2507	1.911 2505	118
119	2.012 0804	1.991 6664	1.951 9386	1.913 6600	119
120	2.015 1994	1.994 6259	1.954 5968	1.916 0404	120
121	2.018 2885	1.997 5555	1.957 2258	1.918 3922	121
122	2.021 3481	2.000 4559	1.959 8259	1.920 7159	122
123	2.024 3788	2.003 3276	1.962 3978	1.923 0119	123
124	2.027 3810	2.006 1708	1.964 9417	1.925 2808	124
125	2.030 3552	2.008 9863	1.967 4584	1.927 5230	125
126	2.033 3018	2.011 7743	1.969 9482	1.929 7389	126
127	2.036 2212	2.014 5354	1.972 4114	1.931 9291	127
128	2.039 1140	2.017 2700	1.974 8488	1.934 0938	128
129	2.041 9806	2.019 9785	1.977 2603	1.936 2336	129
130	2.044 8211	2.022 6613	1.979 6467	1.938 3488	130
131	2.047 6363	2.025 3187	1.982 0083	1.940 4398	131
132	2.050 4264	2.027 9513	1.984 3455	1.942 5071	132
133	2.053 1917	2.030 5593	1.986 6586	1.944 5510	133
134	2.055 9328	2.033 1432	1.988 9480	1.946 5719	134
135	2.058 6498	2.035 7032	1.991 2141	1.948 5700	135
136	2.061 3434	2.038 2399	1.993 4573	1.950 5460	136
137	2.064 0135	2.040 7534	1.995 6779	1.952 5000	137
138	2.066 6608	2.043 2443	1.997 8762	1.954 4324	138
139	2.069 2855	2.045 7127	2.000 0526	1.956 3434	139
140	2.071 8880	2.048 1590	2.002 2074	1.958 2337	140
141	2.074 4685	2.050 5836	2.004 3409	1.960 1032	141
142	2.077 0274	2.052 9867	2.006 4534	1.961 9524	142
143	2.079 5650	2.055 3686	2.008 5453	1.963 7816	143
144	2.082 0816	2.057 7298	2.010 6168	1.965 5912	144
145	2.084 5774	2.060 0704	2.012 6682	1.967 3812	145
146	2.087 0528	2.062 3907	2.014 7000	1.969 1521	146
147	2.089 5081	2.064 6911	2.016 7121	1.970 9043	147
148	2.091 9435	2.066 9717	2.018 7051	1.972 6376	148
149	2.094 3593	2.069 2329	2.020 6792	1.974 3530	149
150	2.096 7558	2.071 4749	2.022 6344	1.976 0502	150

TABLE 7. $a_{\bar{m}i} = \frac{1 - (1 + i)^{-n}}{i}$

n	$\frac{1}{2}\%$	$\frac{3}{8}\%$	$\frac{1}{2}\%$	$\frac{3}{8}\%$	n
151	125.6415 0226	118.4944 6254	105.8208 7082	95.0013 1029	151
152	126.3256 8804	119.0974 7100	106.2894 2371	95.3655 4000	152
153	127.0081 6762	119.6984 7612	106.7556 4548	95.7273 5759	153
154	127.6889 4525	120.2974 8454	107.2195 4774	96.0867 7904	154
155	128.3680 2519	120.8945 0290	107.6811 4203	96.4438 2021	155
156	129.0454 1166	121.4895 3781	108.1404 3983	96.7984 9687	156
157	129.7211 0889	122.0825 9587	108.5974 5257	97.1508 2468	157
158	130.3951 2109	122.6736 8363	109.0521 9161	97.5008 1919	158
159	131.0674 5246	123.2628 0764	109.5046 6827	97.8484 9586	159
160	131.7381 0719	123.8499 7443	109.9548 9380	98.1938 7003	160
161	132.4070 8946	124.4351 9050	110.4028 7940	98.5369 5695	161
162	133.0744 0346	125.0184 6233	110.8486 3622	98.8777 7178	162
163	133.7400 5332	125.5997 9638	111.2921 7534	99.2163 2956	163
164	134.4040 4321	126.1791 9909	111.7335 0780	99.5526 4523	164
165	135.0663 7727	126.7566 7687	112.1726 4458	99.8867 3364	165
166	135.7270 5962	127.3322 3612	112.6095 9660	100.2186 0955	166
167	136.3860 9439	127.9058 8322	113.0443 7473	100.5482 8760	167
168	137.0434 8567	128.4776 2451	113.4769 8978	100.8757 8236	168
169	137.6992 3758	129.0474 6633	113.9074 5251	101.2011 0828	169
170	138.3533 5419	129.6154 1499	114.3357 7365	101.5242 7972	170
171	139.0058 3959	130.1814 7677	114.7619 6383	101.8453 1095	171
172	139.6566 9785	130.7456 5795	115.1860 3366	102.1642 1614	172
173	140.3059 3302	131.3079 6478	115.6079 9369	102.4810 0939	173
174	140.9535 4914	131.8684 0347	116.0278 5442	102.7957 0466	174
175	141.5995 5027	132.4269 8025	116.4456 2629	103.1083 1586	175
176	142.2439 4042	132.9837 0128	116.8613 1969	103.4188 5678	176
177	142.8867 2361	133.5385 7275	117.2749 4496	103.7273 4115	177
178	143.5279 0385	134.0916 0079	117.6865 1240	104.0337 8257	178
179	144.1674 8514	134.6427 9152	118.0960 3224	104.3381 9457	179
180	144.8054 7146	135.1921 5106	118.5035 1467	104.6405 9061	180
181	145.4418 6679	135.7396 8549	118.9089 6982	104.9409 8402	181
182	146.0766 7510	136.2854 0086	119.3124 0778	105.2393 8807	182
183	146.7099 0035	136.8293 0322	119.7138 3859	105.5358 1593	183
184	147.3415 4649	137.3713 9860	120.1132 7222	105.8302 8070	184
185	147.9716 1744	137.9116 9300	120.5107 1863	106.1227 9536	185
186	148.6001 1715	138.4501 9241	120.9061 8769	106.4133 7285	186
187	149.2270 4952	138.9869 0277	121.2996 8925	106.7020 2598	187
188	149.8524 1848	139.5218 3005	121.6912 3308	106.9887 6750	188
189	150.4762 2791	140.0549 8016	122.0808 2894	107.2736 1007	189
190	151.0984 8170	140.5863 5901	122.4684 8650	107.5565 6626	190
191	151.7191 8375	141.1159 7248	122.8542 1543	107.8376 4857	191
192	152.3383 3790	141.6438 2643	123.2380 2530	108.1168 6941	192
193	152.9559 4803	142.1699 2672	123.6199 2567	108.3942 4111	193
194	153.5720 1799	142.6942 7917	123.9999 2604	108.6697 7590	194
195	154.1865 5161	143.2168 8958	124.3780 3586	108.9434 8597	195
196	154.7995 5272	143.7377 6375	124.7542 6454	109.2153 8338	196
197	155.4110 2516	144.2569 0743	125.1286 2143	109.4854 8015	197
198	156.0209 7273	144.7743 2639	125.5011 1585	109.7537 8819	198
199	156.6293 9923	145.2900 2635	125.8717 5707	110.0203 1937	199
200	157.2363 0846	145.8040 1302	126.2405 5430	110.2850 8543	200

TABLE 7. $\log a_{\bar{m}i}$

n	$\frac{1}{4}\%$	$\frac{1}{8}\%$	$\frac{1}{2}\%$	$\frac{3}{8}\%$	n
151	2.099 1331	2.073 6981	2.024 5714	1.977 7296	151
152	2.101 4917	2.075 9026	2.026 4900	1.979 3915	152
153	2.103 8316	2.078 0887	2.028 3909	1.981 0360	153
154	2.106 1533	2.080 2565	2.030 2740	1.982 6636	154
155	2.108 4568	2.082 4066	2.032 1396	1.984 2744	155
156	2.110 7426	2.084 5389	2.033 9882	1.985 8686	156
157	2.113 0106	2.086 6538	2.035 8196	1.987 4465	157
158	2.115 2613	2.088 7514	2.037 6344	1.989 0083	158
159	2.117 4949	2.090 8321	2.039 4326	1.990 5542	159
160	2.119 7115	2.092 8959	2.041 2145	1.992 0844	160
161	2.121 9113	2.094 9432	2.042 9804	1.993 5991	161
162	2.124 0945	2.096 9742	2.044 7304	1.995 0987	162
163	2.126 2615	2.098 9889	2.046 4646	1.996 5832	163
164	2.128 4123	2.100 9877	2.048 1835	1.998 0528	164
165	2.130 5472	2.102 9708	2.049 8869	1.999 5078	165
166	2.132 6664	2.104 9383	2.051 5755	2.000 9484	166
167	2.134 7701	2.106 8905	2.053 2489	2.002 3747	167
168	2.136 8584	2.108 8275	2.054 9078	2.003 7869	168
169	2.138 9315	2.110 7495	2.056 5522	2.005 1853	169
170	2.140 9897	2.112 6566	2.058 1821	2.006 5699	170
171	2.143 0330	2.114 5492	2.059 7979	2.007 9410	171
172	2.145 0617	2.116 4273	2.061 3998	2.009 2988	172
173	2.147 0760	2.118 2910	2.062 9878	2.010 6434	173
174	2.149 0760	2.120 1407	2.064 5623	2.011 9750	174
175	2.151 0619	2.121 9765	2.066 1232	2.013 2937	175
176	2.153 0338	2.123 7984	2.067 6708	2.014 5997	176
177	2.154 9918	2.125 6067	2.069 2052	2.015 8933	177
178	2.156 9363	2.127 4016	2.070 7267	2.017 1744	178
179	2.158 8673	2.129 1831	2.072 2353	2.018 4433	179
180	2.160 7850	2.130 9515	2.073 7312	2.019 7002	180
181	2.162 6895	2.132 7068	2.075 2146	2.020 9452	181
182	2.164 5809	2.134 4493	2.076 6856	2.022 1783	182
183	2.166 4594	2.136 1791	2.078 1443	2.023 3999	183
184	2.168 3253	2.137 8963	2.079 5910	2.024 6100	184
185	2.170 1784	2.139 6011	2.081 0257	2.025 8087	185
186	2.172 0191	2.141 2936	2.082 4485	2.026 9962	186
187	2.173 8475	2.142 9739	2.083 8597	2.028 1727	187
188	2.175 6637	2.144 6422	2.085 2593	2.029 3382	188
189	2.177 4679	2.146 2985	2.086 6475	2.030 4929	189
190	2.179 2601	2.147 9432	2.088 0244	2.031 6369	190
191	2.181 0405	2.149 5762	2.089 3901	2.032 7704	191
192	2.182 8093	2.151 1977	2.090 7447	2.033 8934	192
193	2.184 5664	2.152 8078	2.092 0885	2.035 0062	193
194	2.186 3121	2.154 4066	2.093 4215	2.036 1088	194
195	2.188 0465	2.155 9942	2.094 7437	2.037 2013	195
196	2.189 7697	2.157 5708	2.096 0554	2.038 2838	196
197	2.191 4819	2.159 1366	2.097 3566	2.039 3566	197
198	2.193 1830	2.160 6916	2.098 6476	2.040 4195	198
199	2.194 8732	2.162 2358	2.099 9283	2.041 4729	199
200	2.196 5528	2.163 7694	2.101 1989	2.042 5168	200

TABLE 7. $a_{\bar{n}|i} = \frac{1 - (1 + i)^{-n}}{i}$

n	$\frac{3}{8}\%$	$\frac{7}{8}\%$	1%	1 $\frac{1}{4}\%$	n
1	0.9925 5583	0.9913 2590	0.9900 9901	0.9876 5432	1
2	1.9777 2291	1.9740 5294	1.9703 9506	1.9631 1538	2
3	2.9555 5624	2.9482 5570	2.9409 8521	2.9265 3371	3
4	3.9261 1041	3.9140 0813	3.9019 6555	3.8780 5798	4
5	4.8894 3961	4.8713 8352	4.8534 3124	4.8178 3504	5
6	5.8455 9763	5.8204 5454	5.7954 7647	5.7460 0992	6
7	6.7946 3785	6.7612 9323	6.7281 9453	6.6627 2585	7
8	7.7366 1325	7.6939 7098	7.6516 7775	7.5681 2429	8
9	8.6715 7642	8.6185 5859	8.5660 1758	8.4623 4498	9
10	9.5995 7958	9.5351 2624	9.4713 0453	9.3455 2591	10
11	10.5206 7452	10.4437 4348	10.3676 2825	10.2178 0337	11
12	11.4349 1267	11.3444 7928	11.2550 7747	11.0793 1197	12
13	12.3423 4508	12.2374 0202	12.1337 4007	11.9301 8466	13
14	13.2430 2242	13.1225 7945	13.0037 0304	12.7705 5275	14
15	14.1369 9495	14.0000 7876	13.8650 5252	13.6005 4592	15
16	15.0243 1261	14.8699 6656	14.7178 7378	14.4202 9227	16
17	15.9050 2492	15.7323 0885	15.5622 5127	15.2299 1829	17
18	16.7791 8107	16.5871 7111	16.3982 6858	16.0295 4893	18
19	17.6468 2984	17.4346 1820	17.2260 0850	16.8193 0759	19
20	18.5080 1969	18.2747 1445	18.0455 5297	17.5993 1613	20
21	19.3627 9870	19.1075 2361	18.8569 8313	18.3696 9495	21
22	20.2112 1459	19.9331 0891	19.6603 7934	19.1305 6291	22
23	21.0533 1473	20.7515 3300	20.4558 2113	19.8820 3744	23
24	21.8891 4614	21.5628 5799	21.2433 8726	20.6242 3451	24
25	22.7187 5547	22.3671 4547	22.0231 5570	21.3572 6865	25
26	23.5421 8905	23.1644 5647	22.7952 0366	22.0812 5299	26
27	24.3594 9286	23.9548 5152	23.5596 0759	22.7962 9925	27
28	25.1707 1251	24.7383 9060	24.3164 4316	23.5025 1778	28
29	25.9758 9331	25.5151 3319	25.0657 8530	24.2000 1756	29
30	26.7750 8021	26.2851 3823	25.8077 0822	24.8889 0623	30
31	27.5683 1783	27.0484 6417	26.5422 8537	25.5692 9010	31
32	28.3556 5045	27.8051 6894	27.2695 8947	26.2412 7418	32
33	29.1371 2203	28.5553 0998	27.9896 9255	26.9049 6215	33
34	29.9127 7621	29.2989 4422	28.7026 6589	27.5604 5644	34
35	30.6826 5629	30.0361 2809	29.4085 8009	28.2078 5822	35
36	31.4468 0525	30.7669 1757	30.1075 0504	28.8472 6737	36
37	32.2052 6576	31.4913 6810	30.7995 0994	29.4787 8259	37
38	32.9580 8016	32.2095 3467	31.4846 6330	30.1025 0133	38
39	33.7052 9048	32.9214 7179	32.1630 3298	30.7185 1983	39
40	34.4469 3844	33.6272 3350	32.8346 8611	31.3269 3316	40
41	35.1830 6545	34.3268 7335	33.4996 8922	31.9278 3522	41
42	35.9137 1260	35.0204 4446	34.1581 0814	32.5213 1874	42
43	36.6389 2070	35.7079 9947	34.8100 0806	33.1074 7530	43
44	37.3587 3022	36.3895 9055	35.4554 5352	33.6863 9536	44
45	38.0731 8136	37.0652 6944	36.0945 0844	34.2581 6825	45
46	38.7823 1401	37.7350 8743	36.7272 3608	34.8228 8222	46
47	39.4861 6774	38.3990 9535	37.3536 9909	35.3806 2442	47
48	40.1847 8189	39.0573 4359	37.9739 5949	35.9314 8091	48
49	40.8781 9542	39.7098 8212	38.5880 7871	36.4755 3670	49
50	41.5664 4707	40.3567 6047	39.1961 1753	37.0128 7574	50

TABLE 7. $\log a_{\overline{m}i}$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	$1\frac{1}{4}\%$	n
1	9.996 7550	9.996 2164	9.995 6786	9.994 6049	1
2	0.296 1654	0.295 3587	0.294 5533	0.292 9457	2
3	0.470 6393	0.469 5651	0.468 4928	0.466 3535	3
4	0.593 9625	0.592 6217	0.591 2834	0.588 6142	4
5	0.689 2591	0.687 6523	0.686 0488	0.682 8519	5
6	0.766 8289	0.764 9569	0.763 0891	0.759 3663	6
7	0.832 1663	0.830 0298	0.827 8985	0.823 6519	7
8	0.888 5509	0.886 1505	0.883 7567	0.878 9882	8
9	0.938 0980	0.935 4346	0.932 7790	0.927 4906	9
10	0.982 2522	0.979 3265	0.976 4098	0.970 6037	10
11	1.022 0436	1.018 8562	1.015 6795	1.009 3574	11
12	1.058 2329	1.054 7845	1.051 3485	1.044 5127	12
13	1.091 3976	1.087 6892	1.083 9946	1.076 6473	13
14	1.121 9871	1.118 0192	1.114 0669	1.106 2096	14
15	1.150 3571	1.146 1305	1.141 9215	1.133 5564	15
16	1.176 7946	1.172 3100	1.167 8449	1.158 9740	16
17	1.201 5344	1.196 7924	1.192 0723	1.182 6976	17
18	1.224 7708	1.219 7723	1.214 7980	1.204 9213	18
19	1.246 6667	1.241 4125	1.236 1846	1.225 8081	19
20	1.267 3600	1.261 8506	1.256 3701	1.245 4959	20
21	1.286 9681	1.281 2044	1.275 4721	1.264 1018	21
22	1.305 5924	1.299 5751	1.293 5919	1.281 7277	22
23	1.323 3205	1.317 0502	1.310 8168	1.298 4609	23
24	1.340 2288	1.333 7068	1.327 2237	1.314 3777	24
25	1.356 3845	1.349 6105	1.342 8796	1.329 5457	25
26	1.371 8469	1.364 8221	1.357 8434	1.344 0236	26
27	1.386 6683	1.379 3935	1.372 1681	1.357 8643	27
28	1.400 8955	1.393 3714	1.385 8999	1.371 1144	28
29	1.414 5705	1.406 7979	1.399 0814	1.383 8157	29
30	1.427 7308	1.419 7103	1.411 7494	1.396 0056	30
31	1.440 4103	1.432 1426	1.423 9383	1.407 7186	31
32	1.452 6396	1.444 1255	1.435 6785	1.418 9848	32
33	1.464 4466	1.455 6869	1.446 9981	1.429 8323	33
34	1.475 8568	1.466 8520	1.457 9223	1.440 2864	34
35	1.486 8929	1.477 6440	1.468 4740	1.450 3701	35
36	1.497 5765	1.488 0840	1.478 6748	1.460 1047	36
37	1.507 9269	1.498 1915	1.488 5437	1.469 5094	37
38	1.517 9619	1.507 9844	1.498 0990	1.478 6026	38
39	1.527 6980	1.517 4792	1.507 3570	1.487 4002	39
40	1.537 1506	1.526 6911	1.516 3329	1.495 9178	40
41	1.546 3337	1.535 6343	1.525 0407	1.504 1695	41
42	1.555 2603	1.544 3216	1.533 4937	1.512 1681	42
43	1.563 9427	1.552 7655	1.541 7041	1.519 9260	43
44	1.572 3921	1.560 9772	1.549 6830	1.527 4545	44
45	1.580 6192	1.568 9671	1.557 4411	1.534 7641	45
46	1.588 6337	1.576 7454	1.564 9883	1.541 8647	46
47	1.596 4449	1.584 3210	1.572 3336	1.548 7654	47
48	1.604 0616	1.591 7027	1.579 4858	1.555 4751	48
49	1.611 4917	1.598 8986	1.586 4531	1.562 0017	49
50	1.618 7429	1.605 9163	1.593 2430	1.568 3528	50

TABLE 7. $a_{ni} = \frac{1 - (1 + i)^{-n}}{i}$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	1 $\frac{1}{4}\%$	n
51	42.2495 7525	40.9980 2772	39.7981 3617	37.5435 8099	51
52	42.9276 1812	41.6337 3256	40.3941 9423	38.0677 3431	52
53	43.6006 1351	42.2639 2324	40.9843 5072	38.5854 1660	53
54	44.2685 9902	42.8886 4757	41.5686 6408	39.0967 0776	54
55	44.9316 1193	43.5079 5298	42.1471 9216	39.6016 8667	55
56	45.5896 8926	44.1218 8647	42.7199 9224	40.1004 3128	56
57	46.2428 6776	44.7304 9465	43.2871 2102	40.5930 1855	57
58	46.8911 8388	45.3338 2369	43.8486 3468	41.0795 2449	58
59	47.5346 7382	45.9319 1939	44.4045 8879	41.5600 2419	59
60	48.1733 7352	46.5248 2716	44.9550 3841	42.0345 9179	60
61	48.8073 1863	47.1125 9198	45.5000 3803	42.5033 0054	61
62	49.4365 4455	47.6952 5847	46.0396 4161	42.9662 2275	62
63	50.0610 8640	48.2728 7085	46.5739 0258	43.4234 2988	63
64	50.6809 7906	48.8454 7296	47.1028 7385	43.8749 9247	64
65	51.2962 5713	49.4131 0826	47.6266 0777	44.3209 8022	65
66	51.9069 5497	49.9758 1984	48.1451 5621	44.7614 6195	66
67	52.5131 0667	50.5336 5040	48.6585 7050	45.1965 0563	67
68	53.1147 4607	51.0866 4228	49.1669 0149	45.6261 7840	68
69	53.7119 0677	51.6348 3745	49.6701 9949	46.0505 4656	69
70	54.3046 2210	52.1782 7752	50.1685 1435	46.4696 7562	70
71	54.8929 2516	52.7170 0374	50.6618 9539	46.8836 3024	71
72	55.4768 4880	53.2510 5699	51.1503 9148	47.2924 7431	72
73	56.0564 2561	53.7804 7781	51.6340 5097	47.6962 7093	73
74	56.6316 8795	54.3053 0638	52.1129 2175	48.0950 8240	74
75	57.2026 6794	54.8255 8253	52.5870 5124	48.4889 7027	75
76	57.7693 9746	55.3413 4575	53.0564 8637	48.8779 9533	76
77	58.3319 0815	55.8526 3520	53.5212 7364	49.2622 1761	77
78	58.8902 3141	56.3594 8966	53.9814 5905	49.6416 9640	78
79	59.4443 9842	56.8619 4762	54.4370 8817	50.0164 9027	79
80	59.9944 4012	57.3600 4721	54.8882 0611	50.3866 5706	80
81	60.5403 8722	57.8538 2623	55.3348 5753	50.7522 5389	81
82	61.0822 7019	58.3433 2216	55.7770 8666	51.1133 3717	82
83	61.6201 1930	58.8285 7215	56.2149 3729	51.4699 6264	83
84	62.1539 6456	59.3096 1304	56.6484 5276	51.8221 8532	84
85	62.6838 3579	59.7864 8133	57.0776 7600	52.1700 5958	85
86	63.2097 6257	60.2592 1321	57.5026 4951	52.5136 3909	86
87	63.7317 7427	60.7278 4457	57.9234 1535	52.8529 7688	87
88	64.2499 0002	61.1924 1097	58.3400 1520	53.1881 2531	88
89	64.7641 6875	61.6529 4768	58.7524 9030	53.5191 3611	89
90	65.2746 0918	62.1094 8965	59.1608 8148	53.8460 6035	90
91	65.7812 4981	62.5620 7152	59.5652 2919	54.1689 4850	91
92	66.2841 1892	63.0107 2765	59.9655 7346	54.4878 5037	92
93	66.7832 4458	63.4554 9210	60.3619 5392	54.8028 1518	93
94	67.2786 5467	63.8963 9861	60.7544 0982	55.1138 9154	94
95	67.7703 7685	64.3334 8065	61.1429 8002	55.4211 2744	95
96	68.2584 3856	64.7667 7140	61.5277 0299	55.7245 7031	96
97	68.7428 6705	65.1963 0375	61.9086 1682	56.0242 6698	97
98	69.2236 8938	65.6221 1028	62.2857 5923	56.3202 6368	98
99	69.7009 3239	66.0442 2333	62.6591 6755	56.6126 0610	99
100	70.1746 2272	66.4626 7492	63.0288 7877	56.9013 3936	100

TABLE 7. $\log a_{\overline{m}i}$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	1 $\frac{1}{4}\%$	n
51	1.625 8223	1.612 7630	1.599 8627	1.574 5356	51
52	1.632 7368	1.619 4453	1.606 3189	1.580 5570	52
53	1.639 4926	1.625 9698	1.612 6180	1.586 4232	53
54	1.646 0958	1.632 3423	1.618 7660	1.592 1401	54
55	1.652 5520	1.638 5686	1.624 7686	1.597 7137	55
56	1.658 8666	1.644 6541	1.630 6310	1.603 1490	56
57	1.665 0448	1.650 6037	1.636 3587	1.608 4513	57
58	1.671 0912	1.656 4223	1.641 9560	1.613 6253	58
59	1.677 0105	1.662 1146	1.647 4278	1.618 6757	59
60	1.682 8071	1.667 6848	1.652 7783	1.623 6068	60
61	1.688 4849	1.673 1370	1.658 0117	1.628 4226	61
62	1.694 0481	1.678 4752	1.663 1319	1.633 1271	62
63	1.699 5003	1.683 7031	1.668 1426	1.637 7241	63
64	1.704 8450	1.688 8243	1.673 0473	1.642 2170	64
65	1.710 0857	1.693 8422	1.677 8496	1.646 6094	65
66	1.715 2255	1.698 7599	1.682 5526	1.650 9042	66
67	1.720 2677	1.703 5807	1.687 1593	1.655 1048	67
68	1.725 2151	1.708 3074	1.691 6728	1.659 2141	68
69	1.730 0705	1.712 9428	1.696 0959	1.663 2348	69
70	1.734 8368	1.717 4897	1.700 4311	1.667 1697	70
71	1.739 5164	1.721 9507	1.704 6814	1.671 0212	71
72	1.744 1118	1.726 3283	1.708 8489	1.674 7920	72
73	1.748 6254	1.730 6247	1.712 9362	1.678 4844	73
74	1.753 0595	1.734 8422	1.716 9454	1.682 1006	74
75	1.757 4163	1.738 9833	1.720 8788	1.685 6429	75
76	1.761 6978	1.743 0497	1.724 7385	1.689 1134	76
77	1.765 9062	1.747 0437	1.728 5263	1.692 5139	77
78	1.770 0433	1.750 9671	1.732 2446	1.695 8466	78
79	1.774 1109	1.754 8217	1.735 8949	1.699 1132	79
80	1.778 1110	1.758 6095	1.739 4790	1.702 3155	80
81	1.782 0452	1.762 3321	1.742 9988	1.705 4553	81
82	1.785 9151	1.765 9912	1.746 4558	1.708 5342	82
83	1.789 7225	1.769 5883	1.749 8517	1.711 5538	83
84	1.793 4689	1.773 1251	1.753 1880	1.714 5157	84
85	1.797 1556	1.776 6030	1.756 4662	1.717 4213	85
86	1.800 7842	1.780 0234	1.759 6878	1.720 2721	86
87	1.804 3560	1.783 3879	1.762 8541	1.723 0694	87
88	1.807 8724	1.786 6975	1.765 9665	1.725 8147	88
89	1.811 3348	1.789 9538	1.769 0262	1.728 5091	89
90	1.814 7443	1.793 1579	1.772 0346	1.731 1538	90
91	1.818 1022	1.796 3111	1.774 9927	1.733 7504	91
92	1.821 4095	1.799 4145	1.777 9019	1.736 2997	92
93	1.824 6675	1.802 4692	1.780 7632	1.738 8028	93
94	1.827 8773	1.805 4764	1.783 5778	1.741 2610	94
95	1.831 0399	1.808 4371	1.786 3465	1.743 6754	95
96	1.834 1564	1.811 3523	1.789 0707	1.746 0467	96
97	1.837 2277	1.814 2230	1.791 7511	1.748 3762	97
98	1.840 2547	1.817 0502	1.794 3887	1.750 6646	98
99	1.843 2386	1.819 8348	1.796 9846	1.752 9131	99
100	1.846 1801	1.822 5778	1.799 5395	1.755 1224	100

TABLE 7. $a_{ni} = \frac{1 - (1 + i)^{-n}}{i}$

n	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	2%	$2\frac{1}{2}\%$	n
1	0.9852 2167	0.9828 0098	0.9803 9216	0.9756 0976	1
2	1.9558 8342	1.9486 9875	1.9415 6094	1.9274 2415	2
3	2.9122 0042	2.8979 8403	2.8838 8327	2.8560 2356	3
4	3.8543 8465	3.8309 4254	3.8077 2870	3.7619 7421	4
5	4.7826 4497	4.7478 5508	4.7134 5951	4.6458 2850	5
6	5.6971 8717	5.6489 9762	5.6014 3089	5.5081 2536	6
7	6.5982 1396	6.5346 4139	6.4719 9107	6.3493 9060	7
8	7.4859 2508	7.4050 5297	7.3254 8144	7.1701 3717	8
9	8.3605 1732	8.2604 9432	8.1622 3671	7.9708 6553	9
10	9.2221 8455	9.1012 2291	8.9825 8501	8.7520 6393	10
11	10.0711 1779	9.9274 9181	9.7868 4805	9.5142 0871	11
12	10.9075 0521	10.7395 4969	10.5753 4122	10.2577 6460	12
13	11.7315 3222	11.5376 4097	11.3483 7375	10.9831 8497	13
14	12.5433 8150	12.3220 0587	12.1062 4877	11.6909 1217	14
15	13.3432 3301	13.0928 8046	12.8492 6350	12.3813 7773	15
16	14.1312 6405	13.8504 9677	13.5777 0931	13.0550 0266	16
17	14.9076 4931	14.5950 8282	14.2918 7188	13.7121 9772	17
18	15.6725 6089	15.3268 6272	14.9920 3125	14.3533 6363	18
19	16.4261 6837	16.0460 5673	15.6784 6201	14.9788 9134	19
20	17.1686 3879	16.7528 8130	16.3514 3334	15.5891 6229	20
21	17.9001 3673	17.4475 4919	17.0112 0916	16.1845 4857	21
22	18.6208 2437	18.1302 6948	17.6580 4820	16.7654 1324	22
23	19.3308 6145	18.8012 4764	18.2922 0412	17.3321 1048	23
24	20.0304 0537	19.4606 8565	18.9139 2560	17.8849 8583	24
25	20.7196 1120	20.1087 8196	19.5234 5647	18.4243 7642	25
26	21.3986 3172	20.7457 3166	20.1210 3576	18.9506 1114	26
27	22.0676 1746	21.3717 2644	20.7068 9780	19.4640 1087	27
28	22.7267 1671	21.9869 5474	21.2812 7236	19.9648 8866	28
29	23.3760 7558	22.5916 0171	21.8443 8466	20.4535 4991	29
30	24.0158 3801	23.1858 4934	22.3964 5555	20.9302 9259	30
31	24.6461 4582	23.7698 7650	22.9377 0152	21.3954 0741	31
32	25.2671 3874	24.3438 5897	23.4683 3482	21.8491 7796	32
33	25.8789 5442	24.9079 6951	23.9885 6355	22.2918 8094	33
34	26.4817 2849	25.4623 7789	24.4985 9172	22.7237 8628	34
35	27.0755 9458	26.0072 5100	24.9986 1933	23.1451 5734	35
36	27.6606 8431	26.5427 5283	25.4888 4248	23.5562 5107	36
37	28.2371 2740	27.0690 4455	25.9694 5341	23.9573 1812	37
38	28.8050 5163	27.5862 8457	26.4406 4060	24.3486 0304	38
39	29.3645 8288	28.0946 2857	26.9025 8883	24.7303 4443	39
40	29.9158 4520	28.5942 2955	27.3554 7924	25.1027 7505	40
41	30.4589 6079	29.0852 3789	27.7994 8945	25.4661 2200	41
42	30.9940 5004	29.5678 0135	28.2347 9358	25.8206 0683	42
43	31.5212 3157	30.0420 6522	28.6615 6233	26.1664 4569	43
44	32.0406 2223	30.5081 7221	29.0799 6307	26.5038 4945	44
45	32.5523 3718	30.9662 6261	29.4901 5987	26.8330 2386	45
46	33.0564 8983	31.4164 7431	29.8923 1360	27.1541 6962	46
47	33.5531 9195	31.8589 4281	30.2865 8196	27.4674 8255	47
48	34.0425 5365	32.2938 0129	30.6731 1957	27.7731 5371	48
49	34.5248 8339	32.7211 8063	31.0520 7801	28.0713 6947	49
50	34.9996 8807	33.1412 0946	31.4236 0589	28.3623 1168	50

TABLE 7. $\log a_{\bar{n}i}$

n	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	2%	$2\frac{1}{2}\%$	n
1	9.993 5339	9.992 4656	9.991 3999	9.989 2761	1
2	0.291 3429	0.289 7447	0.288 1510	0.284 9773	2
3	0.464 2213	0.462 0960	0.459 9777	0.455 7618	3
4	0.585 9551	0.583 3056	0.580 6660	0.575 4158	4
5	0.679 6682	0.676 4975	0.673 3398	0.667 0632	5
6	0.755 6605	0.751 9714	0.748 2990	0.741 0038	6
7	0.819 4264	0.815 2218	0.811 0379	0.802 7321	7
8	0.874 2455	0.869 5282	0.864 8362	0.855 5275	8
9	0.922 2332	0.917 0060	0.911 8092	0.901 5055	9
10	0.964 8338	0.959 0998	0.953 4014	0.942 1105	10
11	1.003 0777	0.996 8395	0.990 6429	0.978 3727	11
12	1.037 7254	1.030 9861	1.024 2944	1.011 0528	12
13	1.069 3547	1.062 1170	1.054 9336	1.040 7283	13
14	1.098 4146	1.090 6814	1.083 0096	1.067 8484	14
15	1.125 2611	1.117 0352	1.108 8782	1.092 7690	15
16	1.150 1810	1.141 4654	1.132 8265	1.115 7770	16
17	1.173 4092	1.164 2065	1.155 0891	1.137 1071	17
18	1.195 1399	1.185 4533	1.175 8605	1.156 9537	18
19	1.215 5363	1.205 3683	1.195 3035	1.175 4796	19
20	1.234 7359	1.224 0895	1.213 5558	1.192 8228	20
21	1.252 8563	1.241 7345	1.230 7352	1.209 1006	21
22	1.269 9989	1.258 4043	1.246 9427	1.224 4143	22
23	1.286 2512	1.274 1866	1.262 2660	1.238 8515	23
24	1.301 6897	1.289 1582	1.276 7817	1.252 4886	24
25	1.316 3816	1.303 3859	1.290 5567	1.265 3928	25
26	1.330 3860	1.316 9288	1.303 6504	1.277 6232	26
27	1.343 7555	1.329 8396	1.316 1151	1.289 2323	27
28	1.356 5367	1.342 1651	1.327 9975	1.300 2669	28
29	1.368 7716	1.353 9471	1.339 3398	1.310 7687	29
30	1.380 4978	1.365 2230	1.350 1793	1.320 7753	30
31	1.391 7490	1.376 0269	1.360 5499	1.330 3206	31
32	1.402 5561	1.386 3894	1.370 4823	1.339 4351	32
33	1.412 9467	1.396 3384	1.380 0042	1.348 1467	33
34	1.422 9464	1.405 8990	1.389 1411	1.356 4807	34
35	1.432 5780	1.415 0945	1.397 9160	1.364 4601	35
36	1.441 8629	1.423 9459	1.406 3501	1.372 1061	36
37	1.450 8205	1.432 4729	1.414 4628	1.379 4382	37
38	1.459 4687	1.440 6932	1.422 2720	1.386 4740	38
39	1.467 8238	1.448 6233	1.429 7941	1.393 2302	39
40	1.475 9013	1.456 2784	1.437 0443	1.399 7217	40
41	1.483 7150	1.463 6726	1.444 0368	1.405 9628	41
42	1.491 2783	1.470 8190	1.450 7846	1.411 9665	42
43	1.498 6032	1.477 7297	1.457 2998	1.417 7447	43
44	1.505 7010	1.484 4162	1.463 5938	1.423 3089	44
45	1.512 5822	1.490 8888	1.469 6772	1.428 6696	45
46	1.519 2567	1.497 1574	1.475 5596	1.433 8365	46
47	1.525 7339	1.503 2313	1.481 2503	1.438 8188	47
48	1.532 0221	1.509 1191	1.486 7580	1.443 6252	48
49	1.538 1297	1.514 8290	1.492 0907	1.448 2636	49
50	1.544 0641	1.520 3684	1.497 2560	1.452 7417	50

TABLE 7. $a_{\bar{m}i} = \frac{1 - (1 + i)^{-n}}{i}$

n	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	2%	$2\frac{1}{2}\%$	n
51	35.4676 7298	33.5540 1421	31.7878 4892	28.6461 5774	51
52	35.9287 4185	33.9597 1913	32.1449 4992	28.9230 8072	52
53	36.3829 9690	34.3584 4633	32.4950 4894	29.1932 4948	53
54	36.8305 3882	34.7503 1579	32.8382 8327	29.4568 2876	54
55	37.2714 6681	35.1354 4550	33.1747 8752	29.7139 7928	55
56	37.7058 7863	35.5139 5135	33.5046 9385	29.9648 5784	56
57	38.1338 7058	35.8859 4727	33.8281 3103	30.2096 1740	57
58	38.5555 3751	36.2515 4523	34.1452 2650	30.4484 0722	58
59	38.9709 7292	36.6108 5526	34.4561 0441	30.6813 7290	59
60	39.3802 6889	36.9639 8552	34.7608 8668	30.9086 5649	60
61	39.7835 1614	37.3110 4228	35.0596 9282	31.1303 9657	61
62	40.1808 0408	37.6521 3000	35.3526 4002	31.3467 2836	62
63	40.5722 2077	37.9873 5135	35.6398 4316	31.5577 8377	63
64	40.9578 5298	38.3168 0723	35.9214 1486	31.7636 9148	64
65	41.3377 8618	38.6405 9678	36.1974 6555	31.9645 7705	65
66	41.7121 0461	38.9588 1748	36.4681 0348	32.1605 6298	66
67	42.0808 9125	39.2715 6509	36.7334 3478	32.3517 6876	67
68	42.4442 2783	39.5789 3375	36.9935 6351	32.5383 1099	68
69	42.8021 9490	39.8810 1597	37.2485 9168	32.7203 0340	69
70	43.1548 7183	40.1779 0267	37.4986 1929	32.8978 5698	70
71	43.5023 3678	40.4696 8321	37.7437 4441	33.0710 7998	71
72	43.8446 6677	40.7564 4542	37.9840 6314	33.2400 7803	72
73	44.1819 3771	41.0382 7560	38.2196 6975	33.4049 5417	73
74	44.5142 2434	41.3152 5857	38.4506 5662	33.5658 0895	74
75	44.8416 0034	41.5874 7771	38.6771 1433	33.7227 4044	75
76	45.1641 3826	41.8550 1495	38.8991 3170	33.8758 4433	76
77	45.4819 0962	42.1179 5081	39.1167 9578	34.0252 1398	77
78	45.7949 8485	42.3763 6443	39.3301 9194	34.1709 4047	78
79	46.1034 3335	42.6303 3359	39.5394 0386	34.3131 1265	79
80	46.4073 2349	42.8799 3474	39.7445 1359	34.4518 1722	80
81	46.7067 2265	43.1252 4298	39.9456 0156	34.5871 3875	81
82	47.0016 9720	43.3663 3217	40.1427 4663	34.7191 5976	82
83	47.2923 1251	43.6032 7486	40.3360 2611	34.8479 6074	83
84	47.5786 3301	43.8361 4237	40.5255 1579	34.9736 2023	84
85	47.8607 2218	44.0650 0479	40.7112 8999	35.0962 1486	85
86	48.1386 4254	44.2899 3099	40.8934 2156	35.2158 1938	86
87	48.4124 5571	44.5109 8869	41.0719 8192	35.3325 0671	87
88	48.6822 2237	44.7282 4441	41.2470 4110	35.4463 4801	88
89	48.9480 0234	44.9417 6355	41.4186 6774	35.5574 1269	89
90	49.2098 5452	45.1516 1037	41.5869 2916	35.6657 6848	90
91	49.4678 3696	45.3578 4803	41.7518 9133	35.7714 8144	91
92	49.7220 0686	45.5605 3860	41.9136 1895	35.8746 1604	92
93	49.9724 2055	45.7597 4310	42.0721 7545	35.9752 3516	93
94	50.2191 3355	45.9555 2147	42.2276 2299	36.0734 0016	94
95	50.4622 0054	46.1479 3265	42.3800 2254	36.1691 7089	95
96	50.7016 7541	46.3370 3455	42.5294 3386	36.2626 0574	96
97	50.9376 1124	46.5228 8408	42.6759 1555	36.3537 6170	97
98	51.1700 6034	46.7055 3718	42.8195 2505	36.4426 9434	98
99	51.3990 7422	46.8850 4882	42.9603 1867	36.5294 5790	99
100	51.6247 0367	47.0614 7304	43.0983 5164	36.6141 0526	100

TABLE 7. $\log a_{\bar{m}i}$

n	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	2%	$2\frac{1}{2}\%$	n
51	1.549 8326	1.525 7444	1.502 2611	1.457 0664	51
52	1.555 4419	1.530 9641	1.507 1127	1.461 2445	52
53	1.560 8985	1.536 0335	1.511 8171	1.465 2824	53
54	1.566 2080	1.540 9588	1.516 3803	1.469 1860	54
55	1.571 3764	1.545 7455	1.520 8081	1.472 9608	55
56	1.576 4091	1.550 3989	1.525 1056	1.476 6122	56
57	1.581 3109	1.554 9244	1.529 2779	1.480 1452	57
58	1.586 0868	1.559 3266	1.533 3300	1.483 5646	58
59	1.590 7411	1.563 6099	1.537 2661	1.486 8747	59
60	1.595 2786	1.567 7788	1.541 0909	1.490 0802	60
61	1.599 7031	1.571 8373	1.544 8081	1.493 1847	61
62	1.604 0185	1.575 7895	1.548 4218	1.496 1922	62
63	1.608 2287	1.579 6389	1.551 9357	1.499 1064	63
64	1.612 3371	1.583 3893	1.555 3534	1.501 9309	64
65	1.616 3472	1.587 0438	1.558 6782	1.504 6689	65
66	1.620 2620	1.590 6058	1.561 9132	1.507 3235	66
67	1.624 0849	1.594 0782	1.565 0615	1.509 8980	67
68	1.627 8186	1.597 4640	1.568 1261	1.512 3950	68
69	1.631 4660	1.600 7662	1.571 1098	1.514 8172	69
70	1.635 0298	1.603 9872	1.574 0152	1.517 1676	70
71	1.638 5126	1.607 1297	1.576 8449	1.519 4483	71
72	1.641 9168	1.610 1963	1.579 6013	1.521 6620	72
73	1.645 2448	1.613 1891	1.582 2869	1.523 8108	73
74	1.648 4987	1.616 1105	1.584 9037	1.525 8971	74
75	1.651 6811	1.618 9626	1.587 4540	1.527 9228	75
76	1.654 7937	1.621 7474	1.589 9399	1.529 8900	76
77	1.657 8387	1.624 4671	1.592 2632	1.531 8008	77
78	1.660 8179	1.627 1236	1.594 7260	1.533 6569	78
79	1.663 7332	1.629 7186	1.597 0301	1.535 4600	79
80	1.666 5865	1.632 2540	1.599 2772	1.537 2121	80
81	1.669 3793	1.634 7315	1.601 4689	1.538 9146	81
82	1.672 1135	1.637 1527	1.603 6071	1.540 5692	82
83	1.674 7905	1.639 5190	1.605 6931	1.542 1774	83
84	1.677 4119	1.641 8322	1.607 7285	1.543 7405	84
85	1.679 9792	1.644 0938	1.609 7148	1.545 2602	85
86	1.682 4938	1.646 3050	1.611 6534	1.546 7377	86
87	1.684 9571	1.648 4673	1.613 5456	1.548 1744	87
88	1.687 3704	1.650 5818	1.615 3928	1.549 5715	88
89	1.689 7350	1.652 6500	1.617 1961	1.550 9301	89
90	1.692 0520	1.654 6732	1.618 9568	1.552 2515	90
91	1.694 3229	1.656 6524	1.620 6761	1.553 5369	91
92	1.696 5486	1.658 5888	1.622 3551	1.554 7873	92
93	1.698 7304	1.660 4835	1.623 9950	1.556 0037	93
94	1.700 8692	1.662 3376	1.625 5965	1.557 1871	94
95	1.702 9662	1.664 1522	1.627 1612	1.558 3385	95
96	1.705 0223	1.665 9281	1.628 6895	1.559 4590	96
97	1.707 0385	1.667 6665	1.630 1828	1.560 5493	97
98	1.709 0159	1.669 3684	1.631 6419	1.561 6104	98
99	1.710 9552	1.671 0343	1.633 0675	1.562 6433	99
100	1.712 8575	1.672 6655	1.634 4606	1.563 6484	100

TABLE 7. $a_{\overline{n}|i} = \frac{1 - (1 + i)^{-n}}{i}$

n	3%	3½%	4%	4½%	n
1	0.9708 7379	0.9661 8357	0.9615 3846	0.9569 3780	1
2	1.9134 6970	1.8996 9428	1.8860 9467	1.8726 6775	2
3	2.8286 1135	2.8016 3698	2.7750 9103	2.7489 6435	3
4	3.7170 9840	3.6730 7921	3.6298 9522	3.5875 2570	4
5	4.5797 0719	4.5150 5238	4.4518 2233	4.3899 7674	5
6	5.4171 9144	5.3285 5302	5.2421 3686	5.1578 7248	6
7	6.2302 8296	6.1145 4398	6.0020 5467	5.8927 0094	7
8	7.0196 9219	6.8739 5554	6.7327 4487	6.5958 8607	8
9	7.7861 0892	7.6076 8651	7.4353 3161	7.2687 9050	9
10	8.5302 0284	8.3166 0532	8.1108 9578	7.9127 1818	10
11	9.2526 2411	9.0015 5104	8.7604 7671	8.5289 1692	11
12	9.9540 0399	9.6633 3433	9.3850 7376	9.1185 8078	12
13	10.6349 5533	10.3027 3849	9.9856 4785	9.6828 5242	13
14	11.2960 7314	10.9205 2028	10.5631 2293	10.2228 2528	14
15	11.9379 3509	11.5174 1090	11.1183 8743	10.7395 4573	15
16	12.5611 0203	12.0941 1681	11.6522 9561	11.2340 1505	16
17	13.1661 1847	12.6513 2059	12.1656 6885	11.7071 9143	17
18	13.7535 1308	13.1896 8173	12.6592 9697	12.1599 9180	18
19	14.3237 9911	13.7098 3742	13.1339 3940	12.5932 9359	19
20	14.8774 7486	14.2124 0330	13.5903 2634	13.0079 3645	20
21	15.4150 2414	14.6979 7420	14.0291 5995	13.4047 2388	21
22	15.9369 1664	15.1671 2484	14.4511 1533	13.7844 2476	22
23	16.4436 0839	15.6204 1047	14.8568 4167	14.1477 7489	23
24	16.9355 4212	16.0583 6760	15.2469 6314	14.4954 7837	24
25	17.4131 4769	16.4815 1459	15.6220 7994	14.8282 0896	25
26	17.8768 4242	16.8903 5226	15.9827 6918	15.1466 1145	26
27	18.3270 3147	17.2853 6451	16.3295 8575	15.4513 0282	27
28	18.7641 0823	17.6670 1885	16.6630 6322	15.7428 7351	28
29	19.1884 5459	18.0357 6700	16.9837 1463	16.0218 8853	29
30	19.6004 4135	18.3920 4541	17.2920 3330	16.2888 8854	30
31	20.0004 2849	18.7362 7576	17.5884 9356	16.5443 9095	31
32	20.3887 6553	19.0688 6547	17.8735 5150	16.7888 9086	32
33	20.7657 9178	19.3902 0818	18.1476 4567	17.0228 6207	33
34	21.1318 3668	19.7006 8423	18.4111 9776	17.2467 5796	34
35	21.4872 2007	20.0006 6110	18.6646 1323	17.4610 1240	35
36	21.8322 5250	20.2904 9381	18.9082 8195	17.6660 4058	36
37	22.1672 3544	20.5705 2542	19.1425 7880	17.8622 3979	37
38	22.4924 6159	20.8410 8736	19.3678 6423	18.0499 9023	38
39	22.8082 1513	21.1024 9987	19.5844 8484	18.2296 5572	39
40	23.1147 7197	21.3550 7234	19.7927 7388	18.4015 8442	40
41	23.4123 9997	21.5991 0371	19.9930 5181	18.5661 0949	41
42	23.7013 5920	21.8348 8281	20.1856 2674	18.7235 4975	42
43	23.9819 0213	22.0626 8870	20.3707 9494	18.8742 1029	43
44	24.2542 7392	22.2827 9102	20.5488 4129	19.0183 8305	44
45	24.5187 1254	22.4954 5026	20.7200 3970	19.1563 4742	45
46	24.7754 4907	22.7009 1813	20.8846 5356	19.2883 7074	46
47	25.0247 0783	22.8994 3780	21.0429 3612	19.4147 0884	47
48	25.2667 0664	23.0912 4425	21.1951 3088	19.5356 0654	48
49	25.5016 5693	23.2765 6450	21.3414 7200	19.6512 9813	49
50	25.7297 6401	23.4556 1787	21.4821 8462	19.7620 0778	50

TABLE 7. $\log a_{\overline{m}_i}$

n	3%	3½%	4%	4½%	n
1	9.987 1628	9.985 0596	9.982 9667	9.980 8837	1
2	0.281 8216	0.278 6837	0.275 5635	0.272 4607	2
3	0.451 5733	0.447 4118	0.443 2772	0.439 1691	3
4	0.570 2040	0.565 0303	0.559 8941	0.554 7950	4
5	0.660 8377	0.654 6628	0.648 5378	0.642 4622	5
6	0.733 7742	0.726 6093	0.719 5084	0.712 4706	6
7	0.794 5078	0.786 3640	0.778 2999	0.770 3144	7
8	0.846 3181	0.837 2067	0.828 1922	0.819 2732	8
9	0.891 3205	0.881 2526	0.871 3003	0.861 4621	9
10	0.930 9593	0.919 9461	0.909 0688	0.898 3257	10
11	0.966 2649	0.954 3173	0.942 5277	0.930 8939	11
12	0.997 9978	0.985 1270	0.972 4377	0.959 9273	12
13	1.026 7357	1.012 9527	0.999 3763	0.986 0033	13
14	1.052 9275	1.038 2433	1.023 7924	1.009 5710	14
15	1.076 9292	1.061 3549	1.046 0418	1.030 9859	15
16	1.099 0277	1.082 5742	1.066 4115	1.050 5350	16
17	1.119 4578	1.102 1359	1.085 1360	1.068 4527	17
18	1.138 4136	1.120 2343	1.102 4096	1.084 9332	18
19	1.156 0582	1.137 0323	1.118 3950	1.100 1393	19
20	1.172 5293	1.152 6675	1.133 2299	1.114 2084	20
21	1.187 9442	1.167 2574	1.147 0317	1.127 2579	21
22	1.202 4043	1.180 9033	1.159 9014	1.139 3887	22
23	1.215 9972	1.193 6924	1.171 9265	1.150 6882	23
24	1.228 7991	1.205 7014	1.183 1833	1.161 2325	24
25	1.240 8773	1.216 9971	1.193 7388	1.171 0887	25
26	1.252 2908	1.227 6387	1.203 6520	1.180 3155	26
27	1.263 0921	1.237 6786	1.212 9752	1.188 9651	27
28	1.273 3279	1.247 1633	1.221 7548	1.197 0840	28
29	1.283 0400	1.256 1346	1.230 0327	1.204 7137	29
30	1.292 2658	1.264 6301	1.237 8460	1.211 8914	30
31	1.301 0393	1.272 6833	1.245 2287	1.218 6508	31
32	1.309 3909	1.280 3248	1.252 2109	1.225 0220	32
33	1.317 3485	1.287 5825	1.258 8203	1.231 0326	33
34	1.324 9373	1.294 4813	1.265 0821	1.236 7075	34
35	1.332 1802	1.301 0443	1.271 0190	1.242 0694	35
36	1.339 0985	1.307 2926	1.276 6521	1.247 1392	36
37	1.345 7115	1.313 2454	1.282 0004	1.251 9359	37
38	1.352 0370	1.318 9204	1.287 0818	1.256 4770	38
39	1.358 0913	1.324 3339	1.291 9122	1.260 7785	39
40	1.363 8896	1.329 5011	1.296 5067	1.264 8552	40
41	1.369 4459	1.334 4357	1.300 8791	1.268 7209	41
42	1.374 7733	1.339 1509	1.305 0423	1.272 3882	42
43	1.379 8836	1.343 6585	1.309 0079	1.275 8688	43
44	1.384 7883	1.347 9696	1.312 7873	1.279 1736	44
45	1.389 4977	1.352 0947	1.316 3906	1.282 3127	45
46	1.394 0216	1.356 0434	1.319 8273	1.285 2955	46
47	1.398 3690	1.359 8248	1.323 1064	1.288 1309	47
48	1.402 5487	1.363 4473	1.326 2361	1.290 8269	48
49	1.406 5684	1.366 9190	1.329 2244	1.293 3913	49
50	1.410 4358	1.370 2469	1.332 0784	1.295 8311	50

TABLE 7. $a_{ni} = \frac{1 - (1 + i)^{-n}}{i}$

n	3%	3½%	4%	4½%	n
51	25.9512 2719	23.6286 1630	21.6174 8521	19.8679 5003	51
52	26.1662 3999	23.7957 6454	21.7475 8193	19.9693 3017	52
53	26.3749 9028	23.9572 6043	21.8726 7493	20.0663 4466	53
54	26.5776 6047	24.1132 9510	21.9929 5667	20.1591 8149	54
55	26.7744 2764	24.2640 5323	22.1086 1218	20.2480 2057	55
56	26.9654 6373	24.4097 1327	22.2189 1940	20.3330 3404	56
57	27.1509 3566	24.5504 4760	22.3267 4943	20.4143 8664	57
58	27.3310 0549	24.6864 2281	22.4295 6676	20.4922 3602	58
59	27.5058 3058	24.8177 9981	22.5284 2957	20.5667 3303	59
60	27.6755 6367	24.9447 3412	22.6234 8997	20.6380 2204	60
61	27.8403 5307	25.0673 7596	22.7148 9421	20.7062 4118	61
62	28.0003 4279	25.1858 7049	22.8027 8289	20.7715 2266	62
63	28.1556 7261	25.3003 5796	22.8872 9124	20.8339 9298	63
64	28.3064 7826	25.4109 7388	22.9685 4927	20.8937 7319	64
65	28.4528 9152	25.5178 4916	23.0466 8199	20.9509 7913	65
66	28.5950 4031	25.6211 1030	23.1218 0961	21.0057 2165	66
67	28.7330 4884	25.7208 7951	23.1940 4770	21.0581 0684	67
68	28.8670 3771	25.8172 7489	23.2635 0740	21.1082 3621	68
69	28.9971 2399	25.9104 1052	23.3302 9558	21.1562 0690	69
70	29.1234 2135	26.0003 9664	23.3945 1498	21.2021 1187	70
71	29.2460 4015	26.0873 3975	23.4562 6440	21.2460 4007	71
72	29.3650 8752	26.1713 4275	23.5156 3885	21.2880 7662	72
73	29.4806 6750	26.2525 0508	23.5727 2966	21.3283 0298	73
74	29.5928 8106	26.3309 2278	23.6276 2468	21.3667 9711	74
75	29.7018 2628	26.4066 8868	23.6804 0834	21.4036 3360	75
76	29.8075 9833	26.4798 9244	23.7311 6187	21.4388 8383	76
77	29.9102 8964	26.5506 2072	23.7799 6333	21.4726 1611	77
78	30.0099 8994	26.6189 5721	23.8268 8782	21.5048 9579	78
79	30.1067 8635	26.6849 8281	23.8720 0752	21.5357 8545	79
80	30.2007 6345	26.7487 7567	23.9153 9185	21.5653 4493	80
81	30.2920 0335	26.8104 1127	23.9571 0754	21.5936 3151	81
82	30.3805 8577	26.8699 6258	23.9972 1879	21.6207 0001	82
83	30.4665 8813	26.9275 0008	24.0357 8730	21.6466 0288	83
84	30.5500 8556	26.9830 9186	24.0728 7240	21.6713 9032	84
85	30.6311 5103	27.0368 0373	24.1085 3116	21.6951 1035	85
86	30.7098 5537	27.0886 9926	24.1428 1842	21.7178 0895	86
87	30.7862 6735	27.1388 3986	24.1757 8694	21.7395 3009	87
88	30.8604 5374	27.1872 8489	24.2074 8745	21.7603 1588	88
89	30.9324 7936	27.2340 9168	24.2379 6870	21.7802 0658	89
90	31.0024 0714	27.2793 1564	24.2672 7759	21.7992 4075	90
91	31.0702 9820	27.3230 1028	24.2954 5923	21.8174 5526	91
92	31.1362 1184	27.3652 2732	24.3225 5695	21.8348 8542	92
93	31.2002 0567	27.4060 1673	24.3486 1245	21.8515 6499	93
94	31.2623 3560	27.4454 2680	24.3736 6582	21.8675 2631	94
95	31.3226 5592	27.4835 0415	24.3977 5559	21.8828 0030	95
96	31.3812 1934	27.5202 9387	24.4209 1884	21.8974 1655	96
97	31.4380 7703	27.5558 3948	24.4431 9119	21.9114 0340	97
98	31.4932 7867	27.5901 8308	24.4646 0692	21.9247 8794	98
99	31.5468 7250	27.6233 6529	24.4851 9896	21.9375 9612	99
100	31.5989 0534	27.6554 2540	24.5049 9900	21.9498 5274	100

TABLE 7. $\log a_{\bar{m}i}$

n	3%	3½%	4%	4½%	n
51	1.414 1579	1.373 4383	1.334 8052	1.298 1531	51
52	1.417 7413	1.376 4996	1.337 4109	1.300 3634	52
53	1.421 1923	1.379 4371	1.339 9018	1.302 4681	53
54	1.424 5167	1.382 2566	1.342 2837	1.304 4728	54
55	1.427 7202	1.384 9634	1.344 5614	1.306 3825	55
56	1.430 8078	1.387 5626	1.346 7405	1.308 2021	56
57	1.433 7848	1.390 0594	1.348 8255	1.309 9363	57
58	1.436 6556	1.392 4580	1.350 8209	1.311 5893	58
59	1.439 4247	1.394 7633	1.352 7309	1.313 1652	59
60	1.442 0963	1.396 9788	1.354 5596	1.314 6681	60
61	1.444 6746	1.399 1089	1.356 3107	1.316 1012	61
62	1.447 1633	1.401 1569	1.357 9877	1.317 4682	62
63	1.449 5658	1.403 1266	1.359 5943	1.318 7724	63
64	1.451 8858	1.405 0212	1.361 1335	1.320 0168	64
65	1.454 1264	1.406 8440	1.362 6083	1.321 2043	65
66	1.456 2907	1.408 5979	1.364 0218	1.322 3376	66
67	1.458 3817	1.410 2858	1.365 3766	1.323 4193	67
68	1.460 4023	1.411 9103	1.366 6752	1.324 4520	68
69	1.462 3549	1.413 4742	1.367 9203	1.325 4379	69
70	1.464 2424	1.414 9800	1.369 1139	1.326 3791	70
71	1.466 0670	1.416 4297	1.370 2588	1.327 2780	71
72	1.467 8313	1.417 8259	1.371 3568	1.328 1365	72
73	1.469 5373	1.419 1708	1.372 4099	1.328 9562	73
74	1.471 1873	1.420 4660	1.373 4200	1.329 7395	74
75	1.472 7832	1.421 7139	1.374 3892	1.330 4874	75
76	1.474 3270	1.422 9161	1.375 3189	1.331 2020	76
77	1.475 8206	1.424 0746	1.376 2111	1.331 8850	77
78	1.477 2658	1.425 1910	1.377 0673	1.332 5373	78
79	1.478 6644	1.426 2668	1.377 8889	1.333 1607	79
80	1.480 0178	1.427 3040	1.378 6774	1.333 7563	80
81	1.481 3279	1.428 3034	1.379 4344	1.334 3256	81
82	1.482 5962	1.429 2670	1.380 1609	1.334 8697	82
83	1.483 8238	1.430 1960	1.380 8583	1.335 3897	83
84	1.485 0124	1.431 0916	1.381 5278	1.335 8868	84
85	1.486 1633	1.431 9552	1.382 1707	1.336 3619	85
86	1.487 2778	1.432 7881	1.382 7879	1.336 8160	86
87	1.488 3571	1.433 5912	1.383 3806	1.337 2501	87
88	1.489 4022	1.434 3657	1.383 9497	1.337 6652	88
89	1.490 4147	1.435 1128	1.384 4962	1.338 0620	89
90	1.491 3954	1.435 8335	1.385 0211	1.338 4413	90
91	1.492 3454	1.436 5285	1.385 5251	1.338 8041	91
92	1.493 2657	1.437 1991	1.386 0092	1.339 1510	92
93	1.494 1575	1.437 8460	1.386 4741	1.339 4824	93
94	1.495 0215	1.438 4700	1.386 9209	1.339 7997	94
95	1.495 8586	1.439 0720	1.387 3499	1.340 1029	95
96	1.496 6698	1.439 6529	1.387 7619	1.340 3929	96
97	1.497 4560	1.440 2136	1.388 1578	1.340 6702	97
98	1.498 2178	1.440 7545	1.388 5382	1.340 9354	98
99	1.498 9563	1.441 2766	1.388 9036	1.341 1890	99
100	1.499 6720	1.441 7804	1.389 2547	1.341 4316	100

TABLE 7. $a_{ni} = \frac{1 - (1 + i)^{-n}}{i}$

<i>n</i>	5%	5½%	6%	7%	<i>n</i>
1	0.9523 8095	0.9478 6730	0.9433 9623	0.9345 7944	1
2	1.8594 1043	1.8463 1971	1.8333 9267	1.8080 1817	2
3	2.7232 4803	2.6979 3338	2.6730 1195	2.6243 1604	3
4	3.5459 5050	3.5051 5012	3.4651 0561	3.3872 1126	4
5	4.3294 7667	4.2702 8448	4.2123 6379	4.1001 9744	5
6	5.0756 9206	4.9955 3031	4.9173 2433	4.7665 3966	6
7	5.7863 7340	5.6829 6712	5.5823 8144	5.3892 8940	7
8	6.4632 1276	6.3345 6599	6.2097 9381	5.9712 9851	8
9	7.1078 2168	6.9521 9525	6.8016 9227	6.5152 3225	9
10	7.7217 3493	7.5376 2583	7.3600 8705	7.0235 8154	10
11	8.3064 1422	8.0925 3633	7.8868 7458	7.4986 7434	11
12	8.8632 5164	8.6185 1785	8.3838 4394	7.9426 8630	12
13	9.3935 7299	9.1170 7853	8.8526 8296	8.3576 5074	13
14	9.8986 4094	9.5896 4790	9.2949 8393	8.7454 6799	14
15	10.3796 5804	10.0375 8094	9.7122 4899	9.1079 1401	15
16	10.8377 6956	10.4621 6203	10.1058 9527	9.4466 4860	16
17	11.2740 6625	10.8646 0856	10.4772 5969	9.7632 2299	17
18	11.6895 8690	11.2460 7447	10.8276 0348	10.0590 8691	18
19	12.0853 2086	11.6076 5352	11.1581 1649	10.3355 9524	19
20	12.4622 1034	11.9503 8249	11.4699 2122	10.5940 1425	20
21	12.8211 5271	12.2752 4406	11.7640 7662	10.8355 2733	21
22	13.1630 0258	12.5831 6973	12.0415 8172	11.0612 4050	22
23	13.4885 7388	12.8750 4240	12.3033 7898	11.2721 8738	23
24	13.7986 4179	13.1516 9895	12.5503 5753	11.4693 3400	24
25	14.0939 4457	13.4139 3266	12.7833 5616	11.6535 8318	25
26	14.3751 8530	13.6624 9541	13.0031 6619	11.8257 7867	26
27	14.6430 3362	13.8980 9991	13.2105 3414	11.9867 0904	27
28	14.8981 2726	14.1214 2172	13.4061 6428	12.1371 1125	28
29	15.1410 7358	14.3331 0116	13.5907 2102	12.2776 7407	29
30	15.3724 5103	14.5337 4517	13.7648 3115	12.4090 4118	30
31	15.5928 1050	14.7239 2907	13.9290 8599	12.5318 1419	31
32	15.8026 7667	14.9041 9817	14.0840 4339	12.6465 5532	32
33	16.0025 4921	15.0750 6936	14.2302 2961	12.7537 9002	33
34	16.1929 0401	15.2370 3257	14.3681 4114	12.8540 0936	34
35	16.3741 9429	15.3905 5220	14.4982 4636	12.9476 7230	35
36	16.5468 5171	15.5360 6843	14.6209 8713	13.0352 0776	36
37	16.7112 8734	15.6739 9851	14.7367 8031	13.1170 1660	37
38	16.8678 9271	15.8047 3793	14.8460 1916	13.1934 7345	38
39	17.0170 4067	15.9286 6154	14.9490 7468	13.2649 2846	39
40	17.1590 8635	16.0461 2469	15.0462 9687	13.3317 0884	40
41	17.2943 6796	16.1574 6416	15.1380 1592	13.3941 2041	41
42	17.4232 0758	16.2629 9920	15.2245 4332	13.4524 4898	42
43	17.5459 1198	16.3630 3242	15.3061 7294	13.5069 6167	43
44	17.6627 7331	16.4578 5063	15.3831 8202	13.5579 0810	44
45	17.7740 6982	16.5477 2572	15.4558 3209	13.6055 2159	45
46	17.8800 6650	16.6329 1537	15.5243 6990	13.6500 2018	46
47	17.9810 1571	16.7136 6386	15.5890 2821	13.6916 0764	47
48	18.0771 5782	16.7902 0271	15.6500 2661	13.7304 7443	48
49	18.1687 2173	16.8627 5139	15.7075 7227	13.7667 9853	49
50	18.2559 2546	16.9315 1790	15.7618 6064	13.8007 4629	50

TABLE 7. $\log a_{ni}$

n	5%	5½%	6%	7%	n
1	9.978 8108	9.976 7476	9.974 6942	9.970 6162	1
2	0.269 3752	0.266 3070	0.263 2555	0.257 2028	2
3	0.435 0872	0.431 0312	0.427 0009	0.419 0161	3
4	0.549 7327	0.544 7066	0.539 7165	0.529 8423	4
5	0.636 4354	0.630 4568	0.624 5259	0.612 8047	5
6	0.705 4952	0.698 5816	0.691 7288	0.678 2032	6
7	0.762 4065	0.754 5752	0.746 8195	0.731 5315	7
8	0.810 4485	0.801 7169	0.793 0772	0.776 0688	8
9	0.851 7365	0.842 1220	0.832 6170	0.813 9299	9
10	0.887 7149	0.877 2346	0.866 8829	0.846 5587	10
11	0.919 4136	0.908 0847	0.896 9049	0.874 9845	11
12	0.947 5930	0.935 4326	0.923 4432	0.899 9674	12
13	0.972 8308	0.959 8557	0.947 0749	0.922 0842	13
14	0.995 5756	0.981 8027	0.968 2486	0.941 7830	14
15	1.016 1831	1.001 6291	0.987 3198	0.959 4190	15
16	1.034 9399	1.019 6214	1.004 5748	0.975 2777	16
17	1.052 0806	1.036 0142	1.020 2477	0.989 5932	17
18	1.067 7992	1.051 0010	1.034 5323	1.002 5586	18
19	1.082 2582	1.064 7445	1.047 5910	1.014 3355	19
20	1.095 5950	1.077 3818	1.059 5604	1.025 0606	20
21	1.107 9271	1.089 0301	1.070 5578	1.034 8500	21
22	1.119 3550	1.099 7901	1.080 6836	1.043 8039	22
23	1.129 9661	1.109 7486	1.090 0244	1.052 0082	23
24	1.139 8363	1.118 9819	1.098 6561	1.059 5382	24
25	1.149 0326	1.127 5561	1.106 6449	1.066 4594	25
26	1.157 6135	1.135 5300	1.114 0492	1.072 8298	26
27	1.165 6311	1.142 9554	1.120 9203	1.078 7000	27
28	1.173 1317	1.149 8785	1.127 3045	1.084 1154	28
29	1.180 1567	1.156 3402	1.133 2425	1.089 1161	29
30	1.186 7431	1.162 3776	1.138 7709	1.093 7382	30
31	1.192 9244	1.168 0237	1.143 9226	1.098 0140	31
32	1.198 7306	1.173 3086	1.148 7273	1.101 9722	32
33	1.204 1892	1.178 2593	1.153 2119	1.105 6393	33
34	1.209 3247	1.182 9005	1.157 4007	1.109 0386	34
35	1.214 1599	1.187 2542	1.161 3155	1.112 1917	35
36	1.218 7154	1.191 3411	1.164 9767	1.115 1179	36
37	1.223 0099	1.195 1798	1.168 4026	1.117 8350	37
38	1.227 0609	1.198 7873	1.171 6101	1.120 3592	38
39	1.230 8840	1.202 1793	1.174 6143	1.122 7049	39
40	1.234 4942	1.205 3702	1.177 4297	1.124 8858	40
41	1.237 9047	1.208 3732	1.180 0690	1.126 9142	41
42	1.241 1281	1.211 2007	1.182 5443	1.128 8014	42
43	1.244 1759	1.213 8638	1.184 8666	1.130 5577	43
44	1.247 0589	1.216 3732	1.187 0462	1.132 1927	44
45	1.249 7869	1.218 7383	1.189 0924	1.133 7152	45
46	1.252 3691	1.220 9684	1.191 0140	1.135 1333	46
47	1.254 8142	1.223 0717	1.192 8191	1.136 4545	47
48	1.257 1302	1.225 0560	1.194 5150	1.137 6855	48
49	1.259 3244	1.226 9284	1.196 1090	1.138 8330	49
50	1.261 4037	1.228 6959	1.197 6075	1.139 9026	50

TABLE 7. $a_{\bar{n}|i} = \frac{1 - (1 + i)^{-n}}{i}$

n	5%	5½%	6%	7%	n
51	18.3389 7663	16.9966 9943	15.8130 7607	13.8324 7317	51
52	18.4180 7298	17.0584 8287	15.8613 9252	13.8621 2446	52
53	18.4934 0284	17.1170 4538	15.9069 7408	13.8898 3594	53
54	18.5651 4556	17.1725 5486	15.9499 7554	13.9157 3453	54
55	18.6334 7196	17.2251 7048	15.9905 4297	13.9399 3881	55
56	18.6985 4473	17.2750 4311	16.0288 1412	13.9625 5964	56
57	18.7605 1879	17.3223 1575	16.0649 1898	13.9837 0059	57
58	18.8195 4170	17.3671 2393	16.0989 8017	14.0034 5850	58
59	18.8757 5400	17.4095 9614	16.1311 1337	14.0219 2383	59
60	18.9292 8952	17.4498 5416	16.1614 2771	14.0391 8115	60
61	18.9802 7574	17.4880 1343	16.1900 2614	14.0553 0949	61
62	19.0288 3404	17.5241 8334	16.2170 0579	14.0703 8270	62
63	19.0750 8003	17.5584 6762	16.2424 5829	14.0844 6981	63
64	19.1191 2384	17.5909 6457	16.2664 7009	14.0976 3534	64
65	19.1610 7033	17.6217 6737	16.2891 2272	14.1099 3957	65
66	19.2010 1936	17.6509 6433	16.3104 9314	14.1214 3885	66
67	19.2390 6606	17.6786 3917	16.3306 5390	14.1321 8584	67
68	19.2753 0101	17.7048 7125	16.3496 7349	14.1422 2976	68
69	19.3098 1048	17.7297 3579	16.3676 1650	14.1516 1660	69
70	19.3426 7665	17.7533 0406	16.3845 4387	14.1603 8934	70
71	19.3739 7776	17.7756 4366	16.4005 1308	14.1685 8817	71
72	19.4037 8834	17.7968 1864	16.4155 7838	14.1762 5063	72
73	19.4321 7937	17.8168 8970	16.4297 9093	14.1834 1180	73
74	19.4592 1845	17.8359 1441	16.4431 9899	14.1901 0449	74
75	19.4849 6995	17.8539 4731	16.4558 4810	14.1963 5933	75
76	19.5094 9519	17.8710 4010	16.4677 8123	14.2022 0498	76
77	19.5328 5257	17.8872 4180	16.4790 3889	14.2076 6821	77
78	19.5550 9768	17.9025 9887	16.4896 5933	14.2127 7403	78
79	19.5762 8351	17.9171 5532	16.4996 7862	14.2175 4582	79
80	19.5964 6048	17.9309 5291	16.5091 3077	14.2220 0544	80
81	19.6156 7665	17.9440 3120	16.5180 4790	14.2261 7331	81
82	19.6339 7776	17.9564 2768	16.5264 6028	14.2300 6851	82
83	19.6514 0739	17.9681 7789	16.5343 9649	14.2337 0889	83
84	19.6680 0704	17.9793 1554	16.5418 8348	14.2371 1111	84
85	19.6838 1623	17.9898 7255	16.5489 4668	14.2402 9076	85
86	19.6988 7260	17.9998 7919	16.5556 1008	14.2432 6239	86
87	19.7132 1200	18.0093 6416	16.5618 9630	14.2460 3962	87
88	19.7268 6857	18.0183 5466	16.5678 2670	14.2486 3516	88
89	19.7398 7483	18.0268 7645	16.5734 2141	14.2510 6089	89
90	19.7522 6174	18.0349 5398	16.5786 9944	14.2533 2794	90
91	19.7640 5880	18.0426 1041	16.5836 7872	14.2554 4667	91
92	19.7752 9410	18.0498 6769	16.5883 7615	14.2574 2680	92
93	19.7859 9438	18.0567 4662	16.5928 0769	14.2592 7738	93
94	19.7961 8512	18.0632 6694	16.5969 8839	14.2610 0690	94
95	19.8058 9059	18.0694 4734	16.6009 3244	14.2626 2327	95
96	19.8151 3390	18.0753 0553	16.6046 5325	14.2641 3390	96
97	19.8239 3705	18.0808 5833	16.6081 6344	14.2655 4570	97
98	19.8323 2100	18.0861 2164	16.6114 7494	14.2668 6514	98
99	19.8403 0571	18.0911 1055	16.6145 9900	14.2680 9826	99
100	19.8479 1020	18.0958 3939	16.6175 4623	14.2692 5071	100

TABLE 7. $\log a_{\bar{n}i}$

n	5%	5½%	6%	7%	n
51	1.263 3751	1.230 3646	1.199 0164	1.140 8997	51
52	1.265 2441	1.231 9403	1.200 3412	1.141 8298	52
53	1.267 0168	1.233 4289	1.201 5874	1.142 6972	53
54	1.268 6984	1.234 8348	1.202 7601	1.143 5059	54
55	1.270 2937	1.236 1634	1.203 8631	1.144 2609	55
56	1.271 8077	1.237 4191	1.204 9012	1.144 9650	56
57	1.273 2448	1.238 6060	1.205 8785	1.146 6221	57
58	1.274 6089	1.239 7278	1.206 7983	1.145 2353	58
59	1.275 9042	1.240 7888	1.207 6642	1.146 8076	59
60	1.277 1343	1.241 7918	1.208 4797	1.147 3417	60
61	1.278 3026	1.242 7404	1.209 2476	1.147 8403	61
62	1.279 4121	1.243 6376	1.209 9707	1.148 3058	62
63	1.280 4663	1.244 4866	1.210 6517	1.148 7405	63
64	1.281 4679	1.245 2896	1.211 2933	1.149 1464	64
65	1.282 4197	1.246 0495	1.211 8976	1.149 5251	65
66	1.283 3242	1.246 7683	1.212 4670	1.149 8790	66
67	1.284 1840	1.247 4488	1.213 0036	1.150 2094	67
68	1.285 0011	1.248 0927	1.213 5090	1.150 5178	68
69	1.285 7780	1.248 7023	1.213 9855	1.150 8060	69
70	1.286 5166	1.249 2791	1.214 4342	1.151 0752	70
71	1.287 2188	1.249 8252	1.214 8573	1.151 3266	71
72	1.287 8865	1.250 3424	1.215 2562	1.151 5613	72
73	1.288 5215	1.250 8319	1.215 6320	1.151 7806	73
74	1.289 1254	1.251 2953	1.215 9863	1.151 9856	74
75	1.299 6997	1.251 7342	1.216 3203	1.152 1770	75
76	1.290 2461	1.252 1498	1.216 6350	1.152 3558	76
77	1.290 7656	1.252 5433	1.216 9319	1.152 5228	77
78	1.291 2600	1.252 9161	1.217 2117	1.152 6787	78
79	1.291 7301	1.253 2691	1.217 4754	1.152 8247	79
80	1.292 1776	1.253 6032	1.217 7242	1.152 9609	80
81	1.292 6033	1.253 9200	1.217 9588	1.153 0881	81
82	1.293 0083	1.254 2200	1.218 1797	1.153 2070	82
83	1.293 3937	1.254 5040	1.218 3884	1.153 3181	83
84	1.293 7604	1.254 7732	1.218 5848	1.153 4218	84
85	1.294 1094	1.255 0280	1.218 7704	1.153 5188	85
86	1.294 4413	1.255 2696	1.218 9451	1.153 6094	86
87	1.294 7573	1.255 4984	1.219 1101	1.153 6941	87
88	1.295 0581	1.255 7152	1.219 2656	1.153 7734	88
89	1.295 3444	1.255 9206	1.219 4121	1.153 8471	89
90	1.295 6168	1.256 1149	1.219 5504	1.153 9163	90
91	1.295 8761	1.256 2993	1.219 6809	1.153 9809	91
92	1.296 1228	1.256 4740	1.219 8039	1.154 0412	92
93	1.296 3579	1.256 6395	1.219 9199	1.154 0976	93
94	1.296 5815	1.256 7963	1.220 0293	1.154 1503	94
95	1.296 7943	1.256 9449	1.220 1323	1.154 1993	95
96	1.296 9969	1.257 0857	1.220 2297	1.154 2454	96
97	1.297 1899	1.257 2190	1.220 3216	1.154 2885	97
98	1.297 3735	1.257 3454	1.220 4082	1.154 3287	98
99	1.297 5484	1.257 4652	1.220 4898	1.154 3661	99
100	1.297 7148	1.257 5787	1.220 5670	1.154 4011	100



TABLE 8

$$\frac{1}{a_{\bar{m}i}}$$

TABLE 8. $\frac{1}{a_{\overline{n}|i}} = \frac{i}{1 - (1 + i)^{-n}}$ $\left[\frac{1}{s_{\overline{n}|i}} = \frac{1}{a_{\overline{n}|i}} - i \right]$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
1	1.0025 0000	1.0033 3333	1.0050 0000	1.0066 6667	1
2	0.5018 7578	0.5025 0139	0.5037 5312	0.5050 0554	2
3	0.3350 0139	0.3355 5802	0.3366 7221	0.3377 8762	3
4	0.2515 6445	0.2520 8680	0.2531 3279	0.2541 8051	4
5	0.2015 0250	0.2020 0444	0.2030 0997	0.2040 1772	5
6	0.1681 2803	0.1686 1650	0.1695 9546	0.1705 7709	6
7	0.1442 8928	0.1447 6824	0.1457 2854	0.1466 9198	7
8	0.1264 1035	0.1268 8228	0.1278 2886	0.1287 7907	8
9	0.1125 0462	0.1129 7118	0.1139 0736	0.1148 4763	9
10	0.1013 8015	0.1018 4248	0.1027 7057	0.1037 0321	10
11	0.0922 7840	0.0927 3736	0.0936 5903	0.0945 8572	11
12	0.0846 9370	0.0851 4990	0.0860 6643	0.0869 8843	12
13	0.0782 7595	0.0787 2989	0.0796 4224	0.0805 6052	13
14	0.0727 7510	0.0732 2716	0.0741 3609	0.0750 5141	14
15	0.0680 0777	0.0684 5825	0.0693 6436	0.0702 7734	15
16	0.0638 3642	0.0642 8557	0.0651 8937	0.0661 0049	16
17	0.0601 5587	0.0606 0389	0.0615 0579	0.0624 1546	17
18	0.0568 8433	0.0573 3140	0.0582 3173	0.0591 4030	18
19	0.0539 5722	0.0544 0348	0.0553 0253	0.0562 1027	19
20	0.0513 2288	0.0517 6844	0.0526 6645	0.0535 7362	20
21	0.0489 3947	0.0493 8445	0.0502 8163	0.0511 8843	21
22	0.0467 7278	0.0472 1726	0.0481 1380	0.0490 2041	22
23	0.0447 9455	0.0452 3861	0.0461 3465	0.0470 4123	23
24	0.0429 8121	0.0434 2492	0.0443 2061	0.0452 2729	24
25	0.0413 1298	0.0417 5640	0.0426 5186	0.0435 5876	25
26	0.0397 7312	0.0402 1630	0.0411 1163	0.0420 1886	26
27	0.0383 4736	0.0387 9035	0.0396 8565	0.0405 9331	27
28	0.0370 2347	0.0374 6632	0.0383 6167	0.0392 6983	28
29	0.0357 9093	0.0362 3367	0.0371 2914	0.0380 3789	29
30	0.0346 4059	0.0350 8325	0.0359 7892	0.0368 8832	30
31	0.0335 6449	0.0340 0712	0.0349 0304	0.0358 1316	31
32	0.0325 5569	0.0329 9830	0.0338 9453	0.0348 0542	32
33	0.0316 0806	0.0320 5067	0.0329 4727	0.0338 5898	33
34	0.0307 1620	0.0311 5885	0.0320 5586	0.0329 6843	34
35	0.0298 7533	0.0303 1803	0.0312 1550	0.0321 2898	35
36	0.0290 8121	0.0295 2399	0.0304 2194	0.0313 3637	36
37	0.0283 3004	0.0287 7291	0.0296 7139	0.0305 8680	37
38	0.0276 1843	0.0280 6141	0.0289 6045	0.0298 7687	38
39	0.0269 4335	0.0273 8644	0.0282 8607	0.0292 0354	39
40	0.0263 0204	0.0267 4527	0.0276 4552	0.0285 6406	40
41	0.0255 9204	0.0261 3543	0.0270 3631	0.0279 5595	41
42	0.0251 1112	0.0255 5466	0.0264 5622	0.0273 7697	42
43	0.0245 5724	0.0250 0095	0.0259 0320	0.0268 2509	43
44	0.0240 2855	0.0244 7246	0.0253 7541	0.0262 9847	44
45	0.0235 2339	0.0239 6749	0.0248 7117	0.0257 9541	45
46	0.0230 4022	0.0234 8451	0.0243 8894	0.0253 1439	46
47	0.0225 7762	0.0230 2213	0.0239 2733	0.0248 5399	47
48	0.0221 3433	0.0225 7905	0.0234 8503	0.0244 1292	48
49	0.0217 0915	0.0221 5410	0.0230 6087	0.0239 9001	49
50	0.0213 0099	0.0217 4618	0.0226 5376	0.0235 8416	50

TABLE 8. $\frac{1}{a_{\overline{n}|i}} = \frac{i}{1 - (1 + i)^{-n}}$ $\left[\frac{1}{s_{\overline{n}|i}} = \frac{1}{a_{\overline{n}|i}} - i \right]$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
51	0.0209 0886	0.0213 5429	0.0222 6269	0.0231 9437	51
52	0.0205 3184	0.0209 7751	0.0218 8675	0.0228 1971	52
53	0.0201 6906	0.0206 1499	0.0215 2507	0.0224 5932	53
54	0.0198 1974	0.0202 6592	0.0211 7686	0.0221 1242	54
55	0.0194 8314	0.0199 2958	0.0208 4139	0.0217 7827	55
56	0.0191 5858	0.0196 0529	0.0205 1797	0.0214 5618	56
57	0.0188 4542	0.0192 9241	0.0202 0598	0.0211 4552	57
58	0.0185 4308	0.0189 9035	0.0199 0481	0.0208 4569	58
59	0.0182 5101	0.0186 9856	0.0196 1392	0.0205 5616	59
60	0.0179 6869	0.0184 1652	0.0193 3280	0.0202 7639	60
61	0.0176 9564	0.0181 4377	0.0190 6096	0.0200 0592	61
62	0.0174 3142	0.0178 7984	0.0187 9796	0.0197 4429	62
63	0.0171 7561	0.0176 2432	0.0185 4337	0.0194 9108	63
64	0.0169 2780	0.0173 7681	0.0182 9681	0.0192 4590	64
65	0.0166 8764	0.0171 3695	0.0180 5789	0.0190 0837	65
66	0.0164 5476	0.0169 0438	0.0178 2627	0.0187 7815	66
67	0.0162 2886	0.0166 7878	0.0176 0163	0.0185 5491	67
68	0.0160 0961	0.0164 5985	0.0173 8366	0.0183 3835	68
69	0.0157 9674	0.0162 4729	0.0171 7206	0.0181 2816	69
70	0.0155 8996	0.0160 4083	0.0169 6657	0.0179 2409	70
71	0.0153 8902	0.0158 4021	0.0167 6693	0.0177 2586	71
72	0.0151 9366	0.0156 4518	0.0165 7289	0.0175 3324	72
73	0.0150 0370	0.0154 5553	0.0163 8422	0.0173 4600	73
74	0.0148 1887	0.0152 7103	0.0162 0070	0.0171 6391	74
75	0.0146 3898	0.0150 9147	0.0160 2214	0.0169 8678	75
76	0.0144 6385	0.0149 1666	0.0158 4832	0.0168 1440	76
77	0.0142 9327	0.0147 4641	0.0156 7908	0.0166 4659	77
78	0.0141 2708	0.0145 8056	0.0155 1423	0.0164 8318	78
79	0.0139 6511	0.0144 1892	0.0153 5360	0.0163 2400	79
80	0.0138 0721	0.0142 6135	0.0151 9704	0.0161 6889	80
81	0.0136 5321	0.0141 0770	0.0150 4439	0.0160 1769	81
82	0.0135 0298	0.0139 5781	0.0148 9552	0.0158 7027	82
83	0.0133 5639	0.0138 1156	0.0147 5028	0.0157 2649	83
84	0.0132 1330	0.0136 6881	0.0146 0855	0.0155 8621	84
85	0.0130 7359	0.0135 2944	0.0144 7021	0.0154 4933	85
86	0.0129 3714	0.0133 9333	0.0143 3513	0.0153 1570	86
87	0.0128 0384	0.0132 6038	0.0142 0320	0.0151 8524	87
88	0.0126 7357	0.0131 3046	0.0140 7431	0.0150 5781	88
89	0.0125 4625	0.0130 0349	0.0139 4837	0.0149 3334	89
90	0.0124 2177	0.0128 7936	0.0138 2527	0.0148 1170	90
91	0.0123 0004	0.0127 5797	0.0137 0493	0.0146 9282	91
92	0.0121 8096	0.0126 3925	0.0135 8724	0.0145 7660	92
93	0.0120 6446	0.0125 2310	0.0134 7213	0.0144 6296	93
94	0.0119 5044	0.0124 0944	0.0133 5950	0.0143 5181	94
95	0.0118 3884	0.0122 9819	0.0132 4930	0.0142 4308	95
96	0.0117 2957	0.0121 8928	0.0131 4143	0.0141 3668	96
97	0.0116 2257	0.0120 8263	0.0130 3583	0.0140 3255	97
98	0.0115 1776	0.0119 7818	0.0129 3242	0.0139 3062	98
99	0.0114 1508	0.0118 7585	0.0128 3115	0.0138 3082	99
100	0.0113 1446	0.0117 7559	0.0127 3194	0.0137 3308	100

TABLE 8. $\frac{1}{a_{ni}} = \frac{i}{1 - (1 + i)^{-n}}$ $\left[\frac{1}{s_{ni}} = \frac{1}{a_{ni}} - i \right]$

n	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{2}{3}\%$	n
101	0.0112 1584	0.0116 7734	0.0126 3473	0.0136 3735	101
102	0.0111 1917	0.0115 8103	0.0125 3947	0.0135 4357	102
103	0.0110 2439	0.0114 8660	0.0124 4611	0.0134 5168	103
104	0.0109 3144	0.0113 9401	0.0123 5457	0.0133 6162	104
105	0.0108 4027	0.0113 0320	0.0122 6481	0.0132 7334	105
106	0.0107 5083	0.0112 1413	0.0121 7679	0.0131 8680	106
107	0.0106 6307	0.0111 2673	0.0120 9045	0.0131 0194	107
108	0.0105 7694	0.0110 4097	0.0120 0575	0.0130 1871	108
109	0.0104 9241	0.0109 5680	0.0119 2264	0.0129 3708	109
110	0.0104 0942	0.0108 7417	0.0118 4107	0.0128 5700	110
111	0.0103 2793	0.0107 9306	0.0117 6102	0.0127 7842	111
112	0.0102 4791	0.0107 1340	0.0116 8242	0.0127 0131	112
113	0.0101 6932	0.0106 3518	0.0116 0526	0.0126 2562	113
114	0.0100 9211	0.0105 5834	0.0115 2948	0.0125 5132	114
115	0.0100 1625	0.0104 8285	0.0114 5506	0.0124 7838	115
116	0.0099 4172	0.0104 0868	0.0113 8195	0.0124 0675	116
117	0.0098 6846	0.0103 3579	0.0113 1013	0.0123 3641	117
118	0.0097 9646	0.0102 6416	0.0112 3956	0.0122 6732	118
119	0.0097 2567	0.0101 9374	0.0111 7021	0.0121 9944	119
120	0.0096 5608	0.0101 2451	0.0111 0205	0.0121 3276	120
121	0.0095 8764	0.0100 5645	0.0110 3505	0.0120 6724	121
122	0.0095 2033	0.0099 8951	0.0109 6918	0.0120 0284	122
123	0.0094 5412	0.0099 2367	0.0109 0441	0.0119 3955	123
124	0.0093 8899	0.0098 5892	0.0108 4072	0.0118 7734	124
125	0.0093 2491	0.0097 9521	0.0107 7808	0.0118 1618	125
126	0.0092 6186	0.0097 3253	0.0107 1647	0.0117 5604	126
127	0.0091 9981	0.0096 7085	0.0106 5586	0.0116 9690	127
128	0.0091 3873	0.0096 1015	0.0105 9623	0.0116 3875	128
129	0.0090 7861	0.0095 5040	0.0105 3755	0.0115 8154	129
130	0.0090 1942	0.0094 9159	0.0104 7981	0.0115 2527	130
131	0.0089 6115	0.0094 3368	0.0104 2298	0.0114 6992	131
132	0.0089 0376	0.0093 7667	0.0103 6704	0.0114 1545	132
133	0.0088 4725	0.0093 2053	0.0103 1197	0.0113 6185	133
134	0.0087 9159	0.0092 6524	0.0102 5775	0.0113 0910	134
135	0.0087 3675	0.0092 1079	0.0102 0436	0.0112 5719	135
136	0.0086 8274	0.0091 5715	0.0101 5179	0.0112 0609	136
137	0.0086 2952	0.0091 0430	0.0101 0002	0.0111 5578	137
138	0.0085 7707	0.0090 5223	0.0100 4902	0.0111 0625	138
139	0.0085 2539	0.0090 0093	0.0099 9879	0.0110 5749	139
140	0.0084 7446	0.0089 5037	0.0099 4930	0.0110 0947	140
141	0.0084 2425	0.0089 0054	0.0099 0055	0.0109 6218	141
142	0.0083 7476	0.0088 5143	0.0098 5250	0.0109 1560	142
143	0.0083 2597	0.0088 0301	0.0098 0516	0.0108 6972	143
144	0.0082 7787	0.0087 5528	0.0097 5850	0.0108 2453	144
145	0.0082 3043	0.0087 0822	0.0097 1252	0.0107 8000	145
146	0.0081 8365	0.0086 6182	0.0096 6719	0.0107 3613	146
147	0.0081 3752	0.0086 1607	0.0096 2250	0.0106 9291	147
148	0.0080 9201	0.0085 7094	0.0095 7844	0.0106 5031	148
149	0.0080 4712	0.0085 2643	0.0095 3500	0.0106 0833	149
150	0.0080 0284	0.0084 8252	0.0094 9217	0.0105 6695	150

TABLE 8. $\frac{1}{a_{\overline{n}|i}} = \frac{i}{1 - (1 + i)^{-n}}$ $\left[\frac{1}{s_{\overline{n}|i}} = \frac{1}{a_{\overline{n}|i}} - i \right]$

<i>n</i>	$\frac{1}{4}\%$	$\frac{1}{3}\%$	$\frac{1}{2}\%$	$\frac{3}{4}\%$	<i>n</i>
151	0.0079 5915	0.0084 3921	0.0094 4993	0.0105 2617	151
152	0.0079 1605	0.0083 9648	0.0094 0827	0.0104 8597	152
153	0.0078 7351	0.0083 5432	0.0093 6719	0.0104 4633	153
154	0.0078 3153	0.0083 1273	0.0093 2666	0.0104 0726	154
155	0.0077 9010	0.0082 7167	0.0092 8668	0.0103 6873	155
156	0.0077 4921	0.0082 3116	0.0092 4723	0.0103 3074	156
157	0.0077 0885	0.0081 9118	0.0092 0832	0.0102 9327	157
158	0.0076 6900	0.0081 5171	0.0091 6992	0.0102 5632	158
159	0.0076 2966	0.0081 1275	0.0091 3203	0.0102 1988	159
160	0.0075 9082	0.0080 7429	0.0090 9464	0.0101 8394	160
161	0.0075 5246	0.0080 3631	0.0090 5774	0.0101 4848	161
162	0.0075 1459	0.0079 9882	0.0090 2131	0.0101 1350	162
163	0.0074 7719	0.0079 6180	0.0089 8536	0.0100 7899	163
164	0.0074 4025	0.0079 2524	0.0089 4987	0.0100 4494	164
165	0.0074 0377	0.0078 8913	0.0089 1483	0.0100 1134	165
166	0.0073 6773	0.0078 5347	0.0088 8024	0.0099 7819	166
167	0.0073 3213	0.0078 1825	0.0088 4608	0.0099 4547	167
168	0.0072 9695	0.0077 8346	0.0088 1236	0.0099 1318	168
169	0.0072 6220	0.0077 4909	0.0087 7906	0.0098 8131	169
170	0.0072 2787	0.0077 1513	0.0087 4617	0.0098 4986	170
171	0.0071 9394	0.0076 8158	0.0087 1369	0.0098 1881	171
172	0.0071 6042	0.0076 4844	0.0086 8161	0.0097 8816	172
173	0.0071 2728	0.0076 1568	0.0086 4992	0.0097 5791	173
174	0.0070 9454	0.0075 8332	0.0086 1862	0.0097 2803	174
175	0.0070 6217	0.0075 5133	0.0085 8770	0.0096 9854	175
176	0.0070 3018	0.0075 1972	0.0085 5715	0.0096 6942	176
177	0.0069 9855	0.0074 8847	0.0085 2697	0.0096 4066	177
178	0.0069 6729	0.0074 5759	0.0084 9715	0.0096 1226	178
179	0.0069 3638	0.0074 2706	0.0084 6769	0.0095 8422	179
180	0.0069 0582	0.0073 9688	0.0084 3857	0.0095 5652	180
181	0.0068 7560	0.0073 6704	0.0084 0979	0.0095 2917	181
182	0.0068 4572	0.0073 3754	0.0083 8136	0.0095 0215	182
183	0.0068 1617	0.0073 0838	0.0083 5325	0.0094 7546	183
184	0.0067 8695	0.0072 7954	0.0083 2547	0.0094 4909	184
185	0.0067 5805	0.0072 5102	0.0082 9802	0.0094 2305	185
186	0.0067 2947	0.0072 2281	0.0082 7087	0.0093 9732	186
187	0.0067 0120	0.0071 9492	0.0082 4404	0.0093 7189	187
188	0.0066 7323	0.0071 6734	0.0082 1752	0.0093 4678	188
189	0.0066 4557	0.0071 4005	0.0081 9129	0.0093 2196	189
190	0.0066 1820	0.0071 1307	0.0081 6537	0.0092 9743	190
191	0.0065 9112	0.0070 8637	0.0081 3973	0.0092 7320	191
192	0.0065 6434	0.0070 5996	0.0081 1438	0.0092 4925	192
193	0.0065 3783	0.0070 3384	0.0080 8931	0.0092 2558	193
194	0.0065 1160	0.0070 0799	0.0080 6452	0.0092 0219	194
195	0.0064 8565	0.0069 8242	0.0080 4000	0.0091 7907	195
196	0.0064 5997	0.0069 5711	0.0080 1576	0.0091 5622	196
197	0.0064 3455	0.0069 3208	0.0079 9178	0.0091 3363	197
198	0.0064 0939	0.0069 0730	0.0079 6806	0.0091 1130	198
199	0.0063 8450	0.0068 8278	0.0079 4459	0.0090 8923	199
200	0.0063 5985	0.0068 5852	0.0079 2138	0.0090 6741	200

TABLE 8. $\frac{1}{a_{\bar{n}|i}} = \frac{i}{1 - (1+i)^{-n}}$ $\left[\frac{1}{s_{\bar{n}|i}} = \frac{1}{a_{\bar{n}|i}} - i \right]$

<i>n</i>	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	1 $\frac{1}{4}\%$	<i>n</i>
1	1.0075 0000	1.0087 5000	1.0100 0000	1.0125 0000	1
2	0.5056 3200	0.5065 7203	0.5075 1244	0.5093 9441	2
3	0.3383 4579	0.3391 8361	0.3400 2211	0.3417 0117	3
4	0.2547 0501	0.2554 9257	0.2562 8109	0.2578 6102	4
5	0.2045 2242	0.2052 8049	0.2060 3980	0.2075 6211	5
6	0.1710 6891	0.1718 0789	0.1725 4837	0.1740 3381	6
7	0.1471 7488	0.1479 0070	0.1486 2828	0.1500 8872	7
8	0.1292 5552	0.1299 7190	0.1306 9029	0.1321 3314	8
9	0.1153 1929	0.1160 2868	0.1167 4037	0.1181 7055	9
10	0.1041 7123	0.1048 7538	0.1055 8208	0.1070 0307	10
11	0.0950 5094	0.0957 5111	0.0964 5408	0.0978 6839	11
12	0.0874 5148	0.0881 4860	0.0888 4879	0.0902 5831	12
13	0.0810 2188	0.0817 1669	0.0824 1482	0.0838 2100	13
14	0.0755 1146	0.0762 0453	0.0769 0117	0.0783 0515	14
15	0.0707 3639	0.0714 2817	0.0721 2378	0.0735 2646	15
16	0.0665 5879	0.0672 4965	0.0679 4460	0.0693 4672	16
17	0.0628 7321	0.0635 6346	0.0642 5806	0.0656 6023	17
18	0.0595 9766	0.0602 8756	0.0609 8205	0.0623 8479	18
19	0.0566 6740	0.0573 5715	0.0580 5175	0.0594 5548	19
20	0.0540 3063	0.0547 2042	0.0554 1532	0.0568 2039	20
21	0.0516 4543	0.0523 3541	0.0530 3075	0.0544 3748	21
22	0.0494 7748	0.0501 6779	0.0508 6371	0.0522 7238	22
23	0.0474 9846	0.0481 8921	0.0488 8584	0.0502 9666	23
24	0.0456 8474	0.0463 7604	0.0470 7347	0.0484 8665	24
25	0.0440 1650	0.0447 0843	0.0454 0675	0.0468 2247	25
26	0.0424 7693	0.0431 6959	0.0438 6888	0.0452 8729	26
27	0.0410 5176	0.0417 4520	0.0424 4553	0.0438 6677	27
28	0.0397 2871	0.0404 2300	0.0411 2444	0.0425 4863	28
29	0.0384 9723	0.0391 9243	0.0398 9502	0.0413 2228	29
30	0.0373 4816	0.0380 4431	0.0387 4811	0.0401 7854	30
31	0.0362 7352	0.0369 7068	0.0376 7573	0.0391 0942	31
32	0.0352 6634	0.0359 6454	0.0366 7089	0.0381 0791	32
33	0.0343 2048	0.0350 1976	0.0357 2744	0.0371 6786	33
34	0.0334 3053	0.0341 3092	0.0348 3997	0.0362 8387	34
35	0.0325 9170	0.0332 9324	0.0340 0368	0.0354 5111	35
36	0.0317 9973	0.0325 0244	0.0332 1431	0.0346 6533	36
37	0.0310 5082	0.0317 5473	0.0324 6805	0.0339 2270	37
38	0.0303 4157	0.0310 4671	0.0317 6150	0.0332 1983	38
39	0.0296 6893	0.0303 7531	0.0310 9160	0.0325 5365	39
40	0.0290 3016	0.0297 3780	0.0304 5560	0.0319 2141	40
41	0.0284 2276	0.0291 3169	0.0298 5102	0.0313 2063	41
42	0.0278 4452	0.0285 5475	0.0292 7563	0.0307 4906	42
43	0.0272 9338	0.0280 0493	0.0287 2737	0.0302 0466	43
44	0.0267 6751	0.0274 8039	0.0282 0441	0.0296 8557	44
45	0.0262 6521	0.0269 7943	0.0277 0505	0.0291 9012	45
46	0.0257 8495	0.0265 0053	0.0272 2775	0.0287 1675	46
47	0.0253 2532	0.0260 4228	0.0267 7111	0.0282 6406	47
48	0.0248 8504	0.0256 0338	0.0263 3384	0.0278 3075	48
49	0.0244 6292	0.0251 8265	0.0259 1474	0.0274 1563	49
50	0.0240 5787	0.0247 7900	0.0255 1273	0.0270 1763	50

TABLE 8. $\frac{1}{a_{\overline{m}|i}} = \frac{i}{1 - (1 + i)^{-n}}$ $\left[\frac{1}{s_{\overline{m}|i}} = \frac{1}{a_{\overline{m}|i}} - i \right]$

<i>n</i>	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	1 $\frac{1}{4}\%$	<i>n</i>
51	0.0236 6888	0.0243 9142	0.0251 2680	0.0266 3571	51
52	0.0232 9503	0.0240 1899	0.0247 5603	0.0262 6897	52
53	0.0229 3546	0.0236 6084	0.0243 9956	0.0259 1653	53
54	0.0225 8938	0.0233 1619	0.0240 5658	0.0255 7760	54
55	0.0222 5605	0.0229 8430	0.0237 2637	0.0252 5145	55
56	0.0219 3478	0.0226 6449	0.0234 0823	0.0249 3739	56
57	0.0216 2496	0.0223 5611	0.0231 0156	0.0246 3478	57
58	0.0213 2597	0.0220 5858	0.0228 0573	0.0243 4303	58
59	0.0210 3727	0.0217 7135	0.0225 2020	0.0240 6158	59
60	0.0207 5836	0.0214 9390	0.0222 4445	0.0237 8993	60
61	0.0204 8873	0.0212 2575	0.0219 7800	0.0235 2758	61
62	0.0202 2795	0.0209 6644	0.0217 2041	0.0232 7410	62
63	0.0199 7560	0.0207 1557	0.0214 7125	0.0230 2904	63
64	0.0197 3127	0.0204 7273	0.0212 3013	0.0227 9203	64
65	0.0194 9460	0.0202 3754	0.0209 9667	0.0225 6268	65
66	0.0192 6524	0.0200 0968	0.0207 7052	0.0223 4065	66
67	0.0190 4286	0.0197 8879	0.0205 5136	0.0221 2560	67
68	0.0188 2716	0.0195 7459	0.0203 3888	0.0219 1724	68
69	0.0186 1785	0.0193 6677	0.0201 3280	0.0217 1527	69
70	0.0184 1464	0.0191 6506	0.0199 3282	0.0215 1941	70
71	0.0182 1728	0.0189 6921	0.0197 3870	0.0213 2941	71
72	0.0180 2554	0.0187 7897	0.0195 5019	0.0211 4501	72
73	0.0178 3917	0.0185 9411	0.0193 6706	0.0209 6600	73
74	0.0176 5796	0.0184 1441	0.0191 8910	0.0207 9215	74
75	0.0174 8170	0.0182 3966	0.0190 1609	0.0206 2325	75
76	0.0173 1020	0.0180 6967	0.0188 4784	0.0204 5910	76
77	0.0171 4328	0.0179 0426	0.0186 8416	0.0202 9953	77
78	0.0169 8074	0.0177 4324	0.0185 2488	0.0201 4435	78
79	0.0168 2244	0.0175 8645	0.0183 6984	0.0199 9341	79
80	0.0166 6821	0.0174 3374	0.0182 1885	0.0198 4652	80
81	0.0165 1790	0.0172 8494	0.0180 7180	0.0197 0356	81
82	0.0163 7136	0.0171 3992	0.0179 2851	0.0195 6437	82
83	0.0162 2847	0.0169 9854	0.0177 8886	0.0194 2881	83
84	0.0160 8908	0.0168 6067	0.0176 5273	0.0192 9675	84
85	0.0159 5308	0.0167 2619	0.0175 1998	0.0191 6808	85
86	0.0158 2034	0.0165 9497	0.0173 9050	0.0190 4267	86
87	0.0156 9076	0.0164 6691	0.0172 6417	0.0189 2041	87
88	0.0155 6423	0.0163 4190	0.0171 4089	0.0188 0119	88
89	0.0154 4064	0.0162 1982	0.0170 2056	0.0186 8490	89
90	0.0153 1989	0.0161 0060	0.0169 0306	0.0185 7146	90
91	0.0152 0190	0.0159 8413	0.0167 8832	0.0184 6076	91
92	0.0150 8657	0.0158 7031	0.0166 7624	0.0183 5271	92
93	0.0149 7382	0.0157 5908	0.0165 6673	0.0182 4724	93
94	0.0148 6356	0.0156 5033	0.0164 5971	0.0181 4425	94
95	0.0147 5571	0.0155 4401	0.0163 5511	0.0180 4366	95
96	0.0146 5020	0.0154 4002	0.0162 5284	0.0179 4540	96
97	0.0145 4696	0.0153 3829	0.0161 5284	0.0178 4941	97
98	0.0144 4592	0.0152 3877	0.0160 5503	0.0177 5560	98
99	0.0143 4701	0.0151 4137	0.0159 5936	0.0176 6391	99
100	0.0142 5017	0.0150 4604	0.0158 6574	0.0175 7428	100

TABLE 8. $\frac{1}{a_{\bar{n}|i}} = \frac{i}{1 - (1 + i)^{-n}}$ $\left[\frac{1}{s_{\bar{n}|i}} = \frac{1}{a_{\bar{n}|i}} - i \right]$

n	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	2%	$2\frac{1}{2}\%$	n
1	1.0150 0000	1.0175 0000	1.0200 0000	1.0250 0000	1
2	0.5112 7792	0.5131 6295	0.5150 4950	0.5188 2716	2
3	0.3433 8296	0.3450 6746	0.3467 5467	0.3501 3717	3
4	0.2594 4478	0.2610 3237	0.2626 2375	0.2658 1788	4
5	0.2090 8932	0.2106 2142	0.2121 5839	0.2152 4686	5
6	0.1755 2521	0.1770 2256	0.1785 2581	0.1815 4997	6
7	0.1515 5616	0.1530 3059	0.1545 1196	0.1574 9543	7
8	0.1335 8402	0.1350 4292	0.1365 0980	0.1394 6735	8
9	0.1196 0982	0.1210 5813	0.1225 1544	0.1254 5689	9
10	0.1084 3418	0.1098 7534	0.1113 2653	0.1142 5876	10
11	0.0992 9384	0.1007 3038	0.1021 7794	0.1051 0596	11
12	0.0916 7999	0.0931 1377	0.0945 5960	0.0974 8713	12
13	0.0852 4036	0.0866 7283	0.0881 1835	0.0910 4827	13
14	0.0797 2332	0.0811 5562	0.0826 0197	0.0855 3653	14
15	0.0749 4436	0.0763 7739	0.0778 2547	0.0807 6646	15
16	0.0707 6508	0.0721 9958	0.0736 5013	0.0765 9899	16
17	0.0670 7966	0.0685 1623	0.0699 6984	0.0729 2777	17
18	0.0638 0578	0.0652 4492	0.0667 0210	0.0696 7008	18
19	0.0608 7847	0.0623 2061	0.0637 8177	0.0667 6062	19
20	0.0582 4574	0.0596 9122	0.0611 5672	0.0641 4713	20
21	0.0558 6550	0.0573 1464	0.0587 8477	0.0617 8733	21
22	0.0537 0331	0.0551 5638	0.0566 3140	0.0596 4661	22
23	0.0517 3075	0.0531 8796	0.0546 6810	0.0576 9638	23
24	0.0499 2410	0.0513 8565	0.0528 7110	0.0559 1282	24
25	0.0482 6345	0.0497 2952	0.0512 2044	0.0542 7592	25
26	0.0467 3196	0.0482 0269	0.0496 9923	0.0527 6875	26
27	0.0453 1527	0.0467 9079	0.0482 9309	0.0513 7687	27
28	0.0440 0108	0.0454 8151	0.0469 8967	0.0500 8793	28
29	0.0427 7878	0.0442 6424	0.0457 7836	0.0488 9127	29
30	0.0416 3919	0.0431 2975	0.0446 4992	0.0477 7764	30
31	0.0405 7430	0.0420 7005	0.0435 9635	0.0467 3900	31
32	0.0395 7710	0.0410 7812	0.0426 1061	0.0457 6831	32
33	0.0386 4144	0.0401 4779	0.0416 8653	0.0448 5938	33
34	0.0377 6189	0.0392 7363	0.0408 1867	0.0440 0675	34
35	0.0369 3363	0.0384 5082	0.0400 0221	0.0432 0558	35
36	0.0361 5240	0.0376 7507	0.0392 3285	0.0424 5158	36
37	0.0354 1437	0.0369 4257	0.0385 0678	0.0417 4090	37
38	0.0347 1613	0.0362 4990	0.0378 2057	0.0410 7012	38
39	0.0340 5463	0.0355 9399	0.0371 7114	0.0404 3615	39
40	0.0334 2710	0.0349 7209	0.0365 5575	0.0398 3623	40
41	0.0328 3106	0.0343 8170	0.0359 7188	0.0392 6786	41
42	0.0322 6426	0.0338 2057	0.0354 1729	0.0387 2876	42
43	0.0317 2465	0.0332 8666	0.0348 8993	0.0382 1688	43
44	0.0312 1038	0.0327 7810	0.0343 8794	0.0377 3037	44
45	0.0307 1976	0.0322 9321	0.0339 0962	0.0372 6752	45
46	0.0302 5125	0.0318 3043	0.0334 5342	0.0368 2676	46
47	0.0298 0342	0.0313 8836	0.0330 1792	0.0364 0669	47
48	0.0293 7500	0.0309 6569	0.0326 0184	0.0360 0599	48
49	0.0289 6478	0.0305 6124	0.0322 0396	0.0356 2348	49
50	0.0285 7168	0.0301 7391	0.0318 2321	0.0352 5806	50

TABLE 8. $\frac{1}{a_{\overline{n}|i}} = \frac{i}{1 - (1 + i)^{-n}}$ $\left[\frac{1}{s_{\overline{n}|i}} = \frac{1}{a_{\overline{n}|i}} - i \right]$

<i>n</i>	1½%	1¾%	2%	2½%	<i>n</i>
51	0.0281 9469	0.0298 0269	0.0314 5856	0.0349 0870	51
52	0.0278 3287	0.0294 4665	0.0311 0909	0.0345 7446	52
53	0.0274 8537	0.0291 0492	0.0307 7392	0.0342 5449	53
54	0.0271 5138	0.0287 7672	0.0304 5226	0.0339 4799	54
55	0.0268 3018	0.0284 6129	0.0301 4337	0.0336 5419	55
56	0.0265 2106	0.0281 5795	0.0298 4656	0.0333 7243	56
57	0.0262 2341	0.0278 6606	0.0295 6120	0.0331 0204	57
58	0.0259 3661	0.0275 8503	0.0292 8667	0.0328 4244	58
59	0.0256 6012	0.0273 1430	0.0290 2243	0.0325 9307	59
60	0.0253 9343	0.0270 5336	0.0287 6797	0.0323 5340	60
61	0.0251 3604	0.0268 0172	0.0285 2278	0.0321 2294	61
62	0.0248 8751	0.0265 5892	0.0282 8643	0.0319 0126	62
63	0.0246 4741	0.0263 2455	0.0280 5848	0.0316 8790	63
64	0.0244 1534	0.0260 9821	0.0278 3855	0.0314 8249	64
65	0.0241 9094	0.0258 7952	0.0276 2624	0.0312 8463	65
66	0.0239 7386	0.0256 6813	0.0274 2122	0.0310 9398	66
67	0.0237 6376	0.0254 6372	0.0272 2316	0.0309 1021	67
68	0.0235 6033	0.0252 6596	0.0270 3173	0.0307 3300	68
69	0.0233 6329	0.0250 7459	0.0268 4665	0.0305 6206	69
70	0.0231 7235	0.0248 8930	0.0266 6765	0.0303 9712	70
71	0.0229 8727	0.0247 0985	0.0264 9446	0.0302 3790	71
72	0.0228 0779	0.0245 3600	0.0263 2683	0.0300 8417	72
73	0.0226 3368	0.0243 6750	0.0261 6454	0.0299 3568	73
74	0.0224 6473	0.0242 0413	0.0260 0736	0.0297 9222	74
75	0.0223 0072	0.0240 4570	0.0258 5508	0.0296 5358	75
76	0.0221 4146	0.0238 9200	0.0257 0751	0.0295 1956	76
77	0.0219 8676	0.0237 4284	0.0255 6447	0.0293 8997	77
78	0.0218 3645	0.0235 9806	0.0254 2576	0.0292 6463	78
79	0.0216 9036	0.0234 5748	0.0252 9123	0.0291 4338	79
80	0.0215 4832	0.0233 2093	0.0251 6071	0.0290 2605	80
81	0.0214 1019	0.0231 8828	0.0250 3405	0.0289 1248	81
82	0.0212 7583	0.0230 5936	0.0249 1110	0.0288 0254	82
83	0.0211 4509	0.0229 3406	0.0247 9173	0.0286 9608	83
84	0.0210 1784	0.0228 1223	0.0246 7581	0.0285 9298	84
85	0.0208 9396	0.0226 9375	0.0245 6321	0.0284 9310	85
86	0.0207 7333	0.0225 7850	0.0244 5381	0.0283 9633	86
87	0.0206 5584	0.0224 6636	0.0243 4750	0.0283 0255	87
88	0.0205 4138	0.0223 5724	0.0242 4416	0.0282 1165	88
89	0.0204 2984	0.0222 5102	0.0241 4370	0.0281 2353	89
90	0.0203 2113	0.0221 4760	0.0240 4602	0.0280 3809	90
91	0.0202 1516	0.0220 4690	0.0239 5101	0.0279 5523	91
92	0.0201 1182	0.0219 4882	0.0238 5859	0.0278 7486	92
93	0.0200 1104	0.0218 5327	0.0237 6868	0.0277 9690	93
94	0.0199 1273	0.0217 6017	0.0236 8118	0.0277 2126	94
95	0.0198 1681	0.0216 6944	0.0235 9602	0.0276 4786	95
96	0.0197 2321	0.0215 8101	0.0235 1313	0.0275 7662	96
97	0.0196 3186	0.0214 9480	0.0234 3242	0.0275 0747	97
98	0.0195 4268	0.0214 1074	0.0233 5383	0.0274 4034	98
99	0.0194 5560	0.0213 2876	0.0232 7729	0.0273 7517	99
100	0.0193 7057	0.0212 4880	0.0232 0274	0.0273 1188	100

TABLE 8. $\frac{1}{a_{\bar{n}|i}} = \frac{i}{1 - (1 + i)^{-n}}$ $\left[\frac{1}{s_{\bar{n}|i}} = \frac{1}{a_{\bar{n}|i}} - i \right]$

<i>n</i>	3%	3½%	4%	4½%	<i>n</i>
1	1.0300 0000	1.0350 0000	1.0400 0000	1.0450 0000	1
2	0.5226 1084	0.5264 0049	0.5301 9608	0.5339 9756	2
3	0.3535 3036	0.3569 3418	0.3603 4854	0.3637 7336	3
4	0.2690 2705	0.2722 5114	0.2754 9005	0.2787 4365	4
5	0.2183 5457	0.2214 8137	0.2246 2711	0.2277 9164	5
6	0.1845 9750	0.1876 6821	0.1907 6190	0.1938 7839	6
7	0.1605 0635	0.1635 4449	0.1666 0961	0.1697 0147	7
8	0.1424 5639	0.1454 7665	0.1485 2783	0.1516 0965	8
9	0.1284 3386	0.1314 4601	0.1344 9299	0.1375 7447	9
10	0.1172 3051	0.1202 4137	0.1232 9094	0.1263 7882	10
11	0.1080 7745	0.1110 9197	0.1141 4904	0.1172 4818	11
12	0.1004 6209	0.1034 8395	0.1065 5217	0.1096 6619	12
13	0.0940 2954	0.0970 6157	0.1001 4373	0.1032 7535	13
14	0.0885 2634	0.0915 7073	0.0946 6897	0.0978 2032	14
15	0.0837 6658	0.0868 2507	0.0899 4110	0.0931 1381	15
16	0.0796 1085	0.0826 8483	0.0858 2000	0.0890 1537	16
17	0.0759 5253	0.0790 4313	0.0821 9852	0.0854 1758	17
18	0.0727 0870	0.0758 1684	0.0789 9333	0.0822 3690	18
19	0.0698 1388	0.0729 4033	0.0761 3862	0.0794 0734	19
20	0.0672 1571	0.0703 6108	0.0735 8175	0.0768 7614	20
21	0.0648 7178	0.0680 3659	0.0712 8011	0.0746 0057	21
22	0.0627 4739	0.0659 3207	0.0691 9881	0.0725 4565	22
23	0.0608 1390	0.0640 1880	0.0673 0906	0.0706 8249	23
24	0.0590 4742	0.0622 7283	0.0655 8683	0.0689 8703	24
25	0.0574 2787	0.0606 7404	0.0640 1196	0.0674 3903	25
26	0.0559 3829	0.0592 0540	0.0625 6738	0.0660 2137	26
27	0.0545 6421	0.0578 5241	0.0612 3854	0.0647 1946	27
28	0.0532 9323	0.0566 0265	0.0600 1298	0.0635 2081	28
29	0.0521 1467	0.0554 4538	0.0588 7993	0.0624 1461	29
30	0.0510 1926	0.0543 7133	0.0578 3010	0.0613 9154	30
31	0.0499 9893	0.0533 7240	0.0568 5535	0.0604 4345	31
32	0.0490 4662	0.0524 4150	0.0559 4859	0.0595 6320	32
33	0.0481 5612	0.0515 7242	0.0551 0357	0.0587 4453	33
34	0.0473 2196	0.0507 5966	0.0543 1477	0.0579 8191	34
35	0.0465 3929	0.0499 9835	0.0535 7732	0.0572 7045	35
36	0.0458 0379	0.0492 8416	0.0528 8688	0.0566 0578	36
37	0.0451 1162	0.0486 1325	0.0522 3957	0.0559 8402	37
38	0.0444 5934	0.0479 8214	0.0516 3192	0.0554 0169	38
39	0.0438 4385	0.0473 8775	0.0510 6083	0.0548 5567	39
40	0.0432 6238	0.0468 2728	0.0505 2349	0.0543 4315	40
41	0.0427 1241	0.0462 9822	0.0500 1738	0.0538 6158	41
42	0.0421 9167	0.0457 9828	0.0495 4020	0.0534 0868	42
43	0.0416 9811	0.0453 2539	0.0490 9899	0.0529 8235	43
44	0.0412 2985	0.0448 7768	0.0486 6454	0.0525 8071	44
45	0.0407 8518	0.0444 5343	0.0482 6246	0.0522 0202	45
46	0.0403 6254	0.0440 5108	0.0478 8205	0.0518 4471	46
47	0.0399 6051	0.0436 6919	0.0475 2189	0.0515 0734	47
48	0.0395 7777	0.0433 0646	0.0471 8065	0.0511 8858	48
49	0.0392 1314	0.0429 6167	0.0468 5712	0.0508 8722	49
50	0.0388 6550	0.0426 3371	0.0465 5020	0.0506 0215	50

TABLE 8. $\frac{1}{a_{\overline{n}|i}} = \frac{i}{1 - (1 + i)^{-n}}$ $\left[\frac{1}{s_{\overline{n}|i}} = \frac{1}{a_{\overline{n}|i}} - i \right]$

<i>n</i>	3%	3½%	4%	4½%	<i>n</i>
51	0.0385 3382	0.0423 2156	0.0462 5885	0.0503 3232	51
52	0.0382 1718	0.0420 2429	0.0459 8212	0.0500 7679	52
53	0.0379 1471	0.0417 4100	0.0457 1915	0.0498 3469	53
54	0.0376 2558	0.0414 7090	0.0454 6910	0.0496 0519	54
55	0.0373 4907	0.0412 1323	0.0452 3124	0.0493 8754	55
56	0.0370 8447	0.0409 6730	0.0450 0487	0.0491 8105	56
57	0.0368 3114	0.0407 3245	0.0447 8932	0.0489 8506	57
58	0.0365 8848	0.0405 0810	0.0445 8401	0.0487 9897	58
59	0.0363 5593	0.0402 9366	0.0443 8836	0.0486 2221	59
60	0.0361 3296	0.0400 8862	0.0442 0185	0.0484 5426	60
61	0.0359 1908	0.0398 9249	0.0440 2398	0.0482 9462	61
62	0.0357 1385	0.0397 0480	0.0438 5430	0.0481 4284	62
63	0.0355 1682	0.0395 2513	0.0436 9237	0.0479 9848	63
64	0.0353 2760	0.0393 5308	0.0435 3780	0.0478 6115	64
65	0.0351 4581	0.0391 8826	0.0433 9019	0.0477 3047	65
66	0.0349 7110	0.0390 3031	0.0432 4921	0.0476 0608	66
67	0.0348 0313	0.0388 7892	0.0431 1451	0.0474 8765	67
68	0.0346 4159	0.0387 3375	0.0429 8578	0.0473 7487	68
69	0.0344 8618	0.0385 9453	0.0428 6272	0.0472 6745	69
70	0.0343 3663	0.0384 6095	0.0427 4506	0.0471 6511	70
71	0.0341 9266	0.0383 3277	0.0426 3253	0.0470 6759	71
72	0.0340 5404	0.0382 0973	0.0425 2489	0.0469 7465	72
73	0.0339 2053	0.0380 9160	0.0424 2190	0.0468 8606	73
74	0.0337 9191	0.0379 7816	0.0423 2334	0.0468 0159	74
75	0.0336 6796	0.0378 6919	0.0422 2900	0.0467 2104	75
76	0.0335 4849	0.0377 6450	0.0421 3869	0.0466 4422	76
77	0.0334 3331	0.0376 6390	0.0420 5221	0.0465 7094	77
78	0.0333 2224	0.0375 6721	0.0419 6939	0.0465 0104	78
79	0.0332 1510	0.0374 7426	0.0418 9007	0.0464 3434	79
80	0.0331 1175	0.0373 8489	0.0418 1408	0.0463 7069	80
81	0.0330 1201	0.0372 9894	0.0417 4127	0.0463 0995	81
82	0.0329 1576	0.0372 1628	0.0416 7150	0.0462 5197	82
83	0.0328 2284	0.0371 3676	0.0416 0463	0.0461 9663	83
84	0.0327 3313	0.0370 6025	0.0415 4054	0.0461 4379	84
85	0.0326 4650	0.0369 8662	0.0414 7909	0.0460 9334	85
86	0.0325 6284	0.0369 1576	0.0414 2018	0.0460 4516	86
87	0.0324 8202	0.0368 4756	0.0413 6370	0.0459 9915	87
88	0.0324 0393	0.0367 8190	0.0413 0953	0.0459 5522	88
89	0.0323 2848	0.0367 1868	0.0412 5758	0.0459 1325	89
90	0.0322 5556	0.0366 5781	0.0412 0775	0.0458 7316	90
91	0.0321 8508	0.0365 9919	0.0411 5995	0.0458 3486	91
92	0.0321 1694	0.0365 4273	0.0411 1410	0.0457 9827	92
93	0.0320 5107	0.0364 8834	0.0410 7010	0.0457 6331	93
94	0.0319 8737	0.0364 3594	0.0410 2789	0.0457 2991	94
95	0.0319 2577	0.0363 8546	0.0409 8738	0.0456 9799	95
96	0.0318 6619	0.0363 3682	0.0409 4850	0.0456 6749	96
97	0.0318 0856	0.0362 8995	0.0409 1119	0.0456 3834	97
98	0.0317 5281	0.0362 4478	0.0408 7538	0.0456 1048	98
99	0.0316 9886	0.0362 0124	0.0408 4100	0.0455 8385	99
100	0.0316 4667	0.0361 5927	0.0408 0800	0.0455 5839	100

TABLE 8. $\frac{1}{a_{ni}} = \frac{i}{1 - (1 + i)^{-n}}$ $\left[\frac{1}{s_{ni}} = \frac{1}{a_{ni}} - i \right]$

<i>n</i>	5%	5½%	6%	7%	<i>n</i>
1	1.0500 0000	1.0550 0000	1.0600 0000	1.0700 0000	1
2	0.5378 0488	0.5416 1800	0.5454 3689	0.5530 9179	2
3	0.3672 0856	0.3706 5407	0.3741 0981	0.3810 5166	3
4	0.2820 1183	0.2852 9449	0.2885 9149	0.2952 2812	4
5	0.2309 7480	0.2341 7644	0.2373 9640	0.2438 9069	5
6	0.1970 1747	0.2001 7895	0.2033 6263	0.2097 9580	6
7	0.1728 1982	0.1759 6442	0.1791 3502	0.1855 5322	7
8	0.1547 2181	0.1578 6401	0.1610 3594	0.1674 6776	8
9	0.1406 9008	0.1438 3946	0.1470 2224	0.1534 8647	9
10	0.1295 0458	0.1326 6777	0.1358 6796	0.1423 7750	10
11	0.1203 8889	0.1235 7065	0.1267 9294	0.1333 5690	11
12	0.1128 2541	0.1160 2923	0.1192 7703	0.1259 0199	12
13	0.1064 5577	0.1096 8426	0.1129 6011	0.1196 5085	13
14	0.1010 2397	0.1042 7912	0.1075 8491	0.1143 4494	14
15	0.0963 4229	0.0996 2560	0.1029 6276	0.1097 9462	15
16	0.0922 6991	0.0955 8254	0.0989 5214	0.1058 5765	16
17	0.0886 9914	0.0920 4197	0.0954 4480	0.1024 2519	17
18	0.0855 4622	0.0889 1992	0.0923 5654	0.0994 1260	18
19	0.0827 4501	0.0861 5006	0.0896 2086	0.0967 5301	19
20	0.0802 4259	0.0836 7933	0.0871 8456	0.0943 9293	20
21	0.0779 9611	0.0814 6478	0.0850 0455	0.0922 8900	21
22	0.0759 7051	0.0794 7123	0.0830 4557	0.0904 0577	22
23	0.0741 3682	0.0776 6965	0.0812 7848	0.0887 1393	23
24	0.0724 7090	0.0760 3580	0.0796 7900	0.0871 8902	24
25	0.0709 5246	0.0745 4935	0.0782 2672	0.0858 1052	25
26	0.0695 6432	0.0731 9307	0.0769 0435	0.0845 6103	26
27	0.0682 9186	0.0719 5228	0.0756 9717	0.0834 2573	27
28	0.0671 2253	0.0708 1440	0.0745 9255	0.0823 9193	28
29	0.0660 4551	0.0697 6857	0.0735 7961	0.0814 4865	29
30	0.0650 5144	0.0688 0539	0.0726 4891	0.0805 8640	30
31	0.0641 3212	0.0679 1665	0.0717 9222	0.0797 9691	31
32	0.0632 8042	0.0670 9519	0.0710 0234	0.0790 7292	32
33	0.0624 9004	0.0663 3469	0.0702 7293	0.0784 0807	33
34	0.0617 5545	0.0656 2958	0.0695 9843	0.0777 9674	34
35	0.0610 7171	0.0649 7493	0.0689 7386	0.0772 3396	35
36	0.0604 3446	0.0643 6635	0.0683 9483	0.0767 1531	36
37	0.0598 3979	0.0637 9993	0.0678 5743	0.0762 3685	37
38	0.0592 8423	0.0632 7217	0.0673 5812	0.0757 9505	38
39	0.0587 6462	0.0627 7991	0.0668 9377	0.0753 8676	39
40	0.0582 7816	0.0623 2034	0.0664 6154	0.0750 0914	40
41	0.0578 2229	0.0618 9090	0.0660 5886	0.0746 5962	41
42	0.0573 9471	0.0614 8927	0.0656 8342	0.0743 3591	42
43	0.0569 9333	0.0611 1337	0.0653 3312	0.0740 3590	43
44	0.0566 1625	0.0607 6128	0.0650 0606	0.0737 5769	44
45	0.0562 6173	0.0604 3127	0.0647 0050	0.0734 9957	45
46	0.0559 2820	0.0601 2175	0.0644 1485	0.0732 5996	46
47	0.0556 1421	0.0598 3129	0.0641 4768	0.0730 3744	47
48	0.0553 1843	0.0595 5854	0.0638 9766	0.0728 3070	48
49	0.0550 3965	0.0593 0230	0.0636 6356	0.0726 3853	49
50	0.0547 7674	0.0590 6145	0.0634 4429	0.0724 5985	50

TABLE 8. $\frac{1}{a_{ni}} = \frac{i}{1 - (1 + i)^{-n}}$ $\left[\frac{1}{s_{ni}} = \frac{1}{a_{ni}} - i \right]$

<i>n</i>	5%	5½%	6%	7%	<i>n</i>
51	0.0545 2867	0.0588 3495	0.0632 3880	0.0722 9365	51
52	0.0542 9450	0.0586 2186	0.0630 4617	0.0721 3901	52
53	0.0540 7334	0.0584 2130	0.0628 6551	0.0719 9509	53
54	0.0538 6438	0.0582 3245	0.0626 9602	0.0718 6110	54
55	0.0536 6686	0.0580 5458	0.0625 3696	0.0717 3633	55
56	0.0534 8010	0.0578 8698	0.0623 8765	0.0716 2011	56
57	0.0533 0343	0.0577 2900	0.0622 4744	0.0715 1183	57
58	0.0531 3626	0.0575 8006	0.0621 1574	0.0714 1093	58
59	0.0529 7802	0.0574 3959	0.0619 9200	0.0713 1689	59
60	0.0528 2818	0.0573 0707	0.0618 7572	0.0712 2923	60
61	0.0526 8627	0.0571 8202	0.0617 6642	0.0711 4749	61
62	0.0525 5183	0.0570 6400	0.0616 6366	0.0710 7127	62
63	0.0524 2442	0.0569 5258	0.0615 6704	0.0710 0019	63
64	0.0523 0365	0.0568 4737	0.0614 7615	0.0709 3388	64
65	0.0521 8915	0.0567 4800	0.0613 9066	0.0708 7203	65
66	0.0520 8057	0.0566 5413	0.0613 1022	0.0708 1431	66
67	0.0519 7757	0.0565 6544	0.0612 3454	0.0707 6046	67
68	0.0518 7986	0.0564 8163	0.0611 6330	0.0707 1021	68
69	0.0517 8715	0.0564 0242	0.0610 9625	0.0706 6331	69
70	0.0516 9915	0.0563 2754	0.0610 3313	0.0706 1953	70
71	0.0516 1563	0.0562 5675	0.0609 7370	0.0705 7866	71
72	0.0515 3633	0.0561 8982	0.0609 1774	0.0705 4051	72
73	0.0514 6103	0.0561 2652	0.0608 6505	0.0705 0490	73
74	0.0513 8953	0.0560 6665	0.0608 1542	0.0704 7164	74
75	0.0513 2161	0.0560 1002	0.0607 6867	0.0704 4060	75
76	0.0512 5709	0.0559 5645	0.0607 2463	0.0704 1160	76
77	0.0511 9580	0.0559 0577	0.0606 8315	0.0703 8453	77
78	0.0511 3756	0.0558 5781	0.0606 4407	0.0703 5924	78
79	0.0510 8222	0.0558 1243	0.0606 0724	0.0703 3563	79
80	0.0510 2962	0.0557 6948	0.0605 7254	0.0703 1357	80
81	0.0509 7963	0.0557 2884	0.0605 3984	0.0702 9297	81
82	0.0509 3211	0.0556 9036	0.0605 0903	0.0702 7373	82
83	0.0508 8694	0.0556 5395	0.0604 7998	0.0702 5576	83
84	0.0508 4399	0.0556 1947	0.0604 5261	0.0702 3897	84
85	0.0508 0316	0.0555 8683	0.0604 2681	0.0702 2329	85
86	0.0507 6433	0.0555 5593	0.0604 0249	0.0702 0863	86
87	0.0507 2740	0.0555 2667	0.0603 7956	0.0701 9495	87
88	0.0506 9228	0.0554 9896	0.0603 5795	0.0701 8216	88
89	0.0506 5888	0.0554 7273	0.0603 3757	0.0701 7021	89
90	0.0506 2711	0.0554 4788	0.0603 1836	0.0701 5905	90
91	0.0505 9689	0.0554 2435	0.0603 0025	0.0701 4863	91
92	0.0505 6815	0.0554 0207	0.0602 8318	0.0701 3888	92
93	0.0505 4080	0.0553 8096	0.0602 6708	0.0701 2978	93
94	0.0505 1478	0.0553 6097	0.0602 5190	0.0701 2128	94
95	0.0504 9003	0.0553 4204	0.0602 3758	0.0701 1333	95
96	0.0504 6648	0.0553 2410	0.0602 2408	0.0701 0590	96
97	0.0504 4407	0.0553 0711	0.0602 1135	0.0700 9897	97
98	0.0504 2274	0.0552 9101	0.0601 9935	0.0700 9248	98
99	0.0504 0245	0.0552 7577	0.0601 8803	0.0700 8643	99
100	0.0503 8314	0.0552 6132	0.0601 7736	0.0700 8076	100

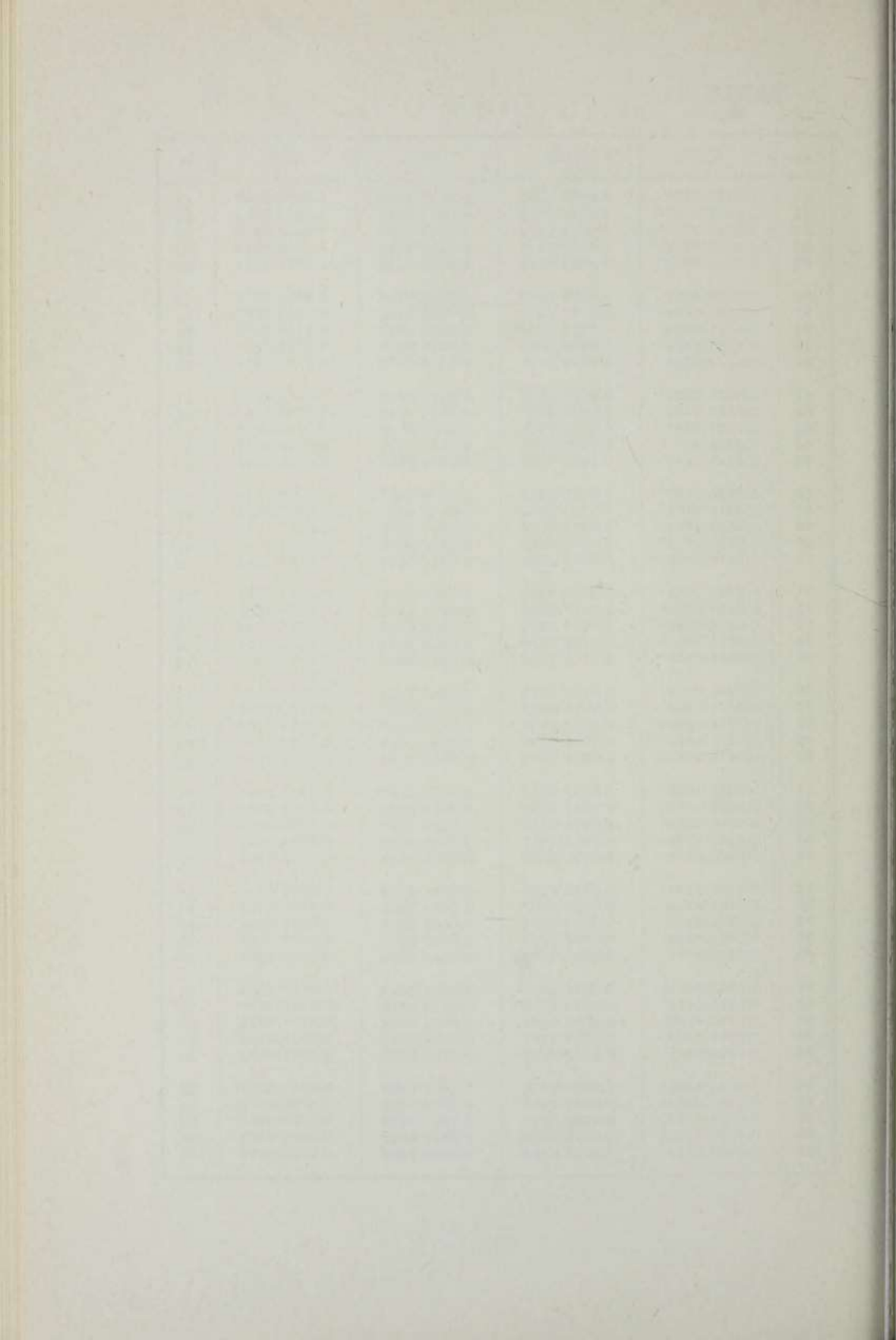


TABLE 9

$$\frac{1}{s_{\frac{1}{p}}|_i} \text{ and } \log \frac{1}{s_{\frac{1}{p}}|_i}$$

TABLE 9. $\frac{1}{S_{\frac{1}{p}} i} = \frac{i}{(1+i)^{\frac{1}{p}} - 1}$

p	1/4%	7/24%	1/3%	5/12%	p
2	2.0012 4922	2.0014 5727	2.0016 6528	2.0020 8117	2
3	3.0024 9861	3.0029 1478	3.0033 3087	3.0041 6282	3
4	4.0037 4805	4.0043 7235	4.0049 9654	4.0062 4459	4
6	6.0062 4696	6.0072 8756	6.0083 2795	6.0104 0824	6
12	12.0137 4384	12.0160 3328	12.0183 2234	12.0228 9946	12
p	1/2%	7/12%	5/8%	2/3%	p
2	2.0024 9688	2.0029 1243	2.0031 2014	2.0033 2780	2
3	3.0049 9446	3.0058 2579	3.0062 4135	3.0066 5682	3
4	4.0074 9221	4.0087 3940	4.0093 6283	4.0099 8616	4
6	6.0124 8788	6.0145 6684	6.0156 0607	6.0166 4513	6
12	12.0274 7526	12.0320 4968	12.0343 3633	12.0366 2270	12
p	3/4%	7/8%	1%	1 1/8%	p
2	2.0037 4300	2.0043 6547	2.0049 8756	2.0056 0927	2
3	3.0074 8755	3.0087 3306	3.0099 7789	3.0112 2203	3
4	4.0112 3249	4.0131 0118	4.0149 6891	4.0168 3567	4
6	6.0187 2276	6.0218 3795	6.0249 5163	6.0280 9600	6
12	12.0411 9435	12.0480 4930	12.0549 0119	12.0618 1437	12
p	1 1/4%	1 3/8%	1 1/2%	1 3/4%	p
2	2.0062 3059	2.0068 5153	2.0074 7208	2.0087 1205	2
3	3.0124 6549	3.0137 0827	3.0149 5037	3.0174 3253	3
4	4.0187 0147	4.0205 6648	4.0224 3021	4.0261 5513	4
6	6.0311 7452	6.0342 8372	6.0373 9144	6.0436 0242	6
12	12.0685 9580	12.0754 3856	12.0822 7820	12.0959 4852	12
p	2%	2 1/4%	2 1/2%	2 3/4%	p
2	2.0099 5049	2.0111 8742	2.0124 2284	2.0136 5675	2
3	3.0199 1199	3.0223 8875	3.0248 6282	3.0273 3422	3
4	4.0298 7623	4.0335 9356	4.0373 0709	4.0410 1686	4
6	6.0498 0747	6.0560 0662	6.0621 9991	6.0683 8735	6
12	12.1096 0670	12.1232 5281	12.1368 8697	12.1505 0916	12
p	3%	3 1/2%	4%	4 1/2%	p
2	2.0148 8916	2.0173 4950	2.0198 0390	2.0222 5241	2
3	3.0298 0294	3.0347 3244	3.0396 5138	3.0445 5985	3
4	4.0447 2289	4.0521 2375	4.0595 0975	4.0668 8103	4
6	6.0745 6894	6.0869 1473	6.0992 3739	6.1115 3716	6
12	12.1641 1941	12.1913 0435	12.2184 4211	12.2455 3306	12
p	5%	5 1/2%	6%	6 1/2%	p
2	2.0246 9508	2.0271 3193	2.0295 6301	2.0319 8837	2
3	3.0494 5791	3.0543 4565	3.0592 2313	3.0640 9043	3
4	4.0742 3769	4.0815 7982	4.0889 0752	4.0962 2091	4
6	6.1238 1418	6.1360 6860	6.1483 0059	6.1605 1031	6
12	12.2725 7753	12.2995 7585	12.3265 2834	12.3534 3534	12
p	7%	7 1/2%	8%	8 1/2%	p
2	2.0344 0804	2.0368 2207	2.0392 3048	2.0416 3333	2
3	3.0689 4762	3.0737 9477	3.0786 3195	3.0834 5922	3
4	4.1035 2009	4.1108 0514	4.1180 7618	4.1253 3329	4
6	6.1726 9791	6.1848 6355	6.1970 0737	6.2091 2954	6
12	12.3802 9716	12.4071 1409	12.4338 8648	12.4606 1463	12

TABLE 9. $\log \frac{1}{s_{\frac{1}{2}}^i}$

p	1/4%	7/24%	1/3%	5/12%	p
2	0.301 3012	0.301 3463	0.301 3915	0.301 4817	2
3	0.477 4829	0.477 5431	0.477 6032	0.477 7235	3
4	0.602 4668	0.602 5345	0.602 6022	0.602 7375	4
6	0.778 6032	0.778 6785	0.778 7537	0.778 9040	6
12	1.079 6783	1.079 7611	1.079 8438	1.080 0093	12
p	1/2%	7/12%	5/8%	2/3%	p
2	0.301 5719	0.301 6620	0.301 7070	0.301 7520	2
3	0.477 8437	0.477 9639	0.478 0239	0.478 0839	3
4	0.602 8727	0.603 0078	0.603 0754	0.603 1429	4
6	0.779 0543	0.779 2044	0.779 2794	0.779 3544	6
12	1.080 1744	1.080 3396	1.080 4221	1.080 5046	12
p	3/4%	7/8%	1%	1 1/8%	p
2	0.301 8420	0.301 9769	0.302 1117	0.302 2463	2
3	0.478 2039	0.478 3837	0.478 5634	0.478 7428	3
4	0.603 2779	0.603 4801	0.603 6822	0.603 8841	4
6	0.779 5044	0.779 7291	0.779 9536	0.780 1779	6
12	1.080 6695	1.080 9167	1.081 1636	1.081 4103	12
p	1 1/4%	1 3/8%	1 1/2%	1 3/4%	p
2	0.302 3809	0.302 5152	0.302 6495	0.302 9177	2
3	0.478 9221	0.479 1013	0.479 2802	0.479 6376	3
4	0.604 0858	0.604 2872	0.604 4885	0.604 8905	4
6	0.780 4019	0.780 6258	0.780 8494	0.781 2959	6
12	1.081 6567	1.081 9029	1.082 1488	1.082 6399	12
p	2%	2 1/4%	2 1/2%	2 3/4%	p
2	0.303 1854	0.303 4525	0.303 7192	0.303 9854	2
3	0.479 9943	0.480 3504	0.480 7057	0.481 0604	3
4	0.605 2917	0.605 6921	0.606 0918	0.606 4907	4
6	0.781 7416	0.782 1864	0.782 6303	0.783 0733	6
12	1.083 1300	1.083 6191	1.084 1073	1.084 5944	12
p	3%	3 1/2%	4%	4 1/2%	p
2	0.304 2512	0.304 7811	0.305 3092	0.305 8354	2
3	0.481 4144	0.482 1204	0.482 8238	0.483 5246	3
4	0.606 8888	0.607 6827	0.608 4736	0.609 2615	4
6	0.783 5155	0.784 3973	0.785 2756	0.786 1505	6
12	1.085 0806	1.086 0501	1.087 0158	1.087 9777	12
p	5%	5 1/2%	6%	6 1/2%	p
2	0.306 3596	0.306 8820	0.307 4025	0.307 9212	2
3	0.484 2227	0.484 9182	0.485 6112	0.486 3016	3
4	0.610 0464	0.610 8283	0.611 6073	0.612 3834	4
6	0.787 0221	0.787 8903	0.788 7551	0.789 6168	6
12	1.088 9357	1.089 8901	1.090 8407	1.091 7877	12
p	7%	7 1/2%	8%	8 1/2%	p
2	0.308 4381	0.308 9531	0.309 4663	0.309 9778	2
3	0.486 9895	0.487 6750	0.488 3578	0.489 0383	3
4	0.613 1566	0.613 9269	0.614 6944	0.615 4590	4
6	0.790 4751	0.791 3302	0.792 1820	0.793 0308	6
12	1.092 7310	1.093 6707	1.094 6069	1.095 5394	12

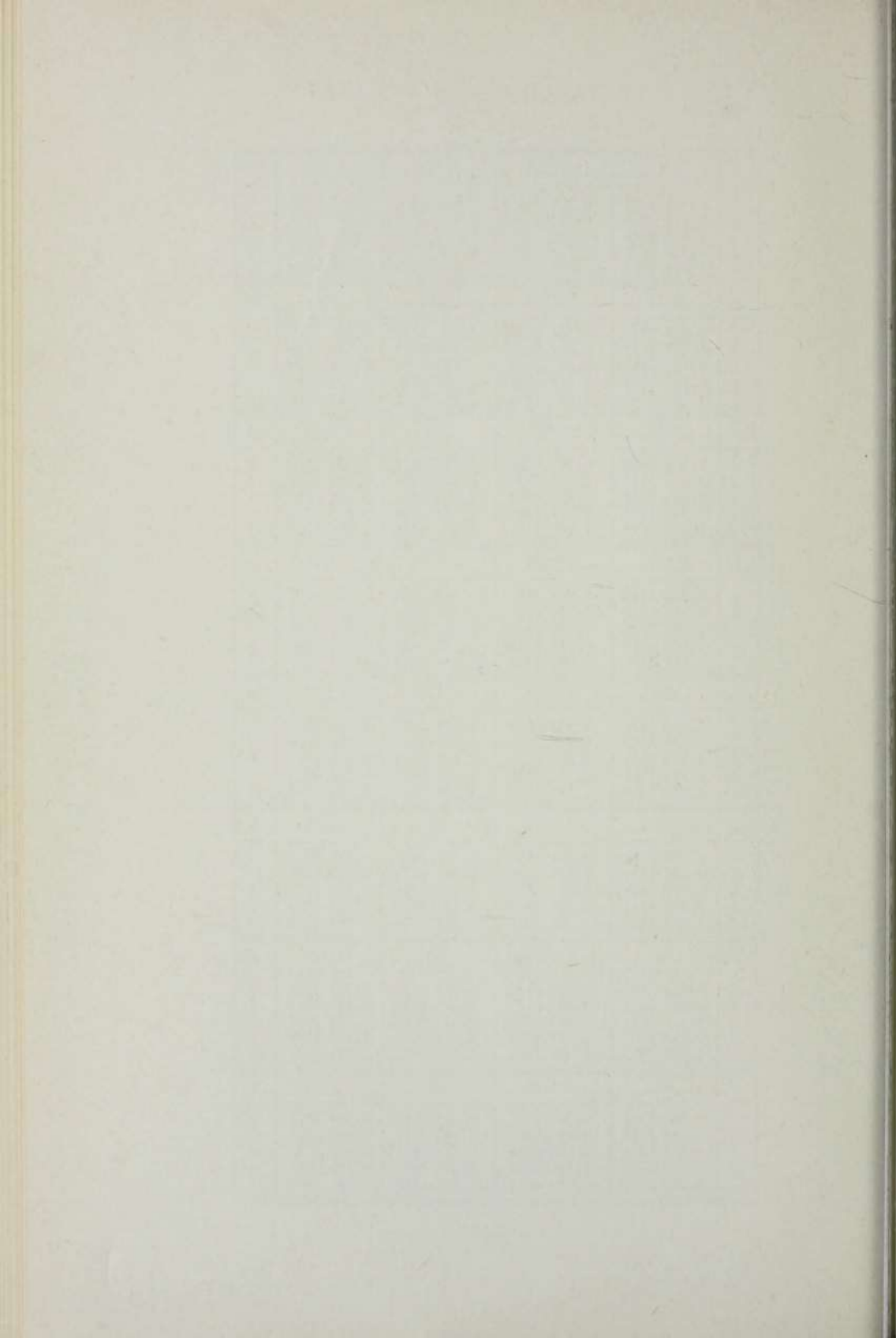


TABLE 10

CSO MORTALITY TABLE

TABLE 10. 1941 CSO Mortality Table

AGE x	l_x	d_x	$1000q_x$	AGE x
0	1 0 2 3 1 0 2	2 3 1 0 2	2 2 .5 8	0
1	1 0 0 0 0 0 0	5 7 7 0	5 .7 7	1
2	9 9 4 2 3 0	4 1 1 6	4 .1 4	2
3	9 9 0 1 1 4	3 3 4 7	3 .3 8	3
4	9 8 6 7 6 7	2 9 5 0	2 .9 9	4
5	9 8 3 8 1 7	2 7 1 5	2 .7 6	5
6	9 8 1 1 0 2	2 5 6 1	2 .6 1	6
7	9 7 8 5 4 1	2 4 1 7	2 .4 7	7
8	9 7 6 1 2 4	2 2 5 5	2 .3 1	8
9	9 7 3 8 6 9	2 0 6 5	2 .1 2	9
10	9 7 1 8 0 4	1 9 1 4	1 .9 7	10
11	9 6 9 8 9 0	1 8 5 2	1 .9 1	11
12	9 6 8 0 3 8	1 8 5 9	1 .9 2	12
13	9 6 6 1 7 9	1 9 1 3	1 .9 8	13
14	9 6 4 2 6 6	1 9 9 6	2 .0 7	14
15	9 6 2 2 7 0	2 0 6 9	2 .1 5	15
16	9 6 0 2 0 1	2 1 0 3	2 .1 9	16
17	9 5 8 0 9 8	2 1 5 6	2 .2 5	17
18	9 5 5 9 4 2	2 1 9 9	2 .3 0	18
19	9 5 3 7 4 3	2 2 6 0	2 .3 7	19
20	9 5 1 4 8 3	2 3 1 2	2 .4 3	20
21	9 4 9 1 7 1	2 3 8 2	2 .5 1	21
22	9 4 6 7 8 9	2 4 5 2	2 .5 9	22
23	9 4 4 3 3 7	2 5 3 1	2 .6 8	23
24	9 4 1 8 0 6	2 6 0 9	2 .7 7	24
25	9 3 9 1 9 7	2 7 0 5	2 .8 8	25
26	9 3 6 4 9 2	2 8 0 0	2 .9 9	26
27	9 3 3 6 9 2	2 9 0 4	3 .1 1	27
28	9 3 0 7 8 8	3 0 2 5	3 .2 5	28
29	9 2 7 7 6 3	3 1 5 4	3 .4 0	29
30	9 2 4 6 0 9	3 2 9 2	3 .5 6	30
31	9 2 1 3 1 7	3 4 3 7	3 .7 3	31
32	9 1 7 8 8 0	3 5 9 8	3 .9 2	32
33	9 1 4 2 8 2	3 7 6 7	4 .1 2	33
34	9 1 0 5 1 5	3 9 6 1	4 .3 5	34
35	9 0 6 5 5 4	4 1 6 1	4 .5 9	35
36	9 0 2 3 9 3	4 3 8 6	4 .8 6	36
37	8 9 8 0 0 7	4 6 2 5	5 .1 5	37
38	8 9 3 3 8 2	4 8 7 8	5 .4 6	38
39	8 8 8 5 0 4	5 1 6 2	5 .8 1	39
40	8 8 3 3 4 2	5 4 5 9	6 .1 8	40
41	8 7 7 8 8 3	5 7 8 5	6 .5 9	41
42	8 7 2 0 9 8	6 1 3 1	7 .0 3	42
43	8 6 5 9 6 7	6 5 0 3	7 .5 1	43
44	8 5 9 4 6 4	6 9 1 0	8 .0 4	44
45	8 5 2 5 5 4	7 3 4 0	8 .6 1	45
46	8 4 5 2 1 4	7 8 0 1	9 .2 3	46
47	8 3 7 4 1 3	8 2 9 9	9 .9 1	47
48	8 2 9 1 1 4	8 8 2 2	1 0 .6 4	48
49	8 2 0 2 9 2	9 3 9 2	1 1 .4 5	49

TABLE 10. 1941 CSO Mortality Table

AGE x	l_x	d_x	$1000q_x$	AGE x
50	8 1 0 9 0 0	9 9 9 0	1 2 .3 2	50
51	8 0 0 9 1 0	1 0 6 2 8	1 3 .2 7	51
52	7 9 0 2 8 2	1 1 3 0 1	1 4 .3 0	52
53	7 7 8 9 8 1	1 2 0 2 0	1 5 .4 3	53
54	7 6 6 9 6 1	1 2 7 7 0	1 6 .6 5	54
55	7 5 4 1 9 1	1 3 5 6 0	1 7 .9 8	55
56	7 4 0 6 3 1	1 4 3 9 0	1 9 .4 3	56
57	7 2 6 2 4 1	1 5 2 5 1	2 1 .0 0	57
58	7 1 0 9 9 0	1 6 1 4 7	2 2 .7 1	58
59	6 9 4 8 4 3	1 7 0 7 2	2 4 .5 7	59
60	6 7 7 7 7 1	1 8 0 2 2	2 6 .5 9	60
61	6 5 9 7 4 9	1 8 9 8 8	2 8 .7 8	61
62	6 4 0 7 6 1	1 9 9 7 9	3 1 .1 8	62
63	6 2 0 7 8 2	2 0 9 5 8	3 3 .7 6	63
64	5 9 9 8 2 4	2 1 9 4 2	3 6 .5 8	64
65	5 7 7 8 8 2	2 2 9 0 7	3 9 .6 4	65
66	5 5 4 9 7 5	2 3 8 4 2	4 2 .9 6	66
67	5 3 1 1 3 3	2 4 7 3 0	4 6 .5 6	67
68	5 0 6 4 0 3	2 5 5 5 3	5 0 .4 6	68
69	4 8 0 8 5 0	2 6 3 0 2	5 4 .7 0	69
70	4 5 4 5 4 8	2 6 9 5 5	5 9 .3 0	70
71	4 2 7 5 9 3	2 7 4 8 1	6 4 .2 7	71
72	4 0 0 1 1 2	2 7 8 7 2	6 9 .6 6	72
73	3 7 2 2 4 0	2 8 1 0 4	7 5 .5 0	73
74	3 4 4 1 3 6	2 8 1 5 4	8 1 .8 1	74
75	3 1 5 9 8 2	2 8 0 0 9	8 8 .6 4	75
76	2 8 7 9 7 3	2 7 6 5 1	9 6 .0 2	76
77	2 6 0 3 2 2	2 7 0 7 1	1 0 3 .9 9	77
78	2 3 3 2 5 1	2 6 2 6 2	1 1 2 .5 9	78
79	2 0 6 9 8 9	2 5 2 2 4	1 2 1 .8 6	79
80	1 8 1 7 6 5	2 3 9 6 6	1 3 1 .8 5	80
81	1 5 7 7 9 9	2 2 5 0 2	1 4 2 .6 0	81
82	1 3 5 2 9 7	2 0 8 5 7	1 5 4 .1 6	82
83	1 1 4 4 4 0	1 9 0 6 2	1 6 6 .5 7	83
84	9 5 3 7 8	1 7 1 5 7	1 7 9 .8 8	84
85	7 8 2 2 1	1 5 1 8 5	1 9 4 .1 3	85
86	6 3 0 3 6	1 3 1 9 8	2 0 9 .3 7	86
87	4 9 8 3 8	1 1 2 4 5	2 2 5 .6 3	87
88	3 8 5 9 3	9 3 7 8	2 4 3 .0 0	88
89	2 9 2 1 5	7 6 3 8	2 6 1 .4 4	89
90	2 1 5 7 7	6 0 6 3	2 8 0 .9 9	90
91	1 5 5 1 4	4 6 8 1	3 0 1 .7 3	91
92	1 0 8 3 3	3 5 0 6	3 2 3 .6 4	92
93	7 3 2 7	2 5 4 0	3 4 6 .6 6	93
94	4 7 8 7	1 7 7 6	3 7 1 .0 0	94
95	3 0 1 1	1 1 9 3	3 9 6 .2 1	95
96	1 8 1 8	8 1 3	4 4 7 .1 9	96
97	1 0 0 5	5 5 1	5 4 8 .2 6	97
98	4 5 4	3 2 9	7 2 4 .6 7	98
99	1 2 5	1 2 5	1 0 0 0 .0 0	99

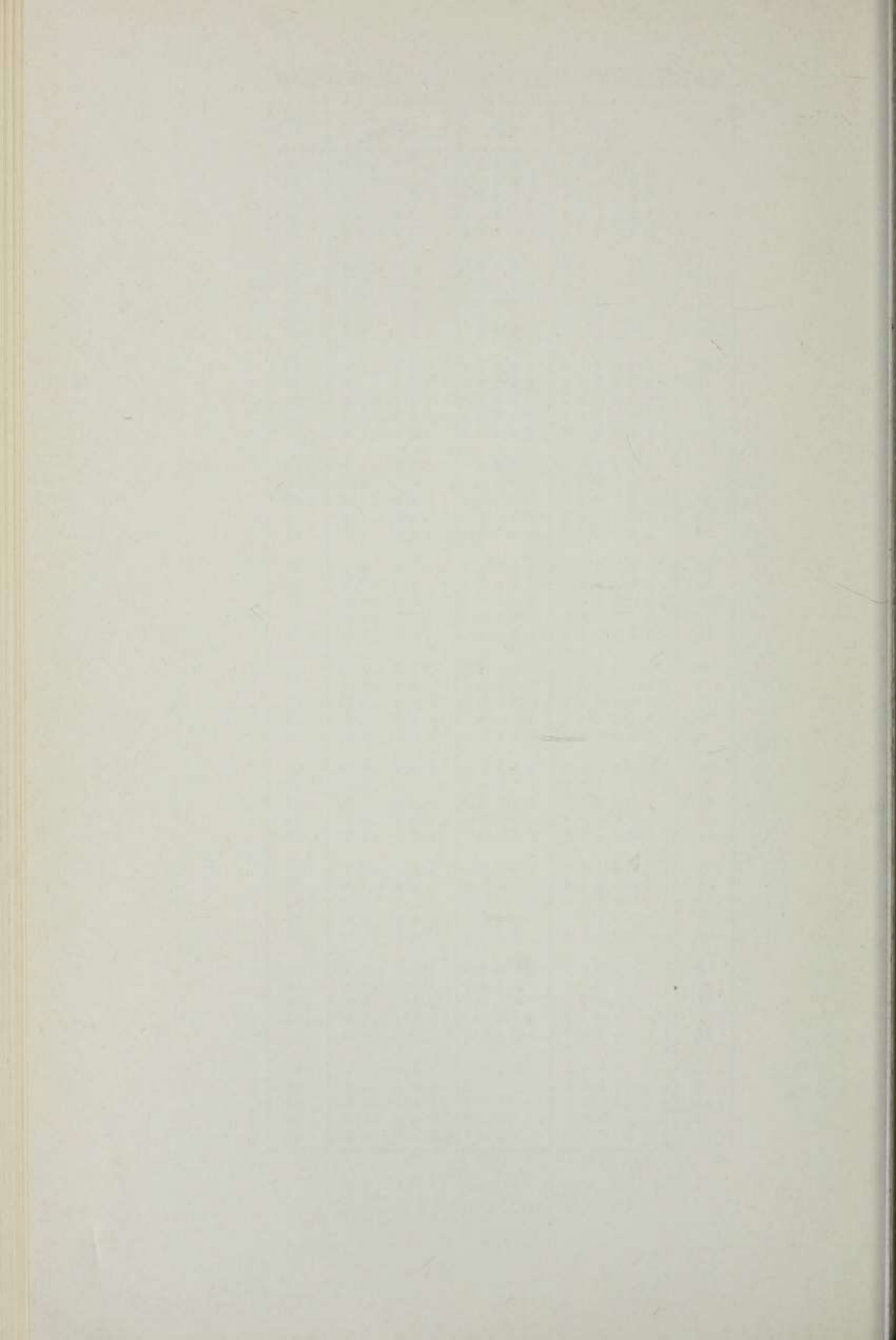


TABLE 11

COMMUTATION COLUMNS

CSO— $2\frac{1}{2}\%$

TABLE 11. Commutation Columns—CSO 2½%

AGE x	D_x	N_x	C_x	M_x	AGE x
0	1023102 .00	31374229 .80	22538 .5366	257876 .8839	0
1	975609 .76	30351127 .80	5491 .9691	235338 .3473	1
2	946322 .43	29375518 .04	3822 .1152	229846 .3782	2
3	919419 .28	28429195 .61	3032 .2168	226024 .2630	3
4	893962 .20	27509776 .33	2607 .3702	222992 .0462	4
5	869550 .88	26615814 .13	2341 .1360	220384 .6760	5
6	846001 .18	25746263 .25	2154 .4803	218043 .5400	6
7	823212 .53	24900262 .07	1983 .7445	215889 .0597	7
8	801150 .42	24077049 .54	1805 .6425	213905 .3152	8
9	779804 .53	23275899 .12	1613 .1747	212099 .6727	9
10	759171 .73	22496094 .59	1458 .7451	210486 .4980	10
11	739196 .60	21736922 .86	1377 .0655	209027 .7529	11
12	719790 .26	20997726 .26	1348 .5565	207650 .6874	12
13	700885 .94	20277935 .90	1353 .8821	206302 .1309	13
14	682437 .28	19577049 .96	1378 .1693	204948 .2488	14
15	664414 .29	18894612 .68	1393 .7300	203570 .0795	15
16	646815 .33	18230198 .39	1382 .0812	202176 .3495	16
17	629657 .27	17583383 .06	1382 .3537	200794 .2683	17
18	612917 .42	16953725 .79	1375 .5355	199411 .9146	18
19	596592 .68	16340808 .37	1379 .2123	198036 .3791	19
20	580662 .42	15744215 .69	1376 .5331	196657 .1668	20
21	565123 .40	15163553 .27	1383 .6196	195280 .6337	21
22	549956 .28	14598429 .87	1389 .5416	193897 .0141	22
23	535153 .17	14048473 .59	1399 .3275	192507 .4725	23
24	520701 .32	13513320 .42	1407 .2700	191108 .1450	24
25	506594 .02	12992619 .10	1423 .4649	189700 .8750	25
26	492814 .61	12486025 .08	1437 .5192	188277 .4101	26
27	479357 .22	11993210 .47	1454 .5491	186839 .8909	27
28	466211 .03	11513853 .25	1478 .2003	185385 .3418	28
29	453361 .83	11047642 .22	1503 .6464	183907 .1415	29
30	440800 .58	10594280 .39	1531 .1580	182403 .4951	30
31	428518 .18	10153479 .81	1559 .6094	180872 .3371	31
32	416506 .91	9724961 .63	1592 .8453	179312 .7277	32
33	404755 .37	9308454 .72	1626 .9874	177719 .8824	33
34	393256 .29	8903699 .35	1669 .0508	176092 .8950	34
35	381995 .63	8510443 .06	1710 .5610	174423 .8442	35
36	370968 .10	8128447 .43	1759 .0801	172713 .2832	36
37	360161 .02	7757479 .33	1809 .6928	170954 .2031	37
38	349566 .90	7397318 .31	1862 .1345	169144 .5103	38
39	339178 .75	7047751 .41	1922 .4869	167282 .3758	39
40	328983 .61	6708572 .66	1983 .5110	165359 .8889	40
41	318976 .11	6379589 .05	2050 .6947	163376 .3779	41
42	309145 .51	6060612 .94	2120 .3381	161325 .6832	42
43	299485 .04	5751467 .43	2194 .1367	159205 .3451	43
44	289986 .39	5451982 .39	2274 .5951	157011 .2084	44
45	280638 .95	5161996 .00	2357 .2099	154736 .6133	45
46	271436 .89	4881357 .05	2444 .1542	152379 .4034	46
47	262372 .33	4609920 .16	2536 .7650	149935 .2492	47
48	253436 .24	4347547 .83	2630 .8594	147398 .4842	48
49	244624 .00	4094111 .59	2732 .5292	144767 .6248	49

TABLE 11. Commutation Columns—CSO $2\frac{1}{2}\%$

AGE x	D_x	N_x	C_x	M_x	AGE x
50	2 3 5 9 2 5 .0 4	3 8 4 9 4 8 7 .5 9	2 8 3 5 .6 2 2 1	1 4 2 0 3 5 .0 9 5 6	50
51	2 2 7 3 3 5 .1 5	3 6 1 3 5 6 2 .5 5	2 9 4 3 .1 3 7 4	1 3 9 1 9 9 .4 7 3 5	51
52	2 1 8 8 4 7 .2 5	3 3 8 6 2 2 7 .4 0	3 0 5 3 .1 7 7 2	1 3 6 2 5 6 .3 3 6 1	52
53	2 1 0 4 5 6 .3 3	3 1 6 7 3 8 0 .1 5	3 1 6 8 .2 2 2 9	1 3 3 2 0 3 .1 5 8 9	53
54	2 0 2 1 5 5 .0 3	2 9 5 6 9 2 3 .8 2	3 2 8 3 .8 1 2 1	1 3 0 0 3 4 .9 3 6 0	54
55	1 9 3 9 4 0 .6 1	2 7 5 4 7 6 8 .7 9	3 4 0 1 .9 1 3 1	1 2 6 7 5 1 .1 2 3 9	55
56	1 8 5 8 0 8 .4 3	2 5 6 0 8 2 8 .1 8	3 5 2 2 .0 9 0 1	1 2 3 3 4 9 .2 1 0 8	56
57	1 7 7 7 5 4 .4 3	2 3 7 5 0 1 9 .7 5	3 6 4 1 .7 8 3 5	1 1 9 8 2 7 .1 2 0 7	57
58	1 6 9 7 7 7 .1 7	2 1 9 7 2 6 5 .3 2	3 7 6 1 .6 9 6 8	1 1 6 1 8 5 .3 3 7 2	58
59	1 6 1 8 7 4 .5 7	2 0 2 7 4 8 8 .1 5	3 8 8 0 .1 8 5 4	1 1 2 4 2 3 .6 4 0 4	59
60	1 5 4 0 4 6 .2 3	1 8 6 5 6 1 3 .5 8	3 9 9 6 .1 9 9 9	1 0 8 5 4 3 .4 5 5 0	60
61	1 4 6 2 9 2 .8 0	1 7 1 1 5 6 7 .3 5	4 1 0 7 .7 0 8 0	1 0 4 5 4 7 .2 5 5 1	61
62	1 3 8 6 1 6 .9 7	1 5 6 5 2 7 4 .5 5	4 2 1 6 .6 7 6 0	1 0 0 0 4 3 9 .5 4 7 1	62
63	1 3 1 0 1 9 .4 0	1 4 2 6 6 5 7 .5 8	4 3 1 5 .4 1 3 8	9 6 2 2 2 2 .8 7 1 1	63
64	1 2 3 5 0 8 .3 9	1 2 9 5 6 3 8 .1 8	4 4 0 7 .8 3 1 2	9 1 9 0 7 7 .4 5 7 3	64
65	1 1 6 0 8 8 .1 5	1 1 7 2 1 2 9 .7 9	4 4 8 9 .4 4 9 7	8 7 4 9 9 9 .6 2 6 1	65
66	1 0 8 7 6 7 .2 9	1 0 5 6 0 4 1 .6 4	4 5 5 8 .7 2 8 2	8 3 0 1 0 1 .1 7 6 4	66
67	1 0 1 5 5 5 .7 0	9 4 7 2 7 4 .3 5	4 6 1 3 .1 8 9 3	7 8 4 5 1 1 .4 4 8 2	67
68	9 4 4 6 5 .5 4 5	8 4 5 7 1 8 .6 5 1	4 6 5 0 .4 5 2 1	7 3 8 3 8 .2 5 8 9	68
69	8 7 5 1 1 .0 5 0	7 5 1 2 5 3 .1 0 6	4 6 7 0 .0 1 4 3	6 9 1 8 7 .8 0 6 8	69
70	8 0 7 0 6 .6 2 5	6 6 3 7 4 2 .0 5 6	4 6 6 9 .2 2 6 0	6 4 5 1 7 .7 9 2 5	70
71	7 4 0 6 8 .9 4 2	5 8 3 0 3 5 .4 3 1	4 6 4 4 .2 3 5 4	5 9 8 4 8 .5 6 6 5	71
72	6 7 6 1 8 .1 4 8	5 0 8 9 6 6 .4 8 9	4 5 9 5 .4 2 8 1	5 5 2 0 4 .3 3 1 1	72
73	6 1 3 7 3 .4 9 8	4 4 1 3 4 8 .3 4 1	4 5 2 0 .6 6 2 7	5 0 6 0 8 .9 0 3 0	73
74	5 5 3 5 5 .9 2 1	3 7 9 9 7 4 .8 4 3	4 4 1 8 .2 4 9 2	4 6 0 8 8 .2 4 0 3	74
75	4 9 5 8 7 .5 2 6	3 2 4 6 1 8 .9 2 2	4 2 8 8 .2 8 6 9	4 1 6 6 9 .9 9 1 1	75
76	4 4 0 8 9 .7 8 7	2 7 5 0 3 1 .3 9 6	4 1 3 0 .2 2 0 2	3 7 3 8 1 .7 0 4 2	76
77	3 8 8 8 4 .2 0 6	2 3 0 9 4 1 .6 0 9	3 9 4 4 .9 6 1 8	3 3 2 5 1 .4 8 4 0	77
78	3 3 9 9 0 .8 5 0	1 9 2 0 5 7 .4 0 3	3 7 3 3 .7 2 5 8	2 9 3 0 6 .5 2 2 2	78
79	2 9 4 2 8 .0 7 7	1 5 8 0 6 6 .5 5 3	3 4 9 8 .6 8 4 1	2 5 5 7 2 .7 9 6 4	79
80	2 5 2 1 1 .6 3 6	1 2 8 6 3 8 .4 7 6	3 2 4 3 .1 1 5 8	2 2 0 7 4 .1 1 2 3	80
81	2 1 3 5 3 .6 0 2	1 0 3 4 2 6 .8 4 0	2 9 7 0 .7 3 6 8	1 8 8 3 0 .9 9 6 5	81
82	1 7 8 6 2 .0 4 7	8 2 0 7 3 .2 3 8	2 6 8 6 .4 0 2 0	1 5 8 6 0 .2 5 9 7	82
83	1 4 7 3 9 .9 8 4	6 4 2 1 1 .1 9 1	2 3 9 5 .3 2 1 2	1 3 1 7 3 .8 5 7 7	83
84	1 1 9 8 5 .1 5 1	4 9 4 7 1 .2 0 7	2 1 0 3 .3 5 6 1	1 0 7 7 8 .5 3 6 5	84
85	9 5 8 9 .4 7 4 6	3 7 4 8 6 .0 5 6 1	1 8 1 6 .1 9 4 6	8 6 7 5 .1 8 0 4	85
86	7 5 3 9 .3 9 0 5	2 7 8 9 6 .5 8 1 5	1 5 4 0 .0 3 9 4	6 8 5 8 .9 8 5 8	86
87	5 8 1 5 .4 6 3 2	2 0 3 5 7 .1 9 1 0	1 2 8 0 .1 4 5 4	5 3 1 8 .9 4 6 4	87
88	4 3 9 3 .4 7 7 3	1 4 5 4 1 .7 2 7 8	1 0 4 1 .5 6 4 6	4 0 3 8 .8 0 1 0	88
89	3 2 4 4 .7 5 4 6	1 0 1 4 8 .2 5 0 5	8 2 7 .6 2 1 5	2 9 9 7 .2 3 6 4	89
90	2 3 3 7 .9 9 2 9	6 9 0 3 .4 9 5 9	6 4 0 .9 3 7 7	2 1 6 9 .6 1 4 9	90
91	1 6 4 0 .0 3 0 9	4 5 6 5 .5 0 3 0	4 8 2 .7 7 3 0	1 5 2 8 .6 7 7 2	91
92	1 1 1 7 .2 5 7 1	2 9 2 5 .4 7 2 1	3 5 2 .7 7 0 7	1 0 4 5 .9 0 4 2	92
93	7 3 7 .2 3 6 3	1 8 0 8 .2 1 5 0	2 4 9 .3 3 9 1	6 9 3 .1 3 3 5	93
94	4 6 9 .9 1 5 8	1 0 7 0 .9 7 8 7	1 7 0 .0 8 8 8	4 4 3 .7 9 4 4	94
95	2 8 8 .3 6 5 7	6 0 1 .0 6 2 9	1 1 1 .4 6 7 8	2 7 3 .7 0 5 6	95
96	1 6 9 .8 6 4 6	3 1 2 .6 9 7 2	7 4 .1 0 9 8	1 6 2 .2 3 7 8	96
97	9 1 .6 1 1 7	1 4 2 .8 3 2 6	4 9 .0 0 1 9	8 8 .1 2 8 0	97
98	4 0 .3 7 5 5	5 1 .2 2 0 9	2 8 .5 4 5 1	3 9 .1 2 6 1	98
99	1 0 .8 4 5 4	1 0 .8 4 5 4	1 0 .5 8 1 0	1 0 .5 8 1 0	99

TABLE 11. Net Single Premiums—Whole Life—CSO $2\frac{1}{2}\%$

AGE x	LIFE INSURANCE $1000A_x$	ANNUITY DUE $\ddot{a}_x = 1 + a_x$	AGE x	LIFE INSURANCE $1000A_x$	ANNUITY DUE $\ddot{a}_x = 1 + a_x$
0	252 .05395	30 .665788	50	602 .03485	16 .316571
1	241 .22183	31 .109905	51	612 .30951	15 .895310
2	242 .88380	31 .041764	52	622 .60932	15 .473018
3	245 .83373	30 .920817	53	632 .92539	15 .050059
4	249 .44239	30 .772862	54	643 .24363	14 .627011
5	253 .44656	30 .608691	55	653 .55639	14 .204188
6	257 .73434	30 .432892	56	663 .85151	13 .782088
7	262 .25192	30 .247671	57	674 .11607	13 .361241
8	266 .99770	30 .053094	58	684 .34015	12 .942054
9	271 .99083	29 .848376	59	694 .51083	12 .525056
10	277 .25807	29 .632419	60	704 .61612	12 .110739
11	282 .77695	29 .406145	61	714 .64388	11 .699601
12	288 .48773	29 .172003	62	724 .58332	11 .292084
13	294 .34480	28 .931863	63	734 .41697	10 .888904
14	300 .31807	28 .686959	64	744 .13939	10 .490285
15	306 .39027	28 .437999	65	753 .73429	10 .096894
16	312 .57197	28 .184549	66	763 .19063	9 .709184
17	318 .89453	27 .925324	67	772 .42676	9 .327631
18	325 .34873	27 .660702	68	781 .64224	8 .952668
19	331 .94570	27 .390226	69	790 .61793	8 .584665
20	338 .67727	27 .114232	70	799 .41137	8 .224134
21	345 .55395	26 .832288	71	808 .01161	7 .871524
22	352 .56805	26 .544710	72	816 .41295	7 .527069
23	359 .72405	26 .251314	73	824 .60517	7 .191188
24	367 .02066	25 .952153	74	832 .58017	6 .864213
25	374 .46329	25 .647005	75	840 .33212	6 .546383
26	382 .04510	25 .336151	76	847 .85405	6 .237984
27	389 .77170	25 .019360	77	855 .14112	5 .939214
28	397 .64253	24 .696656	78	862 .18858	5 .650268
29	405 .65200	24 .368268	79	868 .99307	5 .371284
30	413 .80049	24 .034180	80	875 .55256	5 .102345
31	422 .08788	23 .694397	81	881 .86510	4 .843531
32	430 .55661	23 .348860	82	887 .93073	4 .594840
33	439 .07973	22 .997731	83	893 .74978	4 .356259
34	447 .78151	22 .640958	84	899 .32419	4 .127708
35	456 .61214	22 .278902	85	904 .65651	3 .909093
36	465 .57443	21 .911446	86	909 .75339	3 .700111
37	474 .66049	21 .538920	87	914 .62127	3 .500528
38	483 .86878	21 .161380	88	919 .27207	3 .309845
39	493 .19827	20 .778871	89	923 .71741	3 .127586
40	502 .63868	20 .391814	90	927 .98183	2 .952745
41	512 .19002	20 .000209	91	932 .10266	2 .783791
42	521 .84385	19 .604402	92	936 .13559	2 .618441
43	531 .59700	19 .204523	93	940 .17819	2 .452694
44	541 .44336	18 .800822	94	944 .14254	2 .279086
45	551 .37253	18 .393726	95	949 .16154	2 .084377
46	561 .38058	17 .983396	96	955 .10093	1 .884082
47	571 .45985	17 .570146	97	961 .97298	1 .559108
48	581 .59988	17 .154405	98	969 .05817	1 .266615
49	591 .79649	16 .736344	99	975 .60976	1 .000000

TABLE 12

THE NUMBER OF EACH DAY OF THE YEAR

TABLE 12. The Number of Each Day of the Year

Day of Month	January	February	March	April	May	June	July	August	September	October	November	December	Day of Month
1	1	32	60	91	121	152	182	213	244	274	305	335	1
2	2	33	61	92	122	153	183	214	245	275	306	336	2
3	3	34	62	93	123	154	184	215	246	276	307	337	3
4	4	35	63	94	124	155	185	216	247	277	308	338	4
5	5	36	64	95	125	156	186	217	248	278	309	339	5
6	6	37	65	96	126	157	187	218	249	279	310	340	6
7	7	38	66	97	127	158	188	219	250	280	311	341	7
8	8	39	67	98	128	159	189	220	251	281	312	342	8
9	9	40	68	99	129	160	190	221	252	282	313	343	9
10	10	41	69	100	130	161	191	222	253	283	314	344	10
11	11	42	70	101	131	162	192	223	254	284	315	345	11
12	12	43	71	102	132	163	193	224	255	285	316	346	12
13	13	44	72	103	133	164	194	225	256	286	317	347	13
14	14	45	73	104	134	165	195	226	257	287	318	348	14
15	15	46	74	105	135	166	196	227	258	288	319	349	15
16	16	47	75	106	136	167	197	228	259	289	320	350	16
17	17	48	76	107	137	168	198	229	260	290	321	351	17
18	18	49	77	108	138	169	199	230	261	291	322	352	18
19	19	50	78	109	139	170	200	231	262	292	323	353	19
20	20	51	79	110	140	171	201	232	263	293	324	354	20
21	21	52	80	111	141	172	202	233	264	294	325	355	21
22	22	53	81	112	142	173	203	234	265	295	326	356	22
23	23	54	82	113	143	174	204	235	266	296	327	357	23
24	24	55	83	114	144	175	205	236	267	297	328	358	24
25	25	56	84	115	145	176	206	237	268	298	329	359	25
26	26	57	85	116	146	177	207	238	269	299	330	360	26
27	27	58	86	117	147	178	208	239	270	300	331	361	27
28	28	59	87	118	148	179	209	240	271	301	332	362	28
29	29	*	88	119	149	180	210	241	272	302	333	363	29
30	30		89	120	150	181	211	242	273	303	334	364	30
31	31		90		151		212	243		304		365	31

* For leap years, February 29 is number 60 and the number of each day from March 1 to December 31 inclusive is one greater than the number in this table.

INDEX

(Numbers refer to pages)

A

- Accrued bond interest, 118
- Accumulating
 - at bank discount, 7
 - the discount on a bond, 115
 - at simple interest, 3-4
- Amortization
 - amount of interest or principal in any payment, 93
 - of a bonded debt, 95-96
 - comparison with sinking fund, 102
 - definition, 87
 - finding the size of the payments, 88
 - outstanding principal, 91
 - of the premium or discount on the redemption value of a bond, 113-15
 - schedule, 89
- Amount
 - of annuity due, 76
 - at compound interest, 31
 - of ordinary annuities certain, 57, 76
 - at simple interest, 2
 - of simple interest, 1
- "And interest" price of a bond, 119
- Annual premiums
 - limited payment policy, 162, 164
 - ordinary life policy, 162
 - paid-up life policy, 165
 - whole life insurance, 162
- Annuities
 - certain, 56
 - contingent, 56, 144
 - definition, 56
 - general case, 78-84
 - ordinary, 56
 - payment interval, 56
 - perpetuities, 56, 137
 - simple, 56, 136
 - term of, 62, 75-76
 - whole life, 144
- Annuities certain; *see* Ordinary annuities certain
- Annuity due
 - amount, 76
 - present value, 76
 - term, 76
- Antilogarithms, 23

B

- Bank discount, 7
 - formula, 7
 - relation to simple interest, 12
- Base of logarithms, 17
- Bond
 - accrued interest, 119
 - amortization of premium or discount on redemption value, 113-15
 - "and interest" price, 119
 - book value, 117-18
 - coupon, 107
 - definition, 106
 - face, 106
 - flat price, 118
 - interest payments, 106
 - par value, 107
 - premium-discount method, 112
 - price between interest dates, 117
 - price on an interest date, 108
 - quotation, 119
 - quoted price, 119
 - redemption value, 106
 - registered, 107
 - schedules, 115
 - yield rate, 108, 121-24
- Book value
 - of an asset, 127
 - of a bond, 117-18
 - constant percentage method of depreciation, 132
 - straight line depreciation, 128

C

- Capitalized cost, 140
- Characteristic of logarithm, 19-21
- Commutation symbols, 145, 148, 161
- Comparison date, 51
- Compound interest
 - effective rate, 39
 - equations of value, 49-53
 - equivalence of rates, 37
 - formula, 31-32
 - nomial rate, 33
 - solving for i and n , 46-48
 - use with a fractional period, 43-45

Constant percentage method of depreciation, 131-34

Contingent annuities, 56, 144

D

Date, comparison or focal, 51

Declining-balance method of depreciation, 134

Deferred annuity, 77

period of deferment, 77

Depreciation

constant percentage method, 131

formula, 132

rate of depreciation, 132

declining-balance method, 134

definition, 127

expense, 127, 129

life of asset, 127

reserve for, 127

salvage or scrap value, 127

schedules, 129-30, 134, 136

sinking fund method, 129

straight line method, 128

sum of digits method, 135

wearing value, 127

Discount, 8

bank, 7

bond, 112

accumulation of, 113-15

at compound interest, 41

rate, 7

relation to simple interest, 12

at simple interest, 4

Discounting a note, 11

E

Endowment insurance, 167

Equations of value, 49

comparison date, 51

focal date, 51

Equity

buyer's, 91

seller's, 91

Exact interest, 4

F

Face

of a bond, 106

of a life insurance policy, 160

Flat price of a bond, 118

Focal date, 51

G

General annuity, 78-84

Geometric series, sum of, 58

I

Insurance; *see* Life insurance

Interest

in advance, 7

amount, 1

Interest—*Cont.*

compound, 2, 30

definition, 1, 8

exact, 4

for a fraction of a period, 43-45

ordinary, 4

payments on bonds, 106

rate, 1

ratio, 1

simple, 2

Interest-bearing note, 10

Interest rate, 1

equivalence of simple interest and bank discount rates, 12-13

equivalence of two compound rates, 37

Interpolation

annuities, solving for i , 70-71

compound interest

solving for i , 46-47

solving for n , 47-48

to find book value of bond, 119

to find yield rate of bond, 124

logarithms, 24

L

Laws of logarithms, 18-24

Leap year, 5

Life annuities

deferred temporary, 154

deferred whole, 151

guaranteed, 153

purchased with periodic payments, 156-58

pure endowment, 154

temporary, 152

whole, 144, 147

whole life annuity due, 147, 149

Life insurance

annual premiums

limited payment, 162, 164

ordinary life, 162

paid-up life policy, 165

whole life, 162

beneficiary, 160

company, 160

endowment, 167

policy, 160

premium, 160

reserves, 170

single premium, whole life, 160

term, 166-67

Limited payment life insurance, 162, 164

Loading factor, 146, 160

Logarithms, 17-29

antilogarithms, 23

base, 17

characteristic, 19-21

common, 18

computation, 26-29

definition, 17

Logarithms—*Cont.*

- interpolation, 24
- laws, 18–20
- mantissa, 20, 22–23
- Natural or Napierian, 17
- numerical value, 19

M

- Mantissa of logarithms, 20, 22–23
- Maturity value
 - at compound interest, 31
 - involving a fractional period, 43
 - of a promissory note, 9–10
 - at simple interest, 2
- Mortality tables, 144
 - CSO, 145
 - 1937 Standard Annuity table, 145
 - symbols, 146

N

- Napierian logarithms, 17
- Natural logarithms, 17
- Net indebtedness, 99
- Net premiums, 146, 160
- Nominal rate, 33
- Noninterest-bearing note, 10
- Notes, promissory, 9
 - face, 9
 - interest-bearing, 10
 - maker, 10
 - maturity value, 9
 - noninterest-bearing, 10

O

- Ordinary annuities certain, 56
 - amount, 57
 - formula, 58
 - finding final payment, 73
 - finding number of payments, 72
 - finding unknown interest rate, 70
 - periodic payments, 66
 - present value, 61
 - formula, 62
 - term, 62, 75

- Ordinary interest, 4
- Ordinary life policy, 162
- Outstanding liability, 91
- Outstanding principal, 91

P

- Par value of a bond, 107
- Periodic expense, 98
- Perpetuities
 - definition, 137
 - formula, 138–39
 - present value, 137
- Premium, bond, 112
 - amortization of, 113–15
- Present value
 - annuity due, 76

Present value—*Cont.*

- at compound interest, 31, 40
 - involving a fractional period, 44
- of ordinary annuities, 61, 76
 - formula, 62
- perpetuity, 137
 - formula, 139
- Principal, 1
 - at compound interest, 31
- Proceeds, 12
- Promissory notes, 9
- Purchase price of a bond
 - between interest dates, 117
 - on an interest date, 108
- Pure endowment, 154–55
 - formula, 155

Q

- Quotation on a bond, 119
- Quoted price of a bond, 119

R

- Rate, 1
 - bank discount, 7
 - bond interest, 106
 - compound interest, 31
 - constant percentage depreciation, 132
 - effective, 39
 - equivalence of bank discount and simple interest, 12–13
 - equivalence of two compound rates, 37
 - nominal, 33
 - simple interest, 2
 - yield, of a bond, 108
- Reciprocals of $a_{\overline{n}|i}$ and $s_{\overline{n}|i}$, 67
 - formula relating, 67
- Redemption value of a bond, 106
- Reserve for depreciation, 127
- Reserves, life insurance, 170
- Rounding off decimals, 4

S

- Schedules
 - amortization, 89
 - bonds, 115
 - depreciation reserve, 129–30, 134, 136
 - sinking fund, 99–100
- Simple annuity, 56, 78
- Simple interest, 1–7
 - formula, 2
 - relation to bank discount, 12
- Single premium, whole life insurance, 160
- Sinking fund method of depreciation, 129
- Sinking funds, 97
 - comparison with amortization, 102
 - growth, 99
 - net indebtedness, 99
 - periodic expense, 98
 - rate of interest actually paid by borrower, 103
 - schedule, 99–100

Straight line method of depreciation, 128
Sum of digits method of depreciation, 135

T

Tax Code, 1954, 134
Term
 annuity due, 76
 insurance, 166
 ordinary annuity, 62, 75
Terminal reserve, 171
Time
 approximate, 5
 diagram, 2

Time—*Cont.*

 exact, 5
 line, 2

W

Wearing value, 127
Whole life annuity, 144, 147
 due, 147, 149
 immediate, 147

Y

Yield rate of a bond, 108
 finding the, 121-24

ANSWERS

(In general, the answers are given for the odd-numbered problems.
Some exceptions are made for sake of clarity.)

Exercise 1. Pages 6-7

1. \$515. 3. \$614. 5. \$975.61. 7. \$194.81. 9. 3.60%. 11. (a) \$5,138.88; (b) \$5,136.81; (c) \$5,136.99; (d) \$5,134.93. 13. \$584.58. 15. .33 yr. 17. (a) \$1,020; (b) \$1,019.73. 19. 9.23%. 21. 3.77 yr. 23. \$990.10.

Exercise 2. Pages 8-9

1. \$765.38. 3. \$966.67. 5. \$1,608.75. 7. $\frac{2}{3}$ yr. 9. 5.83%. 11. \$1,477.50. 13. \$5,102.04. 15. 20%. 17. (a) \$1,230; (b) \$1,230.77. 19. \$3,061.22. 21. 9%. 23. 3.275 yr. 25. \$990.

Exercise 3. Pages 13-16

1. (a) 4.01%; (b) 4.08%; (c) 4.17%. 3. 4.88%. 5. (a) bank discount; (b) bank discount; (c) simple interest. 7. (a) \$5,057.50; (b) \$5,057.85. 9. 69 cents. 11. \$1,023. 13. (a) \$990; (b) 6.06%. 15. 6.06%. 17. (a) 5.06%; (b) 5.26%. 19. (a) \$1,030.93; (b) 6.19%. 21. (a) \$1,007.41; (b) \$1,007.25; (c) 5.06%.

Exercise 4. Page 22

1. 1. 3. 3. 5. - 4. 7. 4. 9. - 2. 11. 0. 13. 3.

Exercise 5. Page 26

1. 1.717338. 3. 8.679882 - 10. 5. 0.991226. 7. 3.602060. 9. 9.507316 - 10. 11. 1.672421. 13. 1.329947. 15. 6.794586 - 10. 17. 8.579280 - 10. 19. 6.680598. 21. 3,836. 23. 111.8. 25. .0002512. 27. 999,900. 29. .0007333. 31. 903.82. 33. 3.8905. 35. 1,194,600. 37. .037459. 39. 3.2589.

Exercise 6. Page 29

1. 19,581. 3. .36892. 5. 6.5827. 7. .11572. 9. 2,372.6. 11. .41832. 13. 26,102,000. 15. 1.4903. 17. 284.75. 19. 2.18. 21. 1.51. 23. 57.25. 25. 14.98. 27. 18.10.

Exercise 7. Pages 35-37

1. \$11,509.82. 3. \$1,474.12. 5. \$149.31. 7. \$5,752.88. 9. \$10,626.79.

11. \$12,024.26. 13. \$6,874.70. 15. \$5,744.45. 17. \$8,042.80. 19. \$10,640.50; No. 21. \$534.93.

Exercise 8. Pages 39–40

1. (a) 6.08%, $m = 2$; (b) 6.05%, $m = 2$; (c) 5.91%, $m = 2$. 3. 3.99%, $m = 12$. 5. 3.01%, $m = 2$. 7. (a) 5.06%, $m = 1$; (b) 5.09%, $m = 1$; (c) 5.12%, $m = 1$. 9. 4.89%, $m = 12$. 11. Savings Association. 13. 5.93%, $m = 12$. 15. 4.38%, $m = 12$. 17. 5.64%, $m = 2$. 19. 7.30%, $m = 4$.

Exercise 9. Pages 42–43

1. \$3,108.61. 3. \$3,724.31. 5. \$3,542.79. 7. \$2,971.24. 9. \$575.80. 11. \$3,098.10. 13. \$3,174.63. 15. \$1,071.83. 17. \$4,553.71.

Exercise 10. Pages 45–46

1. (a) \$1,626.43; (b) \$1,626.39. 3. \$4,774.70. 5. \$2,006.35. 7. \$9,266.82. 9. \$2,746.32.

Exercise 11. Pages 48–49

1. 9.205 six-month periods. 3. 4.38%, $m = 12$. 5. 3.85% effective. 7. 8.45 years. 9. 3.87 years. 11. 4.62%, $m = 12$. 13. 10.24 years. 15. 35.00 years. 17. 4.75%, $m = 2$. 19. 3.38%, $m = 12$.

Exercise 12. Pages 53–55

1. \$5,881.78. 3. \$3,544.46. 5. \$750.18. 7. \$102.01. 9. \$1,103.12 and \$2,206.24. 11. \$5,474.21. 13. \$2,488.01 and \$2,988.01. 15. \$3,391.29. 17. \$6,845.69.

Exercise 13. Pages 60–61

1. \$1,211.58. 3. \$2,487.19. 5. \$5,796.36. 7. \$7,606.10. 9. \$9,396.44. 11. \$3,102.90. 13. (a) \$1,964.14; (b) \$2,116.52; (c) \$2,403.19.

Exercise 14. Pages 64–66

1. \$2,271.88. 3. \$1,601.77. 5. \$5,350.86. 7. \$619.48. 9. (a) \$5,742.87; (b) \$6,264.61; (c) \$7,454.60; (d) \$9,676.54. 11. \$4,751.77. 13. \$19,980.33. 15. \$1,539.96. 17. (a) \$9,109.32; (b) \$10,252.62; (c) \$13,778.67; (d) \$15,973.25. 19. \$11,009.12. 21. \$10,818.50.

Exercise 15. Pages 68–70

1. \$273.02. 3. \$266.39. 5. \$1,338.96. 7. \$678.65. 9. (a) \$562.98; (b) \$571.43; (c) \$634.20. 11. \$369.58. 13. \$63.55.

Exercise 16. Pages 74–75

1. 12.37%, $m = 12$. 3. 16.50%, $m = 12$. 5. 4.28%, $m = 2$. 7. (a) 8 full size, one smaller; (b) \$22.37. 9. (a) 16 full size, one smaller; (b) \$90.83. 11. (a) 25 full size, one smaller; (b) \$100.07.

Exercise 17. Page 78

1. \$15,841.33. 3. \$11,920.50. 5. \$2,525.19. 7. \$4,419.10.

Exercise 18. Pages 84–86

1. \$4,002.59. 3. \$26,047.25. 5. \$9,238.20. 7. \$4,342.76. 9. \$249.84.
11. 17.08%, $m = 2$. 13. \$984.17. 15. \$4,343.72. 17. (a) \$2,545.55; (b) \$2,545.91. 19. \$4,011.24. 21. \$2,643.68. 23. \$28.53. 25. \$17,868.80.

Exercise 19. Pages 94–95

1. (a) \$672.16; (b) \$2,488.47; (c) \$597.51. 4. (a) \$214.11; (b) \$6,039.58.
5. (a) \$8,960.42; (b) \$6,039.58. 7. \$10,214.76. 8. \$27,304.17. 9. \$1,638.25. 11. \$330.32 interest; \$2,169.68 principal. 13. \$2,894.93. 15. \$117.28. 17. \$3,849.35.

Exercise 20. Page 97

All problems are schedules.

Exercise 21. Pages 100–102

1. (a) \$411.57; (b) \$711.57. 2. (a) \$3,532.47; (b) \$6,467.53. 5. (a) \$1,019.70; (b) \$2,019.70; (c) \$44,836.44. 7. (a) \$33,934.30; (b) \$33,610.84.
11. (a) \$1,075.33; (b) \$2,275.33; (c) \$6,227.39. 13. (a) \$411.57; (b) \$1,011.57; (c) \$1,763.63.

Exercise 22. Pages 104–5

1. 7.26%, $m = 2$. 3. \$1,821.89 amortization payment; \$197.81 saved.
5. 4.37%, $m = 2$. 7. 6.18%, $m = 4$. 9. 8.44%, $m = 1$. 11. 6.82%, $m = 2$. 13. 6.84%, $m = 1$. 15. 5%, $m = 2$.

Exercise 23. Pages 111–12

1. \$1,195.98. 3. \$1,095.55. 5. \$1,049.74. 7. \$1,047.73. 9. \$9,974.61.
11. \$953.74. 13. \$112.90. 15. \$5,980.33.

Exercise 24. Page 113

1. \$1,195.95. 3. \$4,527.15. 5. \$3,616.22. 7. \$562.98.

Exercise 25. Pages 116–17

1. \$1,039.02 book value. 3. \$1,016.39 book value. 5. \$535.16 book value.
7. \$1,019.51 book value. 9. \$9.43. 11. \$548.48 book value. 13. \$101.95 book value.

Exercise 26. Pages 120–21

1. (a) \$1,098.50; (b) \$1,088.50. 3. \$1,050.19. 5. \$2,287.78 purchase price; \$2,281.11 book value. 7. (a) \$1,049.78; (b) \$1,039.78. 9. (a) \$891.30; (b) \$881.30. 11. \$990.00. 15. (a) \$1,108.85; (b) \$1,093.85.
17. \$98.00.

Exercise 27. Pages 124–26

1. 3.97%, $m = 1$. 3. 3.25%, $m = 2$. 5. 4.90%, $m = 1$. 7. 5.73%, $m = 1$. 9. 5.56%, $m = 2$. 11. 5.53%, $m = 2$. 13. 4.60%, $m = 2$. 15. 3.54%, $m = 2$. 17. 5.17%, $m = 2$. 19. 5.27%, $m = 2$. 21. 2.628%, $m = 12$.

Exercise 28. Pages 136–37

1. (a) \$2,000.00; (b) \$6,000.00. 2. (a) \$1,921.58; (b) \$6,119.18; (c) \$2,039.20. 4. (a) 31.12%; (b) \$4,095.35. 7. \$1,234.25. 10. 23.53%. 11. \$512.99. 15. \$281.50. 17. (a) \$428.57; (b) \$253.71; (c) \$1,678.57. 19. (a) \$225.00; (b) \$225.00; (c) \$2,750.00. 21. (a) \$6,183.90; (b) \$25,871.13; (c) \$24,128.87.

Exercise 29. Pages 142–43

1. \$123,140.55. 3. \$264,681.56. 5. \$63,740.55. 7. \$13,266.78. 9. \$116,183.74. 11. \$5,036.40. 13. \$47,821.92. 15. \$1,994.42.

Exercise 30. Pages 150–51

1. (a) 955,942; (b) 790,282; (c) 181,765; (d) 10,833. 3. (a) 2,531; (b) 4,625; (c) 28,009; (d) 4,681. 5. (a) .198%; (b) .311%; (c) 2.271%; (d) 32.364%. 7. \$70,250.04. 9. \$1,011.23. 11. \$918.37. 13. \$50,209.03. 15. \$20,193.79.

Exercise 31. Pages 155–56

1. \$4,890.42. 3. \$3,258.98. 5. \$7,451.67. 7. \$1,228.76. 9. \$8,587.61. 11. \$58,479.59. 13. \$2,931.90. 15. \$4,749.68. 17. \$13,165.75.

Exercise 32. Pages 158–59

1. \$456.50. 3. \$216.64. 5. \$1,125.51. 7. \$1,021.05. 9. \$281.24. 11. \$251.98.

Exercise 33. Pages 163–64

1. \$3,597.24. 3. \$10,417.66. 5. \$16,220.66. 7. (a) \$8,747.55; (b) \$16,347.83; (c) \$28,785.22. 9. \$137.03. 11. \$831.75. 13. \$16,270.39. 15. \$5,038.40. 17. \$233.92. 19. \$641.83.

Exercise 34. Page 166

1. \$242.26. 3. \$232.02. 5. \$1,321.35. 7. \$237.69. 9. \$140.59. 11. \$12,004.29. 13. \$12,657.03. 15. \$337.14. 17. \$641.83.

Exercise 35. Pages 169–70

1. \$44.65. 3. \$690.18. 5. \$79.90. 7. \$403.04. 9. \$6,229.90. 11. \$18,302.51. 13. \$4,632.66. 15. \$641.83; 25-year-pay is the best policy.

Exercise 36. Pages 172-73

1. \$3,640.14. 3. \$3,372.25. 5. \$4,663.52. 7. \$7,631.91. 9. \$280.44; 0.
11. \$255.09; \$254.21; \$253.14; \$252.07; \$250.90. 13. + \$21.82; + \$0.45;
- \$22.96; - \$48.13; - \$75.65. 15. \$5,166.12.

This book has been set on the Monotype in 11 and 10 point Modern No. 8, leaded 2 points. Chapter numbers are in 14 point and chapter titles in 18 point Alternate Gothic caps. The size of the type page is 27 by 46½ picas.

